

## THE EXCESS OF CONSUMPTION GROWTH\*

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*Recent empirical research often rejects the time-additive expected utility setting of the permanent income hypothesis. This rejection may be due to either the failure of the permanent income hypothesis or the misspecification of the underlying preferences. This paper investigates whether the departure from the time-additive expected utility specification can explain some consumption puzzle as a result of consumers' optimizing behavior, and examines the empirical validity of this argument. We find that empirical explanation of the excess of consumption growth puzzle by the general recursive utility framework is somewhat better than the time-additive expected utility framework. However, the test results indicate that the overall test of the model is rejected, as is the time-additive expected utility specification.*

### I. INTRODUCTION

There have been many advances both theoretical and empirical on the permanent income hypothesis originated by Friedman (1957). The permanent income hypothesis, reinterpreted by Hall (1978) and Flavin (1981) states that consumption is chosen to maximize expected current and future utility given expectations of current and future income. The implied time series process generating consumption suggests that tomorrow's consumption is expected to be the same as today's consumption. This 'martingale' property of consumption is a result of the joint hypothesis composed of the permanent income model, a time-additive 'certainty equivalence' expected utility specification, and rational expectations.

The stochastic implications of the hypothesis have been tested typically by the excess sensitivity test [Hall (1978), Flavin (1981, 1985), Hall and Mishkin (1982)] and the orthogonality test [Hansen and Singleton (1982, 1983), and Mankiw, Ro-

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temberg, and Summers (1985)]. Several authors use the quadratic utility function and derives the certainty equivalence consumption function [also see Flavin (1981,1985) and Hall and Mishkin (1982)]. Hall fails to reject the rational expectation-permanent income hypothesis with lagged consumption and lagged income except that he rejects the hypothesis with lagged stock prices. But most authors reject the rational expectation-permanent income hypothesis in favor of the excess sensitivity of consumption to income [see Flavin (1981), Campbell(1987), Kuehwein (1987) and Zeldes (1989a, b)].

Consumption puzzles are associated with these rejections of the permanent income hypothesis under the certainty equivalence framework. The frequently studied consumption puzzles are the excess of consumption growth – persistent growth of consumption, even when the real interest rate has been negative – [Deaton (1987)], the excess sensitivity of consumption to anticipated changes in income [Hall (1978) and Flavin (1981)], and the excess smoothness of consumption in response to unanticipated changes in income [Campbell and Deaton (1989)].

This rejection may be due to either failure of the permanent income hypothesis or the misspecification of the underlying preferences. As an alternative, Epstein and Zin (1989, 1991), Weil (1990), and Farmer (1990) propose more flexible intertemporal utility frameworks based on the recursive utility structure of Kreps and Porteus (1978). They use their models in order to explain several empirical puzzles related to the asset pricing.

The purpose of this paper is to show that the recursive non-expected utility framework of Epstein and Zin and Weil can resolve the puzzle of the excess of consumption growth, by showing how it may result from optimizing behavior. To be more specific, we focus on the testing of the recursive non-expected utility model with microeconomic data which has not been done before.

The structure of the paper is as follows. In Section II, we show that the non-expected utility framework of Epstein and Zin (1989, 1991) and Weil (1990) can be used to explain consumption puzzle as an optimizing behavior. Section III contains the data description, the sample selection criteria, and the splitting of data into subgroups. In Section IV, we explain econometric methodology concerning estimating equations and testing procedures, taking measurement errors into account. Section V presents the empirical results. Section VI concludes the paper by summarizing the main results and addressing possible directions for future research.

## II. NON-EXPECTED UTILITY MODEL OF CONSUMER'S BEHAVIOR

For explaining the consumption puzzle in this Section and performing the empirical study in Section V, we employ Epstein and Zin's (1989, 1991) model and their Euler equations in the form of household optimizing behavior.

2.1. Model

Consider the consumer's intertemporal optimization problem,

$$(2.1) \quad V_{i,t} = \underset{c_{i,t}}{\text{Max}} U[c_{i,t}, E_t V_{i,t+1}],$$

$$\text{s. t.} \quad A_{i,t+1} = (A_{i,t} - c_{i,t}) w'_{i,t+1} R_{i,t+1},$$

where  $V_{i,t}$  is the family  $i$ 's value function at time  $t$ ,  $U[\cdot, \cdot]$  is a Kreps and Porteus-type recursive utility function (aggregator function),  $c_{i,t}$  is consumption of family  $i$  at time  $t$ ,  $A_{i,t}$  is wealth of family  $i$  at time  $t$ ,  $w_{i,t}$  and  $R_{i,t}$  are P-vector of portfolio weights and P-vector of gross after-tax real returns on individual assets at the beginning of time  $t$ , with the typical element  $w'_{i,t}$  and  $R'_{i,t}$  respectively.<sup>1</sup>

We can rewrite the consumer's problem using Epstein and Zin's (1989, 1991) parameterization which extends the formulation of the space of finite horizon temporal lotteries in the Kreps and Porteus approach to an infinite horizon.<sup>2</sup>

$$(2.2) \quad V(I_{i,t}, A_{i,t}) = \underset{c_{i,t}, w_{i,t+1}}{\text{Max}} [c'_{i,t} + \beta \rho [V(I_{i,t+1}, A_{i,t+1})]^\rho]^\frac{1}{\sigma},$$

$$\text{s. t.} \quad A_{i,t+1} = (A_{i,t} - c_{i,t}) w'_{i,t+1} R_{i,t+1},$$

where  $\rho[V_{i,t+1}] = [E_t(V_{i,t+1})^\rho]^\frac{1}{\rho}$  and where  $\rho(\lt 1)$ ,  $\alpha(\lt 1)$  and  $\beta(= 1/(1+\delta))$  are the intertemporal elasticity of substitution ( $\sigma = 1/(1-\rho)$ ) parameter, the coefficient of relative risk aversion ( $\text{RRA} = 1-\alpha$ ) and the discount factor respectively,  $I_{i,t}$  is the information set available to the consumer in the planning period, and  $\rho[V_{i,t+1}]$  is the certainty equivalent of future utility.

Equation (2.2) says that the parameter of risk aversion ( $\alpha$ ) determines the certainty equivalent of random future utility and the aggregator function combines this certainty equivalent with deterministic current consumption through the parameter of intertemporal substitution ( $\rho$ ).

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<sup>1</sup> When the aggregator function  $U[\cdot, \cdot]$  is linear in the second argument, intertemporal utility is the expected sum of discounted future utilities. Thus time- and state-separable expected utility representation is a special case of Kreps and Porteus-type preferences.

<sup>2</sup> Epstein and Zin (1987) explain the empirical puzzles related to the capital asset pricing model with the nonexpected utility framework whose intertemporal budget constraint does not includes labor income. They also test the overidentifying restrictions of the model. They get some empirical support for their model using the GMM estimation and the  $\chi^2$ -test.

## 2.2. Derivation of Euler Equations

The optimization problem characterized in equation (2.2) can be solved by using the information that the aggregator function is homothetic and the certainty equivalent function is linear homogeneous. By homotheticity of aggregator function, there exists a function  $H(I_{i,t})$  such that  $V(I_{i,t}, A_{i,t}) = H(I_{i,t})A_{i,t}$ , where  $I_{i,t}$  is the information set available at time  $t$ . Deriving the Euler equations from equation (2.2) is equivalent to deriving them from the following two separate optimization problems with respect to consumption and portfolio weights:

$$(2.3) \quad H(I_{i,t})A_{i,t} = \underset{C_{i,t}}{\text{Max}} U[c_{i,t}^e + \beta(A_{i,t} - c_{i,t})^\rho \mu [H(I_{i,t+1})w'_{i,t+1}R_{i,t+1}]^\rho],$$

$$(2.4) \quad \mu^* = \underset{w_{i,t+1}}{\text{Max}} \mu [H(I_{i,t+1})w'_{i,t+1}R_{i,t+1}].$$

From the first order condition with respect to consumption and portfolio weights, we have the following equations directly from Epstein and Zin (1989, 1991):<sup>3)</sup>

$$(2.5) \quad \beta^\gamma E_t[(c_{i,t+1}/c_{i,t})^{\gamma(\rho-1)} M'_{i,t+1}] = 1,$$

$$(2.6) \quad E_t[(c_{i,t+1}/c_{i,t})^{\gamma(\rho-1)} M_{i,t+1}^{-1} (R_{i,t+1}^k - R_{i,t+1}^j)] = 0, \\ k = 1, \dots, P, \quad j = 1, \dots, P, \quad k \neq j,$$

where  $\gamma = \alpha/\rho$  and  $M_{i,t} (= w'_{i,t} R_{i,t})$  is the rate of return on the family  $i$ -th portfolio. Summing [(2.6)  $\times w_{i,t+1}^k$ ] over  $k (= 1, \dots, P)$ , multiplying the resulting equation by  $\beta^\gamma$  and subtracting from (2.5) (i.e., (2.5)  $- \beta^\gamma \sum_{k=1}^P [(2.6) \times w_{i,t+1}^k]$ ), we get the equation (2.7):

$$(2.7) \quad \beta^\gamma E_t[(c_{i,t+1}/c_{i,t})^{\gamma(\rho-1)} M_{i,t+1}^{-1} R_{i,t+1}^j] = 1, \quad j = 1, \dots, P.$$

We will concentrate on equation (2.7) later on, as the validity of equation (2.5) and (2.6) implies the validity of equation (2.7).<sup>4)</sup>

<sup>3)</sup> See Epstein and Zin (1989, 1991) for derivation in detail.

<sup>4)</sup> Several special cases are worth noting. When  $\alpha = \rho(\gamma = 1)$ , the equation (2.7) reduces to the Euler equation of the CRRA expected utility model. In particular, when  $\alpha = \rho = 0$ , the equation (2.7) reduces to the Euler equation of the logarithmic expected utility model. And when  $\alpha = 0$  and  $\rho \neq 0$  ( $\gamma = 0$ ), the equation (2.5) reduces to the Euler equation of the non-expected utility model with logarithmic risk preferences.

### 2.3. Explanation of the Excess of Consumption Growth

The Euler equation (2.7) serves as a basis in explaining the consumption puzzle. Let  $Y_{i,t+1} = \left( \frac{C_{i,t+1}}{C_{i,t}} \right)^{\gamma(\rho-1)} M_{i,t+1}^{-1} R_{i,t+1}^j$  and  $y_{i,t+1} = \ln(Y_{i,t+1})$ . Then we can express equation (2.7) as equation (2.8),

$$(2.8) \quad E_t[Y_{i,t+1}] = (1 + \delta)^\gamma.$$

We assume the joint lognormality between consumption growth, the return on the family  $i$ -th portfolio and the return on individual asset. Thus we can express LHS of equation (2.8) as equation (2.9),

$$(2.9) \quad E_t[Y_{i,t}] = e^{\mu_i + \frac{\sigma_i^2}{2}},$$

where  $\mu_i \equiv E_t[\ln(Y_{i,t+1})]$ , i.e.,  $\ln(Y_{i,t+1}) = \mu_i + \varepsilon_{i,t+1}^j$  and where  $\varepsilon_{i,t+1}^j = \ln(Y_{i,t+1}) - E_t[\ln(Y_{i,t+1})]$  and  $\varepsilon_{i,t+1}^j \sim N(0, \sigma_i)$ . Combining (2.8) and (2.9) and taking the logarithm, we get

$$(2.10) \quad \mu_i + \frac{\sigma_i^2}{2} = \gamma \ln(1 + \delta).$$

Using  $\mu_i = \ln \left[ \left( \frac{C_{i,t+1}}{C_{i,t}} \right)^{\gamma(\rho-1)} M_{i,t+1}^{-1} R_{i,t+1}^j \right] - \varepsilon_{i,t+1}^j$  and solving equation (2.10) for  $\ln \left( \frac{C_{i,t+1}}{C_{i,t}} \right)$ , equation (2.7) can be expressed as equation (2.11), via equation (2.10),

$$(2.11) \quad \ln \left( \frac{C_{i,t+1}}{C_{i,t}} \right) = \frac{-\delta}{1-\rho} + \frac{1}{2\gamma(1-\rho)} \sigma_i^2 + \frac{\gamma-1}{\gamma(1-\rho)} \ln(1 + m_{i,t+1}) \\ + \frac{1}{\gamma(1-\rho)} \ln(1 + r_{i,t+1}^j) - \frac{1}{\gamma(1-\rho)} \varepsilon_{i,t+1}^j,$$

where  $M = 1 + m$  and  $R = 1 + r$ .

In considering the long-run trend in consumption, the aggregate data with the representative consumer assumption is not appropriate, because turnover in the population may generate a trend in aggregate consumption. As Kuehlwein (1987) shows, the steady state aggregate per capita consumption growth rate will be independent of the individual consumption growth rate. Turnover in the sample will determine the observed aggregate trend, assuming constant growth rates for population, individual real income and individual consumption, and no bequests.

For this reason, aggregate growth rates may not be the basis for tests of the excess of the consumption growth.

This argument relies on the overlapping generation type argument, which is analogous to an infinite horizon. Now we turn to the individual model with micro data. As we assume the heterogeneous consumer, the error term can be decomposed into two parts; the time-specific error term ( $v_t$ ) and the idiosyncratic family  $i$ 's error term ( $u_{i,t}$ ) which are assumed to be serially uncorrelated with each other, and normally distributed with a mean zero for simplicity. Then equation (2.11) becomes

$$(2.12) \ln\left(\frac{c_{i,t+1}}{c_{i,t}}\right) = \frac{\delta}{1-\rho} + \frac{1}{2\gamma(1-\rho)}(\sigma_v^2 + \sigma_{u_{i,t}}^2) + \frac{\gamma-1}{\gamma(1-\rho)} \ln(1+m_{i,t+1}) \\ + \frac{1}{\gamma(1-\rho)} \ln(1+r'_{i,t+1}) - \frac{1}{\gamma(1-\rho)}(v'_{t+1} + u'_{i,t+1}).$$

This is a basic equation to explain the excess of consumption growth puzzle by the uncertainty terms.

Although a negative rate of return in our Euler equation may have a negative effect of consumption growth, the uncertainty terms can explain the excess of consumption growth when the corresponding coefficient  $\left(\frac{1}{2\gamma(1-\rho)}\right)$  is positive where  $\gamma = \frac{\alpha}{\rho}$ . Note that  $\frac{1}{2\gamma(1-\rho)}$  is positive if  $0 < \alpha < 1$  and  $0 < \rho < 1$  or  $\alpha < 0$  and  $\rho < 0$ . In other words, the conditions for a positive coefficient of uncertainty terms are that  $0 < \text{RRA} < 1$  and  $\sigma > 1$  or  $\text{RRA} > 1$  and  $0 < \sigma < 1$ , where  $\text{RRA} = 1 - \alpha$  and  $\sigma = \frac{1}{1-\rho}$ . The expected utility case ( $\alpha = \rho$ ) belongs to this region of parameter space, but we can still relax this parameter restriction in order to explain the excess of consumption puzzle. In view of the fact that the expected utility restriction ( $\alpha = \rho$ ) is not well supported empirically and that Kuehlwein's (1987) expected utility explanation of this puzzle is of limited value in the sense that puzzle-explaining uncertainty terms are not detected sufficiently by the data relative to other variance (of measurement error), the explanation of the excess of consumption growth by the non-expected utility framework is hoped to be more relevant empirically than that of the time-additive expected utility model.

### III. DATA

We use the Panel Study of Income Dynamics (PSID) from the Survey of Research Center (SRC) at the University of Michigan. The PSID data have been used in several studies [Hall and Mishkin (1982), Kuehlwein (1987), Hotz, Kydland, and Sedlacek (1988), and Zeldes (1989a)]. The data are collected from

1968 (wave 1) to 1984 (wave 17). The variables we use are as follows.

### 3.1. Description of Data

#### Consumption

Consumption data from PSID are available only in the form of food consumption. Much research uses food consumption to study consumption behavior [Hall and Mishkin (1982), Altonji and Siow (1987), Kuehlwein (1987), Hotz, Kydland, and Sedlacek (1988), Zeldes (1989a), and Runkle (1991)]. For this reason, we define consumption as the sum of annual food expenditure at home, annual food expenditure away from home, and food stamp expenditure. Although Hall and Mishkin (1982) and Kuehlwein (1987) exclude food stamp expenditure, Zeldes (1989a), whom we have chosen to follow, includes this expenditure. These consumption data are deflated by overall food CPI (Consumer Price Index) to obtain real food consumption.

#### Rates of returns

For nominal returns, we use the S&P (Standard and Poor's) 500 stock index: S&P 500 stock market index, indexes on Industry 400, Transportation 20, Utilities 40, and Finance 40. The nominal rate of return on the family's portfolio is not available, so we use the S&P 500 stock market return as a proxy for the family's portfolio return. Later, we analyze the mismeasurement problem associated with using proxy data for the family's portfolio return and examine the implications of this problem for our empirical results. The rates of return on individual assets are indexes of Industry 400, Transportation 20, Utilities 40, Finance 40, and 3-month Treasury Bill.

All nominal returns are converted into real returns using the implicit deflator associated with the measure of overall food CPI between year  $t$  and  $t+1$ . To calculate the real after-tax rate of return, we use the marginal tax rates on unearned income for head and wife, which are reported in PSID. Even if the optimal portfolio weights are the same for all families, the real after-tax rates of return for each family will differ because of the varying marginal tax rates.

#### Disposable income

Disposable income is used to calculate the wealth-income ratio by which to separate data set into the high-wealth group and the low-wealth group. Disposable income is calculated by total money income minus total federal income taxes of head and wife minus total federal income taxes of all extra earners minus social security taxes of head and wife, which is deflated by the annual average of the NIPA (National Income and Product Accounts) personal consumption expenditure deflator. Total money income includes taxable income of head and wife, taxable income of other family members, transfers of head and wife, and transfers

of other family members. In PSID, there is no information available on social security tax data, so we calculate social security taxes according to Zeldes (1989a). He imputes the social security taxes by multiplying the appropriate social security tax by the lesser of annual wages and the ceiling on wages taxable by social security. We use the self-employed tax rate for the head if he is self-employed. The regular rate is used for the wife's wages because we have no information on whether the wife is self-employed in PSID.

### **Wealth**

The micro data set may enable us to identify consumers who might be liquidity constrained. Recent tests by Hall and Mishkin (1982), Zeldes (1989a), Flavin (1985), and Mariger (1987) suggest that a fraction of the population does not conform to the permanent income hypothesis. Even though no liquidity constraints are implied by the model, we can still test, in loose form, whether liquidity constraints exist. This can be accomplished by examining whether the permanent income hypothesis can be rejected for different groups of data.

We need wealth for the separation of the sample. The wealth data is only used in order to separate the data set into two groups. In PSID, there is no wealth data. Therefore, we can only estimate wealth from the asset income in a somewhat crude way. Asset income is defined as the sum of the head's income from rent, interest, and dividends, the wife's other income from assets including rent, interest, dividends, alimony, trust funds, and royalties, and the asset income of all other family members. Wealth is estimated by summing the first \$250 of asset income divided by the passbook rate at the commercial banks, with the resulting asset income divided by the annual average rate of return on a 3-month Treasury Bill. Also real wealth is obtained by dividing nominal wealth by the NIPA personal consumption expenditure deflator.

### **3.2. Sample Selection Criteria and Splitting of Data into Subgroups**

We use several sample selection criteria in determining our data set. The PSID data include the 'poverty sample' which was collected by the U.S. Census Bureau for the 1966~67 Survey of Economic Opportunity (SEO). These households have a 1967 income-to-needs ratio of less than 2.0. This poverty sample is excluded in order to focus on the representative sample. Some data (e.g., marginal tax rate, head's annual income from rent, interest, and dividends, and total annual asset income of all other family members) have missing variables for several years. So we collect the data from 1976 to 1983. We eliminate the observations estimated by the Survey interviewer when he could not get a response from the member family. When there is a major change in family composition, we exclude these observations because it is not clear to which family the questions for the preceding year refer. We exclude the extreme outliers of the consumption

[Table 1] Descriptive Statistics

Variable	Total	High-wealth	Low-wealth
	Mean(s.e.)	Mean(s.e.)	Mean(s.e.)
$c_{it+1}/c_{it}$	1.0335(0.0080)	1.0201(0.0087)	1.0454(0.0130)
$m_{it}$	0.0439(0.0022)	0.0396(0.0030)	0.0477(0.0031)
$r_{it}^1$	0.0449(0.0022)	0.0405(0.0031)	0.0488(0.0032)
$r_{it}^2$	0.0742(0.0032)	0.0684(0.0045)	0.0793(0.0046)
$r_{it}^3$	0.0290(0.0018)	0.0259(0.0025)	0.0318(0.0025)
$r_{it}^4$	0.0405(0.0021)	0.0372(0.0029)	0.0435(0.0029)
$r_{it}^5$	0.0112(0.0008)	0.0077(0.0012)	0.0142(0.0012)
$y_{it+1}/y_{it}$	1.0310(0.0058)	1.0345(0.0082)	1.0278(0.0080)
Income	\$17,740(177)	\$20,449(269)	\$15,341(213)
Wealth	\$11,671(633)	\$24,486(1,246)	\$327(40)
Consumption	\$2,911(28)	\$3,028(43)	\$2,808(37)
MTR	0.27(0.0025)	0.31(0.0034)	0.24(0.0032)
Observation	2,498	1,173	1,325
Sample period	1978~1983	1978~1983	1978~1983

Note:  $r_{it}^1$ ,  $r_{it}^2$ ,  $r_{it}^3$  and  $r_{it}^4$  are after-tax real rates of return on stock indexes of Industry 400, Transportation 20, Utilities 40, and Finance 40.  $r_{it}^5$  is after-tax real rate of return on Treasury Bill(3 month). MTR is marginal tax rate.

growth  $\left(\frac{c_{i,t+1}}{c_{i,t}}\right)$  which lies outside 1/3 ~ 3 range. Finally, if the head's age is greater than 64, we exclude these observations following Kuehlwein's (1987) argument that declining health may have adversely affected their consumption growth and that standard Euler equation no longer holds in the presence of significant mortality rates [Yaari (1965)].

We split the sample into two groups, i.e., the high-wealth group and the low-wealth group. We follow Zeldes (1989a) in separating families into the high-wealth and low-wealth group. If the wealth is greater than 2-months' worth of income, we call these families the high-wealth group, otherwise, we call them the low-wealth group. If the null hypothesis of no liquidity constraints is not rejected for the high-wealth group and is rejected for the low-wealth group, a possible explanation is that the low-wealth group is liquidity constrained. Because our model does not consider liquidity constraints, we expect that the two groups' behavior concerning consumption will be similar.

## IV. ECONOMETRIC METHODOLOGY

### 4.1. Measurement Errors

The Euler equation for consumption and portfolio weights is as follows,

$$(4.1) \quad E_t \left[ \beta^y \left( \frac{c_{i,t+1}^*}{c_{i,t}^*} \right)^{\gamma(\rho-1)} M_{i,t+1}^{*\gamma-1} R_{i,t+1}^j \right] = 1, \quad j = 1, \dots, P,$$

where all variables and parameters are as above and the ' \* ' denotes optimal value. The hypothesis we want to test is

$$(4.2) \quad H_0 : c_{i,t} = c_{i,t}^* \text{ and } w_{i,t} = w_{i,t}^*,$$

where  $c_{i,t}$  and  $w_{i,t}$  are actual consumption and portfolio weights. The measured consumption ( $\hat{c}_{i,t}$ ) and measured portfolio weights ( $\hat{w}_{i,t}$ ) are subject to measurement errors. We consider the measurement error problem for measuring consumption and later for measuring portfolio weights. We then investigate how the estimating equations are modified.

#### Measurement Error for Consumption

As is well recognized, the micro data set is subject to measurement error, and for identifying the uncertainty terms, we assume the multiplicative measurement error for consumption, which is also assumed to be lognormally distributed with mean 1. The multiplicative measurement error implies that the error is likely to be positively correlated with the level of consumption. We then have

$$(4.3) \quad \ln \left( \frac{c_{i,t+1}}{c_{i,t}} \right) = \ln \left( \frac{\hat{c}_{i,t+1}}{\hat{c}_{i,t}} \right) - \omega_{i,t+1} + \omega_{i,t},$$

where  $\omega_{i,t}$  is the log of measurement error for consumption.

#### Measurement Error for Portfolio Weights

Now, we analyze the measurement error for the family's portfolio weights. In empirical work, we do not have data for the family's portfolio return which is expressed by

$$(4.4) \quad m_{i,t} = \sum_{j=1}^P w_{i,t}^j r_{i,t}^j.$$

The available data is but the market portfolio data,

$$(4.5) \quad m_t = \sum_{j=1}^P b_t^j r_t^j,$$

where  $b_t^j$  and  $r_t^j$  are the market portfolio weights and market rate of return on  $j$ -th asset, respectively. We adjust the market rate of return data by family-specific marginal tax rates ( $MTR_{i,t}$ ), which is our measured portfolio and is used as a proxy for the family's portfolio,

$$(4.6) \quad \hat{m}_{i,t} = \sum_{j=1}^P b_t^j r_{i,t}^j,$$

where  $\hat{m}_{i,t} = m_t(1 - MTR_{i,t})$ , and  $r_{i,t}^j = r_t^j(1 - MTR_{i,t})$ . Comparing (4.4) and (4.6), we know that, in general, the family's actual portfolio weights ( $w_{i,t}$ ) and measured portfolio weights ( $b_t^j$ ) are not the same. Also,  $m_{i,t} \neq \hat{m}_{i,t}$ . Therefore we should take the associated measurement error into consideration.

We assume that aggregate data for  $m_t$  and  $r_t^j$  and individual data for  $MTR_{i,t}$  have no measurement errors, and define measurement errors ( $B_{i,t}$ ) associated with the family's portfolio return as

$$(4.7) \quad w_{i,t}^j = b_t^j + B_{i,t}^j, \quad j = 1, \dots, P,$$

where  $\sum_{j=1}^P w_{i,t}^j = 1$ ,  $\sum_{j=1}^P b_t^j = 1$ ,  $\sum_{j=1}^P B_{i,t}^j = 0$  and measured portfolio weights ( $b_t^j$ ) are uncorrelated with measurement errors ( $B_{i,t}$ ). We assume that the family's actual portfolio weights ( $w_{i,t}$ ) are distributed around the market portfolio weights ( $b_t^j$ ); i.e., the market portfolio weights are regarded as the mean of the family's actual portfolio weights.

The relationship between the family's actual rate of return on the portfolio and the measured one is the following,

$$(4.8) \quad \begin{aligned} 1 + m_{i,t} &= 1 + \sum_{j=1}^P w_{i,t}^j r_{i,t}^j = 1 + \sum_{j=1}^P (b_t^j + B_{i,t}^j) r_{i,t}^j \\ &= \left[ 1 + \sum_{j=1}^P b_t^j r_{i,t}^j \right] + \left[ \sum_{j=1}^P B_{i,t}^j r_{i,t}^j \right]. \end{aligned}$$

The portfolio term in the Euler equation can be rewritten as

$$(4.9) \quad \begin{aligned} \ln(1 + m_{i,t}) &\simeq m_{i,t} = \left[ \sum_{j=1}^P b_t^j r_{i,t}^j \right] + \left[ \sum_{j=1}^P B_{i,t}^j r_{i,t}^j \right] \\ &= \hat{m}_{i,t} + \eta_{i,t} \simeq \ln(1 + \hat{m}_{i,t}) + \eta_{i,t} \end{aligned}$$

where  $\eta_{i,t} = \sum_{j=1}^P B_{i,t}^j r_{i,t}^j$  and  $\hat{m}_{i,t}$  is uncorrelated with  $\eta_{i,t}$ : the family's portfolio returns are assumed to be distributed around the market return. We assume that these measurement errors are different across families, but independent across time;  $E_t(\eta_{i,t+1}) = \bar{d}_i \neq 0$ . In other words, the average measurement error for the family's portfolio return is family-specific, but this average remains the same over time.<sup>5)</sup>

### Modifications of Euler equations

Because we use panel data with the measurement errors for consumption and portfolio weights, we estimate the parameters of the model from linearized version of (4.1). However, we can do without the lognormality assumption which is useful in simplifying the error structure of the Euler equation and in getting the estimated uncertainty terms without considering higher-order error terms greater than the second order. Then, the estimating equation for uncertainty terms and the estimating equation for the parameters of the model turn out to be of the equivalent form except the error structure.

From the consideration of measurement errors associated with consumption and portfolio weights, our final Euler equation for estimating the uncertainty terms is modified as

$$(4.10) \quad \ln \left( \frac{\hat{c}_{i,t+1}}{\hat{c}_{i,t}} \right) = \frac{-\delta}{1-\rho} + \frac{1}{2\gamma(1-\rho)} (\sigma_v^2 + \sigma_u^2) + \frac{1}{\gamma(1-\rho)} \ln(R_{i,t+1}) \\ + \frac{\gamma-1}{\gamma(1-\rho)} \ln(\hat{M}_{i,t-1}) + \omega_{i,t+1} - \omega_{i,t} - \frac{1}{\gamma(1-\rho)} \\ (v_{i,t+1}^j + u_{i,t+1}^j) + \frac{\gamma-1}{\gamma(1-\rho)} \eta_{i,t+1},$$

where  $v_{i,t+1}$  is the time-specific error,  $u_{i,t+1}$  is the family specific error,  $\omega_{i,t+1}$  is the measurement error associated with consumption,  $\eta_{i,t+1}$  is the measurement error associated with portfolio return,  $\eta_{i,t+1}$  is uncorrelated with  $\hat{m}_{i,t+1}$  but is correlated with  $r_{i,t+1}^j$ , and all error terms are assumed to be uncorrelated with each other and serially uncorrelated.

Next, if we consider measurement errors for consumption and family's portfolio weights, we have the final Euler equation for estimating the parameters of

<sup>5</sup> This assumption is made in order to get the zero mean error structure by differencing the Euler equation, making possible the application of the GMM estimation. This assumption should be regarded as an approximating and simplifying assumption of the real economy. Note that although this procedure gives the same portfolio weights to all families, we can correct this mismeasurement problem by adding family-specific measurement error for the family's portfolio return.

the model, using rational expectations assumption and second-order Taylor expansion for the expectation error ( $\ln(1 + \varepsilon_{i,t}) = \varepsilon_{i,t} - \frac{1}{2} \varepsilon_{i,t}^2$ ),

$$\begin{aligned}
 (4.11) \quad \ln \left( \frac{\hat{c}_{i,t+1}}{\hat{c}_{i,t}} \right) &= \frac{-\delta}{1-\rho} + \frac{\sigma_i^2}{2\gamma(1-\rho)} + \frac{1}{\gamma(1-\rho)} \ln(R'_{i,t+1}) \\
 &+ \frac{\gamma-1}{\gamma(1-\rho)} \ln(\hat{M}_{i,t+1}) + \omega_{i,t+1} - \omega_{i,t} \\
 &+ \frac{1}{\gamma(1-\rho)} \left( \frac{1}{2} \varepsilon_{i,t+1}^2 - \frac{1}{2} \sigma_i^2 - \varepsilon_{i,t+1}^j \right) \\
 &+ \frac{\gamma-1}{\gamma(1-\rho)} \eta_{i,t+1},
 \end{aligned}$$

where  $E_t(\varepsilon_{i,t+1}^2) = \sigma_i^2$ .

## 4.2. Methods of Estimation

### Estimation of Euler equation

We use the differenced form of the Euler equation for estimation. To delete the variable intercept term and to get the zero mean error term, we will use the differenced form of (4.10),

$$\begin{aligned}
 (4.12) \quad \Delta \ln \left( \frac{\hat{c}_{i,t+1}}{\hat{c}_{i,t}} \right) &= \frac{1}{\gamma(1-\rho)} \Delta \ln(R'_{i,t+1}) + \frac{\gamma-1}{\gamma(1-\rho)} \Delta \ln(\hat{M}_{i,t+1}) + \omega_{i,t+1} \\
 &- 2\omega_{i,t} + \omega_{i,t-1} - \frac{1}{\gamma(1-\rho)} (v'_{i+1} - v'_i + u'_{i,t+1} - u'_{i,t}) \\
 &+ \frac{\gamma-1}{\gamma(1-\rho)} (\eta_{i,t+1} - \eta_{i,t}).
 \end{aligned}$$

From equation (4.12), we know that  $\eta_{i,t}$  and  $r'_{i,t+1}$  are correlated, so OLS estimation results in biasedness of parameter estimates. The alternative is to use the instrumental variable estimation such that the instruments are uncorrelated with error terms and correlated with explanatory variables. We consider lagged variables either within the model or outside the model and use them as our instruments. To estimate the uncertainty terms in equation (4.10), we follow a two-step procedure. First, we use the estimation results about coefficients of the model from the 2SLS estimation. In this stage, we do not need estimates of the parameters of the model themselves because our major concern is the estimates of errors

which are used to identify the uncertainty terms in the second step. From the lognormality assumption, we can avoid the higher order error terms and restrict ourselves up to the second order error terms. On the other hand, this convenience requires some cost. We note that the lognormality is a strong assumption and thus places some limitations on the validity of empirical results.

### Estimation of uncertainty terms

Then from equation (4.12), define  $x_{i,t+1}$  as

$$\begin{aligned}
 (4.13) \quad x_{i,t+1} &\equiv \Delta \left( \frac{\hat{c}_{i,t+1}}{\hat{c}_{i,t}} \right) - \frac{1}{\gamma(1-\rho)} \Delta \ln(R_{i,t+1}^i) - \frac{\gamma-1}{\gamma(1-\rho)} \Delta(\hat{M}_{i,t+1}) \\
 &= \omega_{i,t+1} - 2\omega_{i,t} + \omega_{i,t-1} - \frac{1}{\gamma(1-\rho)} (v_{i,t+1}^i - v_{i,t}^i + u_{i,t+1}^i - u_{i,t}^i) \\
 &\quad + \frac{\gamma-1}{\gamma(1-\rho)} (\eta_{i,t+1} - \eta_{i,t}).
 \end{aligned}$$

When  $\gamma = 1$ , equation (4.13) reduces to the time-additive expected utility result of Kuehlwein (1987). We use variance and covariance structure of  $x_{i,t+1}$  to identify variance terms,  $\sigma_v^2$ ,  $\sigma_u^2$ ,  $\sigma_\omega^2$ , and  $\sigma_\eta^2$ , where all the variances are calculated across both time and cross section.

$$\begin{aligned}
 (4.14) \quad \text{i) } \text{Var}(x_{i,t+1}) &= 6\sigma_\omega^2 + \left[ \frac{1}{\gamma(1-\rho)} \right]^2 (2\sigma_v^2 + 2\sigma_u^2) + \left[ \frac{\gamma-1}{\gamma(1-\rho)} \right]^2 (2\sigma_\eta^2), \\
 \text{ii) } \text{Cov}(x_{i,t+1}, x_{i,t}) &= -4\sigma_\omega^2 + \left[ \frac{1}{\gamma(1-\rho)} \right]^2 (-\sigma_v^2 - \sigma_u^2) + \left[ \frac{\gamma-1}{\gamma(1-\rho)} \right]^2 (-\sigma_\eta^2), \\
 \text{iii) } \text{Cov}(x_{i,t+1}, x_{j,t+1}) &= \left[ \frac{1}{\gamma(1-\rho)} \right]^2 (2\sigma_v^2).
 \end{aligned}$$

From (4.14) i) and ii), we can identify the variance of measurement error for consumption ( $\sigma_\omega^2$ ) and from (4.14) iii), we can identify the variance of time-specific error term ( $\sigma_v^2$ ). However, the variance of family-specific error term ( $\sigma_u^2$ ) and the variance of measurement error for the family's portfolio return ( $\sigma_\eta^2$ ) are not identified separately. We only have the following relation between  $\sigma_u^2$  and  $\sigma_\eta^2$ ,

$$(4.15) \quad D = E\sigma_u^2 + F\sigma_\eta^2,$$

where D, E, and F are positive numbers resulting from manipulating equations

of (4.14). Under nonnegative restriction of the variance terms, we can get the maximum values of  $\sigma_u^2$  and  $\sigma_v^2$  from equation (4.15). As we know from (4.13), if  $\gamma = 1$  (expected utility case), we do not have to worry about measurement error for the portfolio weights, but the non-expected utility case ( $\gamma \neq 1$ ) is subject to this measurement error problem. In this regard, the ratios of uncertainty terms to total variances would be overestimated in case of no measurement error for the family's portfolio weights;

$$(4.16) \quad \frac{\sigma_v^2 + \sigma_u^2}{\sigma_v^2 + \sigma_u^2 + \sigma_w^2} > \frac{(\sigma_v^2)' + (\sigma_u^2)'}{(\sigma_v^2)' + (\sigma_u^2)' + (\sigma_w^2)' + (\sigma_r^2)'},$$

where ‘ ’ denotes variances resulting from considering measurement errors for the family's portfolio weights,  $\sigma_v^2 = (\sigma_v^2)'$ ,  $\sigma_w^2 = (\sigma_w^2)'$ , and  $\sigma_u^2 > (\sigma_u^2)'$ . To get the more concrete implications of measurement errors for the portfolio weights for empirical results, we calculate four cases where the measurement error for the portfolio return is one, two, five, and ten times larger than the measurement error for consumption, together with two extreme cases where  $\sigma_u^2$  takes on maximum value and  $\sigma_r^2 = 0$ , and  $\sigma_v^2$  takes on maximum value and  $\sigma_u^2 = 0$ .

The existence of measurement errors is assumed as in Altonji and Siow (1987) and Kuehlwein (1987, 1991), because it is generally agreed that the panel data is noisy, containing measurement errors due to incorrect responses from the interviewees. Still, the direct test of the existence of measurement errors may be derivable. We estimate the measurement errors by calculating variances and derive the ratios of uncertainty terms out of total variances. As we show later, the magnitudes of variances of measurement error range widely from about 99.9% to 0.2% across different instruments, suggesting the existence of measurement error in some cases. Also, as our purpose is to show the existence of sizable uncertainty terms, we may offer our arguments in the more conservative way, by assuming the existence of measurement errors. However, more exhaustive analysis of measurement error problem in the noisy panel data will be investigated in the future. This investigation includes the test of the existence of measurement errors (e.g., Hausman test) and the derivation of the standard error of uncertainty terms by using simulation technique.

### Estimation of Parameters of the Model

In the equation (4.11), the expectation of total error terms is not zero, so we cannot get the consistent estimates of the parameters of the model. Thus, we use the differenced form of (4.11) to get the mean zero error. The basic equation for the GMM estimation is

$$(4.17) \quad v_{i,t+1}^j = \Delta \ln \left( \frac{\hat{c}_{i,t+1}}{\hat{c}_{i,t}} \right) - \frac{1}{\gamma(1-\rho)} \Delta \ln R_{i,t+1}^j - \frac{\gamma-1}{\gamma(1-\rho)} \Delta \ln \hat{M}_{i,t+1},$$

$$E(v_{i,t+1}^j \mid I_{i,t-2}) = 0,$$

where

$$v_{i,t+1}^j = \omega_{i,t+1} - 2\omega_{i,t} + \omega_{i,t-1} + \frac{1}{\gamma(1-\rho)} \left( \frac{1}{2} \varepsilon_{i,t+1}^{j^2} - \frac{1}{2} \varepsilon_{i,t}^{j^2} - \varepsilon_{i,t+1}^j + \varepsilon_{i,t}^j \right) + \frac{\gamma-1}{\gamma(1-\rho)} (\eta_{i,t+1} - \eta_{i,t}).$$

We form the GMM estimators for  $\gamma$  and  $\rho$ , exploiting the fact that the family  $i$ -th information set at time  $t$ ,  $I_{i,t}$ , is of no help in forecasting future economy-wide shocks. Generally, the GMM estimation can be applied to a set of stochastic Euler equations which in turn implies a set of population orthogonality conditions [see Hansen (1982)]. In our linearized version of the Euler equation, we have the nonlinearity in parameters.<sup>6</sup>

It will be noted that the nonstationarity of variables makes the estimation of Euler equation and parameters of the model biased and inefficient. However, the variables used are in log-differenced form, not in levels form. Also we use micro panel data. These two points may make the nonstationarity problem of the time series data somewhat mitigated.

## V. EMPIRICAL RESULTS

### 5.1. Estimates of Uncertainty Terms

We present the magnitude of uncertainty terms as a key to explaining the excess of consumption growth puzzle. The total sample includes 2,498 observations. By the wealth-income criterion, we have the high-wealth sample with 1,173 observations and the low-wealth sample with 1,325 observations.

#### High-wealth sample

This sample represents the families whose wealth is greater than 2-months' worth of income. Our breakdown of the total sample into two groups depends on the wealth-income ratio. The more specific concept of liquidity (or borrowing)

<sup>6</sup> Because equation (4.17) implies MA(2) error structure, instruments at time  $t-2$  or earlier can be used to estimate equation (4.17). To get the positive semidefinite weighting matrix under MA(2) error structure, we use the Newey-West method [see Newey and West (1987)].

constraints does not correspond exactly to our splitting of the sample. Small wealth is a necessary but not sufficient condition for liquidity constraints. Zeldes (1989a) defines liquidity constraints as wealth being nonnegative in every period ( $A_t \geq 0, \forall t$ ), but he uses the wealth-income ratio to classify the sample into the liquidity constrained and unconstrained groups. Our model does not allow liquidity constraints, so, loosely speaking, the splitting of the sample may be served as a basis for another type of alternative hypothesis against our model specification. We have a 2.0% consumption growth rate and 3.5% income growth rate. Average real income is \$20,449 and average real wealth is \$24,486. Marginal tax rate is 31% compared to 24% for the low-wealth group.

The estimation results are presented in Table 2. The first set is  $\Delta \ln(R_{i,t-2}^j)$ ,  $j = 1, \dots, 5$ ,  $\Delta \ln(y_{i,t-2})$ ,  $\Delta MRT_{i,t-2}$ ,  $\Delta HRYR_{i,t-2}$  and  $\Delta \ln(PAS_{i,t-2})$ , where  $y_{i,t}$  is the disposable income,  $HRYR_{i,t}$  is hours worked, and  $PAS_{i,t}$  is after-tax gross passbook rate of family  $i$  in period  $t$ . The second set has  $\ln(R_{i,t-k}^j)$ ,  $\ln(y_{i,t-k})$ ,  $MRT_{i,t-k}$  and  $\ln(PAS_{i,t-k})$ ,  $k = 2, 3$ . Finally, we will use as the third set,  $\Delta \ln(R_{i,t-2}^j)$ ,  $\Delta \ln(\hat{M}_{i,t-2})$ ,  $\Delta \ln(y_{i,t-2})$ ,  $\Delta MRT_{i,t-2}$  and  $\Delta HRYR_{i,t-2}$ . The estimates of the Euler equation coefficients are insignificantly different from zero.

[Table 2] Estimates of uncertainty -high-wealth sample

A. Estimates of Euler equation

Eq's	Instrument 1		Instrument 2		Instrument 3	
	$\beta_1$	$\beta_2$	$\beta_1$	$\beta_2$	$\beta_1$	$\beta_2$
$r^1$	-0.3801 (-0.174)	0.4051 (0.175)	0.4061 (0.181)	-0.4801 (-0.202)	-0.4551 (-0.206)	0.1750 (0.075)
$r^2$	0.1112 (0.460)	-0.1927 (-0.432)	0.0541 (0.220)	-0.1119 (-0.248)	0.0962 (0.300)	-0.1702 (-0.298)
$r^3$	-0.0847 (-0.211)	0.0209 (0.135)	-0.0193 (-0.048)	-0.0342 (-0.215)	0.2247 (0.478)	-0.3520 (-1.109)
$r^4$	0.0417 (0.206)	-0.0253 (-0.132)	0.0568 (0.279)	-0.0287 (-0.150)	0.3112 (0.672)	-0.2979 (-0.653)
$r^5$	0.1582 (0.268)	0.0171 (0.122)	0.1770 (0.297)	0.0082 (0.057)	1.2385 (1.143)	-0.2346 (-1.185)

Note: t-statistics are in parentheses.

\*: significant at 10% level      \*\*: significant at 5% level

Instrument 1 =  $\{\Delta \ln(R_{i,t-2}^j), j = 1, \dots, 5, \Delta \ln(y_{i,t-2}), \Delta MTR_{i,t-2}$ ,

Instrument 2 =  $\{\Delta \ln(R_{i,t-k}^j), \ln(y_{i,t-k}), MTR_{i,t-k}, \ln(PAS_{i,t-k}), k = 2, 3\}$ ,

Instrument 3 =  $\{\Delta \ln(R_{i,t-2}^j), \hat{M}_{i,t-2}, \Delta \ln(y_{i,t-2}), \Delta MTR_{i,t-2}, \Delta HRYR_{i,t-2}\}$

$$\beta_1 = \frac{1}{r(\rho - 1)}, \beta_2 = \frac{r - 1}{r(\rho - 1)}$$

## B. Portion of uncertainty terms out of total variances

	Eq's	Ratio-A	Ratio-1	Ratio-2	Ratio-5	Ratio-10	Ratio-Z
INST 1	$r^1$	54.8	3.8	n.a	n.a	n.a	0.1
	$r^2$	93.4	84.8	73.0	n.a	n.a	1.0
	$r^3$	95.9	92.1	88.6	79.4	67.5	0.0
	$r^4$	99.0	98.1	97.1	94.4	89.9	0.1
	$r^5$	87.7	78.0	70.3	54.0	38.9	0.0
INST 2	$r^1$	49.9	n.a	n.a	n.a	n.a	0.4
	$r^2$	98.3	96.5	94.4	86.2	59.3	0.5
	$r^3$	99.8	99.6	99.3	98.7	97.5	1.0
	$r^4$	98.2	96.5	94.9	90.1	82.9	0.1
	$r^5$	85.1	74.1	65.6	48.8	34.1	0.0
INST 3	$r^1$	50.7	30.6	19.7	4.6	n.a	1.5
	$r^2$	94.9	88.7	80.7	34.3	n.a	0.8
	$r^3$	80.8	46.9	n.a	n.a	n.a	17.3
	$r^4$	69.9	41.3	14.1	n.a	n.a	3.0
	$r^5$	22.5	11.3	6.8	1.8	n.a	0.4

Note: All values are in percent. Ratio-A is the case for  $\sigma_u^2 = 0$ , ratio-1, ratio-2, ratio-5, and ratio-10 are the cases for  $\sigma_\eta^2 = \sigma_w^2$ ,  $\sigma_\eta^2 = 2\sigma_w^2$ ,  $\sigma_\eta^2 = 5\sigma_w^2$ , and  $\sigma_\eta^2 = 10\sigma_w^2$ , and ratio-Z is the case for  $\sigma_u^2 = 0$ . The n.a. denotes the case where maximum  $\sigma_\eta^2$  is less than  $n\sigma_w^2$ ,  $n = 1, 2, 5$ , and 10.  $r^1 \sim r^5$  are the individual rates of return on stock indexes of Industry 400, Transportation 20, Utilities 40, Finance 40, and on Treasury Bill, which are entered into each Euler equation.

To increase the significance of the estimates deserves further investigation. The family-specific variance term constitutes a large portion of complete variance terms in our model. Kuehlwein's (1987) estimated variances of uncertainty terms are unrealistically small; relative to the variance of measurement error, they constitute only 3% of the variances of the total error terms. In our model, however, in the case  $\sigma_\eta^2 = \sigma_w^2$  and  $\sigma_\eta^2 = 5\sigma_w^2$ , the variances of uncertainty terms, on the average, 67.3% and 59.2% of the variances of the total error terms, respectively. They are 71.6% and 58.5% if we choose the cases where the coefficient of uncertainty terms  $\left(\frac{1}{2\gamma(1-\rho)}\right)$  is positive. Thus, in our model, uncertainty in the future contributes to explaining the excess of consumption growth puzzle.

To be more specific, if we rewrite (4.10) as equation (4.18), and compare it with the Euler equation (4.19) of Kuehlwein (1987,1991), we get

$$(4.18) \quad E_t \left[ \ln \left( \frac{c_{i,t+1}}{c_{i,t}} \right) \right] = \frac{-\delta}{1-\rho} + \frac{1}{2\gamma(1-\rho)} \sigma^2 + \frac{\gamma-1}{\gamma(1-\rho)} E_t [\ln(1 + m_{i,t+1})] \\ + \frac{1}{\gamma(1-\rho)} E_t [\ln(1 + r'_{i,t+1})],$$

$$(4.19) \quad E_t \left[ \ln \left( \frac{c_{i,t+1}}{c_{i,t}} \right) \right] = \frac{-\delta}{1-\rho} + \frac{1}{2(1-\rho)} \sigma^2 + \frac{1}{1-\rho} E_t [\ln(1 + r'_{i,t+1})].$$

If we consider the cases where we exclude negative estimates of  $\frac{1}{2\gamma(1-\rho)}$ , the estimates of  $\frac{1}{2\gamma(1-\rho)}$  range from 0.0209 to 0.6193 in (4.18). In (4.19), the hypothetical values of  $\frac{1}{2(1-\rho)}$  are from 0.05 to 1.00. The corresponding average estimate of  $\sigma^2$  is 0.9014 in case  $\sigma_n^2 = \sigma_w^2$  and 1.0520 in case  $\sigma_n^2 = 5\sigma_w^2$  in (4.18), and 0.0330 in (4.19). Therefore, the uncertainty terms implicit in (4.18) contribute much more than those in (4.19) to explaining the positive consumption growth. The explanation of the excess of consumption growth puzzle by the future uncertainty seems to be better supported by our model than by the time-additive CRRA expected utility model employed by Kuehlwein (1987, 1991) and Zeldes (1989a). The factors which might generate the Euler equation errors will directly affect the estimates of our uncertainty terms.<sup>7</sup>

### Low-wealth sample

This sample represents the families whose wealth is less than 2-months' worth of income. We get a consumption growth rate of 4.5% and income growth rate of 2.8%. In Table 3, the estimates of the Euler equation coefficients and estimates of uncertainty are presented. One sixth of the estimates of the regression coefficients are statistically significant at the 5% level, and one third of them are significant at the 10% level. The estimated fraction of the total variances due to measurement errors is only 10% in case  $\sigma_n^2 = \sigma_w^2$  and 25% in case  $\sigma_n^2 = 5\sigma_w^2$ , which is contrasted with 84% obtained by Kuehlwein (1987) using the CRRA expected utility model. Also the fraction is 2% and 7% respectively for the case

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<sup>7</sup> The general limitations of these results are that we do not estimate the individual parameters of the model and do not test the overidentifying restrictions of the model. Also, we do not get the standard errors of the uncertainty terms. The insignificant estimates of the Euler equation may indicate that rates of return may have little effect on intertemporal consumption decisions, or that the data are simply not informative of the true values of the parameters. Later, we will present the estimates of the individual parameters of the model as a system by the efficient estimation technique of GMM and test the overidentifying restrictions of the model. So, our explanation of positive consumption growth by uncertainty terms in the Euler equation is suggestive.

**[Table 3]** Estimates of uncertainty - low-wealth sample

## A. Estimates of Euler equation

Eq's	Instrument 1		Instrument 2		Instrument 3	
	$\beta_1$	$\beta_2$	$\beta_1$	$\beta_2$	$\beta_1$	$\beta_2$
$r^1$	3.3803 (1.536)	-3.8649* (-1.657)	3.3146 (1.475)	-3.7882 (-1.588)	3.6788* (1.660)	-3.9406* (-1.679)
$r^2$	-0.4941** (-2.071)	0.5884 (1.329)	-0.4886** (-2.021)	0.5780 (1.294)	-0.2582 (-0.811)	0.1869 (0.327)
$r^3$	0.1551 (0.380)	-0.3201** (-2.074)	0.1365 (0.334)	-0.3027* (-1.911)	-0.2285 (-0.481)	0.1078 (0.339)
$r^4$	-0.3252 (-1.622)	-0.0695 (-0.371)	-0.3318* (-1.650)	-0.0659 (-0.351)	0.2294 (0.486)	-0.6183 (-1.344)
$r^5$	-0.9628 (-1.590)	-0.3649** (-2.637)	-0.9388 (-1.542)	-0.3508** (-2.505)	-0.2520 (-0.202)	-0.4587** (-2.072)

Note: See Table 2-A.

## B. Portion of uncertainty terms out of total variances

	Eq's	Ratio-A	Ratio-1	Ratio-2	Ratio-5	Ratio-10	Ratio-Z
INST 1	$r^1$	24.6	n.a	n.a	n.a	n.a	0.1
	$r^2$	93.7	87.1	80.1	56.7	6.4	0.3
	$r^3$	99.4	98.7	98.0	95.8	91.4	5.2
	$r^4$	97.3	94.8	92.4	85.8	76.6	0.0
	$r^5$	79.0	64.4	53.7	33.7	17.5	0.0
INST 2	$r^1$	25.3	n.a	n.a	n.a	n.a	0.1
	$r^2$	93.9	87.4	80.6	57.9	10.4	0.3
	$r^3$	99.5	99.0	98.5	96.8	93.4	5.9
	$r^4$	97.2	94.6	92.1	85.3	75.9	0.0
	$r^5$	79.8	65.6	55.1	35.2	18.9	0.0
INST 3	$r^1$	20.5	n.a	n.a	n.a	n.a	0.3
	$r^2$	98.2	96.4	94.7	89.6	81.7	0.2
	$r^3$	98.4	96.9	95.4	91.1	84.6	0.8
	$r^4$	98.7	97.2	95.5	87.4	31.7	16.7
	$r^5$	98.4	96.7	94.9	88.5	72.8	3.5

Note: See Table 2-B.

of the positive coefficient of uncertainty terms.

With a positive coefficient of  $\frac{1}{\gamma(1-\rho)}$ , in (4.18), the estimates of  $\frac{1}{2\gamma(1-\rho)}$  are from 0.0683 to 1.8394, and in (4.19) the hypothetical values of  $\frac{1}{2(1-\rho)}$  are between 0.05 and 1.00. And the corresponding average estimate of  $\sigma^2$  is 3.0290 in case  $\sigma_\eta^2 = \sigma_w^2$  and 2.5637 in case  $\sigma_\eta^2 = 5\sigma_w^2$  in (4.18), and 0.4410 in (4.19).

Therefore, our model has the advantage over the time-additive CRRA expected utility model of being able to explain the excess consumption growth by means of future uncertainty for the case of positive coefficient of uncertainty terms. Overall, we may conclude that future uncertainty from the general recursive utility framework partially serves to explain the excess of consumption growth puzzle in the sense that uncertainty terms are sizable relative to measurement error terms in the Euler equation.

## 5.2. Estimates of Parameters of the Model and Test Results

Here, we estimate the parameters of the model,  $(\gamma, \sigma)$ , using the GMM estimation, and test the hypothesis of the expected utility and overidentifying restrictions of the model. We use four equations, which include returns on Industry 400, Utilities 40, Finance 40, and Treasury Bill, respectively.

### Estimates of Parameters of the Model

In our framework with MA(2) error structure, (4.17), variables at time  $t-2$  or earlier are valid instruments. We choose  $\{\text{constant}, \Delta \ln(R_{i,t-2}^I), \Delta \ln(R_{i,t-2}^T)\}$  as instrument 1, where  $R^I$  and  $R^T$  are gross returns on Industry 400 and Treasury Bill,  $\{\text{constant}, \Delta \ln(R_{i,t-2}^I), \Delta \ln(PAS_{i,t-2})\}$  as instrument 2, where  $PAS$  is the after-tax gross passbook savings rate, and  $\{\text{constant}, \Delta \ln(R_{i,t-2}^I), HWMTR_{i,t-2}\}$  as instrument 3, where  $HWMTR$  is the marginal tax rate for GMM estimation.

The main results are as follows. In the high-wealth sample case presented in Table 4, the elasticity of intertemporal substitution is estimated to be greater than one in all cases.<sup>81</sup>

The estimates of  $\alpha$  and  $\rho$  indicate that both parameters are significantly different from zero, so, like Bufrman and Leiderman (1990), the logarithmic expected utility function ( $\alpha = \rho = 0$ ) and the non-expected utility function with logarithmic

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<sup>81</sup> This result is quite different from that of Hall (1988) and Epstein and Zin (1987b, 1991) but is similar to the results of Mankiw, Rotemberg, and Summers (1985), Shapiro (1984), Singleton (1990), and Bufrman and Leiderman (1990). The use of micro data set might be partially responsible for this high estimated value of the elasticity of intertemporal substitution. Also we may conjecture that these different magnitudes of estimates of the elasticity of intertemporal substitution between the aggregate and individual data are due to aggregation bias.

**[Table 4]** GMM estimation and test results - high-wealth sample

	INST 1	INST 2	INST 3
$\gamma \left( = \frac{\alpha}{\rho} \right)$	0.2371 (0.0073)	0.2040 (0.0139)	0.2972 (0.0180)
$\sigma \left( = \frac{1}{1-\rho} \right)$	1.2904 (7.2556)	4.0002 (9.2824)	4.0007 (2.2574)
RRA(= 1- $\alpha$ )	0.9466 (1.0332)	0.8470 (0.1245)	0.7770 (0.0799)
$\rho$	0.2251 (4.3571)	0.7500 (0.5801)	0.7500 (0.1410)
J(10)	400.81 [0.00]	393.82 [0.00]	395.16 [0.00]
t-value ( $H_0: \gamma = 1$ )	-104.76**	-57.19**	-39.00*
observation	768	768	768
sample period	1978~1983	1978~1983	1978~1983

Note: Asymptotic standard errors are in parenthesis and p-values are in square brackets. INST 1 = {constant,  $\Delta \ln(R_{i,t-2}^1)$ ,  $\Delta \ln(R_{i,t-2}^4)$ }, INST 2 = {constant,  $\Delta \ln(R_{i,t-2}^1)$ ,  $\Delta \ln(\text{PAS}_{i,t-2})$ }, INST 3 = {constant,  $\Delta \ln(R_{i,t-2}^1)$ ,  $\text{HWMTR}_{i,t-2}$ }.

\*: significant at 5% level      \*\*: significant at 1% level

mic preference ( $\alpha = 0$ ,  $\rho \neq 0$ ) are not reflected by the data. The RRA coefficient, which is estimated to be statistically significant, is very close to unity.

Overall, we find that with the PSID data, the elasticity of intertemporal substitution is greater than one, that the coefficient of relative risk aversion is generally less than one but fairly close to one, and that consumers reveal that they prefer the early resolution of uncertainty ( $\alpha < \rho$ ). That is, they dislike risk more than intertemporal fluctuations. In contrast, Epstein and Zin (1991) report that consumers typically exhibit the late resolution of uncertainty, using the time series aggregate data. We might argue that the implication of the consumer's behavior is quite different between the aggregate data and individual data. Thus, the more exhaustive analysis of the aggregation problem should be an item for further research.<sup>9)</sup>

<sup>9)</sup> The striking difference from the existing literature [Hall (1988), Epstein and Zin (1991), and Giovannini and Jorion (1989)] is that the elasticity of intertemporal substitution is greater than one, implying much more shift in consumption between periods according to the change in the environment. However, our results are consistent with Bufman and Leiderman (1990) and nearly correspond to Mankiw, Rotemberg, and Summers (1985), Shapiro (1984), and Singleton (1990).

**[Table 5]** GMM estimation and test results - low-wealth sample

	INST 1	INST 2	INST 3
$\gamma \left( = \frac{\alpha}{\rho} \right)$	0.2218 (0.0112)	0.1616 (0.0053)	0.2844 (0.0192)
$\sigma \left( = \frac{1}{1-\rho} \right)$	3.3054 (4.1513)	8.0708 (3.4817)	4.0007 (5.3500)
RRA (= $1-\alpha$ )	0.8453 (0.0904)	0.8585 (0.0113)	0.7867 (0.1032)
$\rho$	0.6975 (0.3800)	0.8761 (0.0535)	0.7500 (0.3343)
J(10)	424.28 [0.00]	437.01 [0.00]	448.69 [0.00]
t-value ( $H_0: \gamma = 1$ )	-69.40**	-159.25*	-37.21*
observaton	852	852	852
sample period	1978~1983	1978~1983	1978~1983

Note: See Table 4.

In the low-wealth sample, presented in Table 5, the elasticity of intertemporal substitution is slightly greater than that in the high-wealth sample and other estimates are very similar to those of the high-wealth sample.

### Test Results

In high-wealth sample, from the estimate of  $\gamma$ , we can test the hypothesis of the expected utility,  $H_0: \gamma = 1$ . All the cases show the rejection of the expected utility hypothesis. These results may encourage us to use the non-expected utility model for empirical study.

The discouraging result is that the test of the model's overidentifying restrictions using Hansen's J-statistic leads to the rejection across all instruments. Epstein and Zin (1991) show that the test of the overidentifying restrictions is sensitive to the choice of the instruments. We conclude that although the nested test of the hypothesis of the expected utility is clearly rejected in favor of the non-expected utility model, the Euler equation violation for which the expected utility model is criticized, is not overcome by the general recursive utility structure of Epstein and Zin (1989, 1991) with our micro data set. Hence, the empirical

performance of the non-expected utility model should not be exaggerated.<sup>10</sup>

In low-wealth sample, the expected utility hypothesis,  $H_0: \gamma = 1$ , is rejected and Hansen's J-statistic shows the rejection of the model. The above results are somewhat insensitive to the instruments.

### **Informal Test for Liquidity Constraints**

As we reject the model in both cases, the explanation of the rejection of the model by liquidity constraints does not seem to be appropriate in our case.

## **VI. CONCLUDING REMARKS**

The simple permanent income hypothesis with the certainty equivalence-quadratic utility implies the martingale property of consumption. Recent empirical research often rejects the time-additive expected utility model of the permanent income hypothesis. This rejection may be due to either the failure of the permanent income hypothesis or the misspecification of the underlying preferences.

The failure of the simple permanent income hypothesis leads to some consumption puzzles; the excess of consumption growth - persistent growth of consumption, even when the real interest rate has been negative - and the excess sensitivity of consumption to income. This failure is remedied by dropping certainty equivalence and using the CRRA or CARA time-additive expected utility function. Until now, their theoretical explanation has had little empirical support.

In this respect, we focus on the specification of the preferences in order to see if the departure from the time-additive expected utility can explain the consumption puzzle theoretically and empirically. We expect that the application of the non-expected utility framework to consumption puzzle may be fruitful, and that this explanation by the non-expected utility model may be useful in understanding consumer behavior. The main results of the paper are as follows.

First, we investigate the compatibility of the permanent income hypothesis with consumption growth. Specifically, we try to explain the excess of consumption growth puzzle, first posed by Deaton (1987). Using the stochastic intertemporal optimization framework with the general recursive utility, we can better explain the steady positive consumption growth under the negative real rates of return. Although Kuehlwein (1987) can explain this puzzle by considering the ef-

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<sup>10</sup> Giovannini and Jorion (1989) argue that the relaxation of the reciprocity between the coefficient of relative risk aversion and the elasticity of intertemporal substitution does not improve the fit of the model. Whenever the overidentifying restrictions of the expected utility model are rejected, they are also rejected in the non-expected utility model. Bufman and Leiderman (1990) show that the overidentifying restrictions of the non-expected utility model are not clearly rejected by the Israel data at the standard significance level. In view of the performance of the model, Epstein and Zin's (1991) result lies between the results of Giovannini and Jorion (1989) and Bufman and Leiderman (1990).

fects of uncertainty in the form of variance terms in the Euler equation for consumption, these uncertainty terms constitute only 3% of variances of the total error terms in the high-wealth group, and 16% in the low-wealth group. In our model, if the coefficients of the uncertainty terms are positive, these uncertainty terms constitute 72% in the case where  $\sigma_\eta^2 = \sigma_\varepsilon^2$  and 59% in the case where  $\sigma_\eta^2 = 5\sigma_\varepsilon^2$  of variance of the complete error terms in the high-wealth group, and 98% in the case where  $\sigma_\eta^2 = \sigma_\varepsilon^2$  and 93% in the case where  $\sigma_\eta^2 = 5\sigma_\varepsilon^2$  in the low-wealth group.

In our estimation of the parameters of the Euler equation and uncertainty terms, we consider measurement errors for consumption and the family's portfolio, which are a typical problem in using the micro data set. The limitations of these results are that we do not estimate the individual parameters of the model and do not test the overidentifying restrictions of the model. In addition, we do not derive the standard errors of the uncertainty terms. Overall, by using the general recursive utility structure under the stochastic intertemporal optimization framework, we can explain the excess of consumption growth puzzle better than the CRRA expected utility model, in the sense that uncertainty terms relative to measurement error terms are more sizable than those in the CRRA expected utility model of Kuehlwein (1987, 1991).

Second, we estimate the parameters of the model, the coefficient of relative risk aversion and the elasticity of intertemporal substitution, and test the hypothesis of the expected utility and the overidentifying restrictions of the model, using the non-expected utility model of Epstein and Zin (1989, 1991) with the PSID micro data set. The GMM estimates of the elasticity of intertemporal substitution are greater than one, which are contrasted with the results of Hall (1988) and Epstein and Zin (1991) where almost all estimates are less than one, and the GMM estimates of RRA coefficients are slightly less than one, whereas Epstein and Zin (1991) estimates these coefficients around one. The results of test of the model are mixed. The expected utility hypothesis is clearly rejected in favor of the non-expected utility model, however, the test of the overidentifying restrictions of the model using Hansen's J-statistic leads to the rejection across all instruments. The question of whether the violation of the Euler equation is due to either the intertemporal optimizing model itself or some auxiliary assumptions for deriving testable implications remains open for future research.

Although the analysis in this paper provides a useful basis for understanding consumer behavior, it also suggests some directions for future research. First, in general, the intertemporal model in this paper does not include labor income explicitly. However, this inclusion enables us to study the effect on consumption of the time series property of income process and to investigate the excess smoothness of consumption to unanticipated income. Second, more exhaustive analysis of measurement error problem in the noisy panel data merits investigation. Generally, micro data are considered more inaccurate than aggregate data. We think

that there is room for the empirical work in this paper to be improved. Third, we do not derive the standard error of uncertainty terms. So, some simulation technique may be useful to provide us with estimates of standard errors of uncertainty terms, and with some economic implication for consumption given noisy data.

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