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INTRODUCTION OF STOCK MARKET INTO SOCIALIST ECONOMY

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The decline of communism has shown that the command economy is no longer viable. This realization has given rise to new interest in market socialism. I define a general equilibrium model of a market socialist economy under uncertainty. Every consumer is given shares of firms but is not allowed to trade with others. Instead, each firm is given interfirm shareholdings to trade with other firms. Citizen shares may be distributed according to political considerations to satisfy criterion of justice. By prohibiting individuals from trading their citizen shares, we can avoid concentration of shares in a few hands and preserve equality in the distribution of wealth. Through the interfirm trade of shares, every firm's performance can be evaluated; the value of the firm on the stock market can be used to compensate managerial effort and induce managers to work hard.

I. INTRODUCTION

Existing socialist countries are characterized by i) dictatorship in politics ii) central planning and command in resource allocation without market iii)state ownership of the means of production. In early stage of economic development, heavy industry was crucial and it was more important to mobilize capital than to find efficient way to utilize existing capital. Therefore, in early experiment socialist economies seemed to prevail capitalist ones because the central government commanded the means of production. However, as economies accumulated industrial capital and the investment opportunity became scarce it became more important to use the capital efficiently.

Two reasons have been mentioned for the inefficiencies in a command economy. i)Without the private ownership of means of production, and appropriation of production surplus, the managers do not have incentive to use the means of production efficiently and ii)without market, the managers could not realize the

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true value of limited resources and production plan.

Recently many proposals have been made to reform socialist countries. These proposals focus on how to privatize state enterprises in order to construct an effective structure of ownership to monitor managers while maintaining equality in distribution of wealth. (See E. Borensztein and M. S. Kumar:1991 for the discussion of proposals.)

Finance theorists (for example, Jensen and Meckling, 1976; Fama:1980) were concerned with the principal-agent problem and claimed that managers can be disciplined through i) the stock market and ii) the managerial labor market. The latter can be compatible with socialist economy, but the introduction of full scale stock market cannot be introduced into socialist economy because of ideological limitation. In a socialist economy, means of production should be publicly owned but trade of stock by individual assumes the private ownership of the means of production.

Ortuño-Ortin, Roemer, and Silvestre (1990) showed the feasibility of a market socialist economy in a deterministic world that satisfies three requirements of a market socialist economy - government's commanding investment, resource allocation through the market, and public ownership of the means of production - through the manipulation of interest rates, direct investment, or sales taxes.

Under uncertainty, if insurance markets were completely developed, then the economy would behave like the one in a deterministic world. I suppose the reduction to certainty through the insurance market is limited. In a capitalist economy, the stock market fills the gap a little, even if not completely. The stock market i) allows efficient risk sharing between households ii) provides information about the valuation of uncertain income streams and iii) helps firms choose efficient production plans. Thus, the stock market in a capitalist economy helps to enhance the decentralized efficient use of resources under uncertainty. In this paper, I will show that a market socialist economy can have this efficiency without allowing private ownership of the means of production.

Some economists, such as P. Bardhan (1991), and M. Nuti (1987) have suggested a blueprint to achieve productive efficiency by imitating a capital market among firms and making public firms independent from the state. They assert that the continuous valuation of capital on the stock exchange and the pressure of takeovers could harden the budget constraint and enhance managerial performance in a socialist economy. In particular, Bardhan believes that public firms should be turned into a Joint Stock Company in which some financial institutions (e.g., Investment Bank, Pension Funds, etc.) hold and trade shares in physical good-producing firms.

I will extend the ideas of Bardhan and Nuti to include efficient risk sharing in a socialist economy as in capitalist economy. My challenge is to design a market socialist economy that i) is as efficient as the capitalist stock market economy and ii) satisfies some justice criterion with the ideological limit of non stock market.

This paper is organized as follows: In section II, the model is setup. In section III and IV, capitalist stock market equilibria with managers are defined and properties are explained in both capitalist and market socialist economies. And in section IV, the equilibria are compared.

II. MODEL

I assume one period and S possible states of nature. The true state of nature is unknown at the beginning of the period but has the probability of occurrence p_s, $s=1, \cdots, S$, $0 \leq p_s \leq 1$, $\sum_{s=1}^{S} p_s = 1$, which is common knowledge to every economic agent.

There is one consumption good x_s in each state $s=1, \cdots, S$, produced by M firms indexed by $j=1, \cdots, M$. Each firm's output depends on the state of nature, manager's effort e^j, and labor demand $l_D^j : y_s^j = f_s^j(e^j, l_D^j)$ $s=1, \cdots, S$. Each firm j chooses labor demand l_D^j and, μ^j the weight on state-dependent wage rate, to have a linear combination of fixed wage rate $\overline{w}$, and state-dependent wage rate w_s, which is indexed on aggregate national output $\overline{Y}$: $w_s \equiv \delta \overline{Y_s}$. Then the profit in state s is defined as $\pi_s^j = y_s^j - l_D^j[(1-\mu^j)\overline{w} + \mu^j \delta \overline{Y_s}]$.

There are N consumers indexed by $i=1, \cdots, N$: consumers j is a manager of firm j for $j=1, \cdots, M$, and consumers indexed by $i=M+1, \cdots, N$ are workers endowed with one unit of labor. A worker i can choose linear combination of fixed wage rate $\overline{w}$ and state dependent wage rate $w_s \equiv \delta \overline{Y_s}$, and his labor income becomes $\overline{w}(1-\lambda^i) + \overline{Y_s} \delta \lambda^i$. A manager j receives managerial fee $(\alpha^j + \beta^j \nu^j)$ determined as a linear function of the value of the firm(ν^j) she is managing with effort level e^j.

For each firm $j=1, \cdots, M$, the managerial fee schedule is determined subject to the Individual Rationality (IR) condition and the Incentive Compatibility(IC) condition. The Individual Rationality condition requires that the manager does not gain by leaving the managerial position. The Incentive Compatibility condition states how the managerial fee schedule should be determined in order to induce the second best production decision to maximize the firm's value.

Each manager j derives utility from his/her state-dependent consumption $x^j = (x_1^j, \cdots, x_s^j, \cdots, x_S^j) \in R_+^S$ and disutility from the effort level e^j to make decisions about the firm j's activity. Each worker i derives utility from his/her consumption $x^i = (x_1^i, \cdots, x_s^i, \cdots, x_S^i) \in R_+^S$[1]

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[1] Since I am concerned with the manager's incentive, I assumed the workers do not derive disutility from the labor. It may interpreted in the way that the difference of manager's level is the factor that goes into utility function.

Assumption 1 For each consumer $i=1,\cdots,N$, preferences are represented by expected utility of quadratic function: $EU(x^i, e^i)=\sum_{s=1}^{S} p_s u^i(x^i_s, e^i)$ for $i=1,\cdots,N$ where $u^i(x^i_s, e^i)=c^i(x^i_s-e^i)-\frac{1}{2}(x^i_s-e^i)^2$ for $s=1,\cdots,S$ while $e^i=0$ for non-managers $i=M+1,\cdots,N$.

Assumption 2 For each firm j, the production function in each state is monotonic and concave in labor: $\partial f^j_s/\partial l^j_D\geq 0$, and $\partial^2 f^j_s/\partial (l^j_D)^2\leq 0$.

Assumption 3 "Each firm has a conjecture about how the value of its shares will vary as it varies its production plan, and firms' conjectures are both correct and competitive."[2]

Define the covariance matrix of firms' profit in equilibrium:

$$Cov(\Pi, \Pi)=\begin{bmatrix} Cov(\overline{\pi}^1, \overline{\pi}^1) & \cdots & Cov(\overline{\pi}^1, \overline{\pi}^M) \\ \vdots & \ddots & \vdots \\ Cov(\overline{\pi}^M, \overline{\pi}^1) & \cdots & Cov(\overline{\pi}^M, \overline{\pi}^M) \end{bmatrix} : \text{an } M\times M \text{ square matrix.}$$

where $Cov(\overline{\pi}^j, \overline{\pi}^k)=\sum_{s=1}^{S} p_s(\overline{\pi}^j_s-E\overline{\pi}^j)(\overline{\pi}^k_s-E\overline{\pi}^k)$, a covariance profit at equilibrium of firms j and k, and $E\overline{\pi}^j=\sum_{s=1}^{S} p_s\overline{\pi}^j_s$, firm j's expected profit.

Assumption 4 The covariance matrix of profits in equilibrium is non-singular. Each consumer i is endowed with a vector of initial share holdings in M firms: $\theta^i_0=(\theta^{1i}_0,\cdots,\theta^{ji}_0,\cdots,\theta^{Mi}_0)$ with $\sum_{i=1}^{N}\theta^{ji}_0=1$, $i=1,\cdots,N$ $j=1,\cdots,M$. In a capitalist stock market economy, each consumer can trade his shares at the stock price $v\equiv(v^1,\cdots,v^j,\cdots,v^M)$ and can diversify his state dependent consumption and consumption in state s becomes

$$x^j_s=v(\theta^j_0-\theta^j_1)+\Pi_s\theta^j_1+(\alpha^j+\beta v^j) \text{ for } j=1,\cdots,M,$$
$$x^i_s=v(\theta^i_0-\theta^i_1)+\Pi_s\theta^i_1+[\overline{w}(1-\lambda^i)+\overline{Y}_s\delta\lambda^i] \text{ for } i=M+1,\cdots,N$$

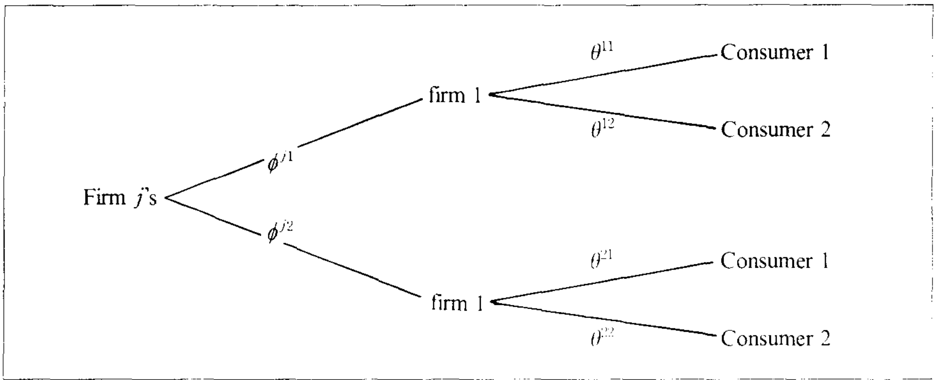
where

$$\Pi_s\equiv(\pi^1_s,\cdots,\pi^M_s).$$

In a market socialist economy, each consumer is given share holdings according to political reason such as the number of households, and is not allowed to trade. Instead, each firm j is given interfirm shares $\phi^j_0\equiv(\phi^{1j}_0,\cdots,\phi^{Mj}_0)$ and can trade shares among firms at stock price $v\equiv(v^1,\cdots,v^M)$ with final share holding

[2] Makowski, L. (1983. a), p. 309. Competitive conjecture means that the firm faces a perfectly elastic demand for its shares in the sense that if the firm raises the price of its shares by any small positive amount, then no one will be willing to buy any of its shares.

[Figure 1] Illustration of market socialist profit distribution



$\phi_1^{\cdot j} \equiv (\phi_1^{1j}, \cdots, \phi_1^{Mj})$. The firms' profits are distributed among the firms according to interfirm share holdings and are distributed again among citizens according to citizen shareholdings.

To illustrate, consider the 2-consumer, 2-firm case. In the figure above, firm j's profit is distributed between firm 1 and firm 2 by ϕ^{j1} and ϕ^{j2}, respectively. Firm 1's receipt of that profit is distributed among consumers again by θ^{11} and θ^{12}, and firm 2's receipt of that by θ^{21} and θ^{22}. Then, consumer 1's dividend of firm 1's profit is determined as $\phi^{11}\theta^{11}+\phi^{12}\theta^{21}$, and consumer 2's dividend of firm 1's profit is determined as $\phi^{11}\theta^{12}+\phi^{12}\theta^{22}$. In general N-consumer and M-firm case, consumer i's dividend of firm j's profit is determined as $\phi^{j\cdot}\theta^{\cdot i}$, $i=1, \cdots, N$, $j=1, \cdots, M$. And consumer's state-dependent consumption is determined as

$$x_s^j = v(\phi_0 - \phi_1)\theta^{\cdot j} + \Pi_s \phi_1 \theta^{\cdot j} + (\alpha^j + \beta^j v^j) \text{ for } j=1, \cdots, M,$$
$$x_s^i = v(\phi_0 - \phi_1)\theta^{\cdot i} + \Pi_s \phi_1 \theta^{\cdot i} + [\overline{w}\,(1-\lambda^i) + \overline{Y}_s \delta \lambda^i] \text{ for } i=M+1, \cdots, N$$

where

$$\Pi_s \equiv (\pi_s^1, \cdots, \pi_s^M).$$

Ⅲ. CAPITALIST STOCK MARKET EQUILIBRIUM

3.1 Definition: Capitalist Stock Market Equilibrium(CSE)

$(\overline{x}, \overline{\theta}_1, \overline{\lambda}, \overline{y}, \overline{e}, \overline{l}_D, \overline{\mu}, \overline{\beta}, \overline{\alpha})$[3] is a CSE relative to θ_0 if there exist a vector of stock price $v \in R^M$, fixed wage rate $\overline{w}$, and an index δ of state-dependent wage on aggregate output such that

[3] $\overline{x} \in R_+^{S\times N}$, $\overline{\theta_1} \in R^{M\times N}$, $\overline{\lambda} \in R^{N-M}$, $\overline{y} \in R^{S\times M}$, $\overline{e} \in R_+^M$, $\overline{l_D} \in R_+^M$, $\overline{\mu} \in R^M$, $\overline{\beta} \in R_+^M$, $\overline{\alpha} \in R_+^M$.

(CSE. 1) for each manager $j=1,\cdots, M$, given managerial fee schedule $(\overline{\alpha^j}, \overline{\beta^j})$, $(\overline{x^j}, \overline{\theta_1^{\cdot j}}, \overline{y^j}, \overline{e^j}, \overline{l_D^j}, \overline{\mu^j})$ is a solution to
(CSE. P. 1) choose $(x^j, \theta_1^{\cdot j}, y^j, e^j, l_D^j, \mu^j)$ to maximize

$$EU^j \equiv \sum_{s=1}^{S} p_s[c^j(x_s^j - e^j) - \frac{1}{2}(x_s^j - e^j)^2] \text{ subject to}$$
$$x_s^j = v(\theta_0^{\cdot j} - \theta_1^{\cdot j}) + \Pi_s \theta_1^{\cdot j} + (\overline{\alpha^j} + \overline{\beta^j}\, v^j) \tag{1}$$
$$y_s^j = f_s^j(e^j, l_D^j) \text{ for } s=1,\cdots, S$$

where $\Pi_s \equiv (\overline{\pi_s^1}, \cdots, \overline{\pi_s^M})$

$$\pi_s^k \equiv y_s^k - l_D^k[(1-\mu^k)\overline{w} + \mu^k \delta \overline{Y_s}] - \beta^k v^k - \alpha^k \text{ for } s=1,\cdots, S, \quad k=1,\cdots, M$$

$$\overline{Y_s} = \sum_{j=1}^{M} \overline{y_s^j} \text{ for } s=1, \cdots, S.$$

(CSE. 2) for each non-manager $i=M+1, \cdots, N$, $(\overline{x^i}, \overline{\theta_1^{\cdot i}}, \overline{\lambda^i})$ is a solution to
(CSE. P. 2) choose $(x^i, \theta_1^{\cdot i}, \lambda^i)$ to maximize

$$EU^i \equiv \sum_{s=1}^{S} (c^i x_s^i) - \frac{1}{2}(x_s^i)^2) \text{ subject to}$$
$$x_s^i = v(\theta_0^{\cdot i} - \theta_1^{\cdot i}) + \Pi_s \theta_1^{\cdot i} + (1-\lambda^i)\overline{w} + \lambda^i \delta \overline{Y_s} \tag{2}$$

(CSE. 3) for each firm $j=1, \cdots, M$, $(\overline{y^j}, \overline{e^j}, \overline{l_D^j}, \overline{\mu^j}, \overline{\beta^j}, \overline{\alpha^j})$ is a solution to
(CSE. P. 3.) choose $(y^j, e^j, l_D^j, \mu^j, \beta^j, \alpha^j)$ to maximize

$$v(y^j, e^j, l_D^j, \mu^j, \beta^j, \alpha^j) \text{ subject to}$$
$$y_s^j = f_s^j(e^j, l_D^j)$$

Incentive Compatibility (IC) and
Individual Rationality (IR)

and (CSE. 4) market clears, i. e.,

$$\sum_{i=1}^{N} \overline{x_s^i} = \sum_{j=1}^{M} \overline{y_s^j} \text{ for } s=1, \cdots, S \tag{3}$$

$$\sum_{j=1}^{M} \overline{l_D^j} = N - M \tag{4}$$

$$\sum_{i=1}^{N} \overline{\theta_1^{ji}} = 1 \text{ for } j=1, \cdots, M \tag{5}$$

$$\sum_{j=1}^{M} \overline{l_D^j}\, \overline{\mu^j} = \sum_{i=M+1}^{N} \overline{\lambda^i} \tag{6}$$

3.2 Equilibrium Share Holding and Linear Risk Sharing Rule

Both stocks and state-dependent wage can be seen as financial assets. Note that each asset price is determined at the level of expected value of the asset minus covariance of the asset and the aggregate output discounted by absolute aggregate risk tolerance.

Given risky assets, Borch (1968) showed that if utility functions are quadratic, then the risk sharing rule is linear if and only if the risk sharing rule is pareto efficient. In equilibrium, each individual is holding the same share in every firm such that the deviation of consumption in each state $s=1, \cdots, S$ from the expected consumption is linear in the deviation of state-dependent aggregate output from the expected aggregate output.

Define the absolute risk tolerance of consumer i by

$$t^i \equiv -\sum_{s=1}^{S} p_s \frac{\partial u^i / \partial x_s^i}{\partial^2 u^i / \partial (x_s^i)^2} \tag{7}$$

$$= (c^i - E\overline{x^i} + \overline{e^i}) \text{ for } i=1, \cdots, M, M+1, \cdots, N$$

where $\overline{e^i} = 0$ for $i = M+1, \cdots, N$.

Property III. 1 In CSE

(1) (Value of firm) for each firm $j=1, \cdots, M$, the value of firm v^j satisfies

$$v^j = E\overline{\pi^j} - \frac{Cov(\overline{Y}, \overline{\pi^j})}{\sum_{i=1}^{N} t^i} \tag{8}$$

(2) (Equilibrium share holding) for each manager $j=1, \cdots, M$,

$$\overline{\theta_1^{kj}} = \frac{t^j}{(1-\sum_{h=1}^{N} \overline{\lambda^h}) \sum_{h=1}^{N} t^h} \quad \text{for } k= 1,\cdots, M \tag{9}$$

and for each non-manager $i=M+1,\cdots, N$

$$\overline{\theta_1^{ki}} = \frac{\dfrac{t^i}{\sum_{h=1}^{N} t^h} - \overline{\lambda^i}\delta}{1-\sum_{h=1}^{N} \overline{\lambda^h}} \quad \text{for } k= 1,\cdots, M$$

(3) (Linear Risk Sharing Rule) for each consumer $i=1,\cdots, M, M+1,\cdots, N$

$$\overline{x_s^i} = E\overline{x^i} + (\overline{Y_s} - E\overline{Y}) \frac{t^i}{\sum_{h=1}^{N} t^h} \text{ for } s=1,\cdots, S \tag{10}$$

Proof: See Appendix 1

Grossman-Hart (1979) assumed Competitive Price Perception which implies $\sum_{s=1}^{S} p_s \frac{\partial EU^i}{\partial x_s^i}(-\Delta v^j + \Delta \pi_s^j)=0$. However, it holds in this model without assuming it.

Property III. 2. (Competitive Price Perception) At equilibrium, for every consumer $i=1,\cdots, N$, the change in the value of each firm is equal to the product of change in profit and marginal utility in each state.

$$\sum_{s=1}^{S} p_s \frac{\partial EU^i}{\partial x_s^i}(-\Delta v^j + \Delta \pi_s^j) = 0 \qquad \text{for } i=1,\cdots, N \tag{11}$$

Proof: Appendix 1.

Note that in the asset market for state-dependent wage firms participate as suppliers and non-managers act as demanders. Then, the price of state-dependent wage is determined as the expected value minus the covariance between the asset and the aggregate output divided by the absolute risk tolerance.

Property III. 3 At equilibrium, δ, the index of fixed wage on aggregate output satisfies

$$\overline{w} = \delta \left(E\overline{Y} - \frac{Cov(\overline{Y}, \sum_{i=M+1}^{N} \overline{x^i})}{\sum_{i=M+1}^{N} t^i} \right) \tag{12}$$

Proof: Appendix 1.

3.3 The Principal Agent Problem

Given managerial fee schedule, a manager makes a decision on the firm's choice of labor demand, effort level, and ratio of state-dependent wage and fixed wage. Given the relationship between the managerial fee schedule and the manager's decision, shareholders choose the managerial fee schedule that maximizes share holders' utility.

Among the manager's actions, the choice of state-dependent wage expenditure (μ^j) and the level of labor demand (l^j_D) is observable, and the interests of the principal and the agent do not conflict.

The equilibrium fixed and state-dependent wage rates ($\overline{w}$, $\delta\overline{Y}_s$) are such that the change of μ^j does not affect expected profit minus covariance discounted by aggregate risk tolerance. So, the firm's value does not depend upon the firm's choice of wage indexation μ^j, and the share holders do not have interest in μ^j. For each firm $j=1, \cdots, M$,

$$\frac{\partial v^j}{\partial \mu^j} = \frac{\partial E\overline{\pi}^j}{\partial \mu^j} - \frac{\dfrac{\partial\, Cov(\overline{Y}, \overline{\pi}^j)}{\partial \mu^j}}{\sum_{i=1}^{N} t^i} \tag{13}$$

$$= -\overline{l}^j_D \left((-\overline{w} + \delta E\overline{Y}) - \frac{\delta\, Cov(\overline{Y}, \overline{Y})}{\sum_{i=1}^{N} t^i} \right) = 0$$

The manager's first order condition with respect to labor demand is equivalent to maximizing the value of the firm.

$$\frac{\partial EU^j}{\partial l^j_D} = \sum_{s=1}^{S} p_s(c^j - x^j_s + e^j) \frac{\partial v^j}{\partial l^j_D} (\theta^j_0 + \overline{\beta}^j) = 0 \Leftrightarrow \frac{\partial v^j}{\partial l^j_D} = 0.$$

However, the level of effort by the manager is not observable to shareholders, and the principal is trying to induce the second best effort level to maximize the firm's value, given the relationship between the managerial fee schedule and the manager's effort.

In equation (8), the firm's value is a function of expected profit and covariance of profit and aggregate output, and expected profit is expected output minus wage expenditure and managerial fee, which is a function of the firm's value. If we rearrange equation (8), we have an explicit function. Since

$$E\pi^j = Ey^j - l^j_D\overline{w} - l^j_D\mu^j(-\overline{w} + \delta E\overline{Y}) - \beta^j v^j - \alpha^j$$

$$= Ey^j - l^j_D\overline{w} - \beta^j v^j - \alpha^j - \frac{Cov(\overline{Y}, \overline{Y})}{\sum_{i=1}^{N} t^i} l^j_D \mu^j \delta$$

and $Cov(\overline{Y}, \pi^j) = Cov(\overline{Y}, y^j) - Cov(\overline{Y}, \overline{Y}) l^j_D \mu^j \delta$[1], the equation (8) can be expressed as

[1] $\overline{\pi}^j_s - E\overline{\pi}^j = (y^j_s - Ey^j) - \overline{l}^j_D\, \mu^j \delta (\overline{Y}_s - E\overline{Y})$

$$v^j = E\overline{\pi^j} - \frac{Cov(\overline{Y}, \overline{\pi^j})}{\sum_{i=1}^{N} t^i}$$
$$= Ey^j - l_D^j\overline{w} - \beta^j v^j - \alpha^j - \frac{Cov(\overline{Y}, y^j)}{\sum_{i=1}^{N} t^i}$$

$$\Leftrightarrow v^j = \frac{Ey^j - l_D^j\overline{w} - \alpha^j - \frac{Cov(\overline{Y}, y^j)}{\sum_{i=1}^{N} t^i}}{1+\beta^j} \quad \text{for } j=1, \cdots, M \tag{14}$$

As for the Individual Rationality condition, a difficulty arises on how each individual will perceive the value of the firm and the price of fixed wage, after the current manager leaves and a new manager is recruited. Since the value of the firm depends on the managerial effort, if the new manager chooses a different production plan and effort level, then the value of the firm would be different from that when the current manager holds the position. To remove this difficulty, the following assumption is made:

Assumption 5 Every individual assumes that the value of the firm (v) and the price of fixed wage (δ) would remain the same in the event that the current manager is replaced with a new manager.

If the managerial fee is such that the expected value minus the covariance of fee discounted by aggregated absolute risk tolerance is equal to the fixed wage $\overline{w}$ plus the effort level, then the manager would not leave the managerial position.

Property III. 4 The Individual Rationality condition for each firm $j=1, \cdots, M$ is equivalent to $\alpha^j + \beta^j v^j - e^j \geq \overline{w}$ (15)

Proof: Appendix 1.

From the first order condition with respect to effort level e^j in the manager's program (CSE. P. 1), we have

$$\frac{\partial EU^j}{\partial e^j} = \sum_{s=1}^{S} p_s(c^j - \overline{x_s^j} + e^j)\left(\frac{\partial v^j}{\partial e^j}\theta_0^{jj} + \left(-\frac{\partial v^j}{\partial e^j} + \frac{\partial \pi_s^j}{\partial e^j}\right)\overline{\theta_1^{jj}}\right.$$
$$\left. + \beta^j\frac{\partial v^j}{\partial e^j} - 1\right)$$

where $\sum_{s=1}^{S} p_s(c^j - \overline{x_s^j} + e^j)\left(-\frac{\partial v^j}{\partial e^j} + \frac{\partial \pi_s^j}{\partial e^j}\right) = 0$ by (11).

So given managerial fee schedule β^j, the manager chooses his/her effort level such that

i) $e^j=0$ if $\frac{\partial EU^j}{\partial e^j} < 0$ for all $e^j \geq 0$

ii) otherwise e^j is determined such that

$$\frac{\partial EU^j}{\partial e^j} = 0 \Leftrightarrow \frac{\partial}{\partial e^j}\left[Ey^j - \frac{Cov(\overline{Y}, y^j)}{\sum_{i=1}^{N} t^i}\right]\frac{\theta_0^{jj}+\beta^j}{1+\beta^j} = 1.$$

Assuming interior solution, the Incentive Compatibility condition is expressed as

$$\frac{\partial}{\partial e^j}\left[Ey^j - \frac{Cov(\overline{Y}, y^j)}{\sum_{i=1}^{N} t^i}\right] = \frac{1+\beta^j}{\theta_0^{jj}+\beta^j} \tag{16}$$

If we examine the above Incentive Compatibility condition, it can be observed that the manager's effort level does not depend on how much the manager is risk averse or how wealthy the manager is from his/her share holding in other firms. Instead, it depends on how much initial share the manager has in the firm he/she is managing.

Now, let's assume the firm's value is monotonic and concave in the level of managerial effort.

Assumption 6 For each firm $j=1, \cdots, M$, define

$$\Psi^j = Ey^j - \overline{l^j_D}\,\overline{w} - \frac{Cov(\overline{Y}, y^j)}{\sum_{i=1}^{N} t^i}$$

where $\overline{l^j_D}$ is such that $\frac{\partial}{\partial l^j_D}\left[Ey^j - \frac{Cov(\overline{Y}, y^j)}{\sum_{i=1}^{N} t^i}\right] = \overline{w}$ (17)

and $\Psi^j_e = \frac{\partial \Psi^j}{\partial e^j}$ and $\Psi^j_{ee} = \frac{\partial^2 \Psi^j_e}{\partial (e^j)^2}$, and assume (i) $\Psi^j_e = 0$ and (ii) $\Psi^j_{ee} < 0$ for $e^j \geq 0$.

Everything being equal (managerial fee schedule, production technology and distribution of aggregate output), the more initial share the manager has in the firm, the more effort level he/she will expend. If we differentiate the above Incentive Compatibility condition with respect to effort level and initial share holding, we have

$$\frac{de^j}{d\theta_0^{jj}} = -\frac{\Psi^j_e}{\Psi^j_{ee}(\theta_0^{jj}+\beta^j)} \geq 0 \quad \text{if } \Psi^j_e \geq 0, \quad \text{and } \Psi^j_{ee} < 0. \tag{18}$$

To induce the same effort level, the shareholders can pay less fee the more initial share holding the manager has:

$$\frac{d\beta^j}{d\theta_0^{jj}} = -\frac{\Psi_e^j}{\Psi_e^j - 1} < 0. \tag{19}$$

The Principal-Agent problem can be rewritten as to

(CSE. P. 3′.) choose (e^j, β^j) to maximize $\dfrac{\Psi^j(e^j) - \alpha^j}{1+\beta^j}$

$$\text{subject to } \Psi_e^j \frac{\theta_0^{jj} + \beta^j}{1+\beta^j} = 1 \tag{IC}$$

$$\alpha^j + \frac{\beta^j}{1+\beta^j}(\Psi^j - \alpha^j) - e^j \geq \overline{w} \tag{IR}$$

and $\alpha^j, \beta^j, e^j \geq 0$

and the solution is as follows.

Property III. 5 (Solution to P-A problem) For each firm $j=1, \cdots, M$ the equilibrium effort level and managerial fee are characterized as follows:

(i) $\overline{\alpha^j} = 0$

(ii) if there is e^j such that

$$(\Psi_e^j)^2(\Psi_e^j - 1) + \Psi^j \, \Psi_{ee}^j = 0 \tag{20}$$

and

$$\frac{(1 - \Psi_e^j \theta_0^{jj})}{\Psi_e^j (1 - \theta_0^{jj})} \Psi^j - e^j - \overline{w} \geq 0$$

then $\overline{e^j}$ is determined by (20), if not, $\overline{e^j}$ is determined by

$$\frac{(1 - \Psi_e^j \theta_0^{jj})}{\Psi_e^j (1 - \theta_0^{jj})} \Psi^j - e^j - \overline{w} = 0 \tag{21}$$

and (iii)

$$\overline{\beta^j} = \frac{1 - \Psi_e^j \theta_0^{jj}}{\Psi_e^j (1 - \theta_0^{jj})} \tag{22}$$

Proof: Appendix 1.

The constant term (α^j) is set to zero, and the coefficient (β^j) is determined by Incentive Compatibility. The optimal level of effort is determined to maximize objective function if the firm's value is enough to satisfy the Individual Rationality condition. If not, it is determined by the Individual Rationality condition.

If it is the case that the value of the firm is enough to satisfy the Individual Rationality condition, then the equilibrium effort level is determined independent of the manager's characteristics (c^j, θ_0^j). Instead, the larger initial share holding the manager has in the firm he/she is managing, the larger stake he/she has in the firm. Thus the coefficient of the managerial fee schedule on the firm's value can be less.

Ⅳ. MARKET SOCIALIST EQUILIBRIUM (MSE)

$(\overline{x}, \overline{\phi_1}, \overline{\lambda}, \overline{y}, \overline{e}, \overline{l_D^j}, \overline{\mu}, \overline{\beta}, \overline{\alpha})$[5] is a MSE relative to (ϕ_0, θ) if there exist a vector of stock price $v \in R^M$, a fixed wage $\overline{w}$, and an index δ of state-dependent wage on aggregate output such that:

(MSE. 1) for each manager $j=1, \cdots, M$, given the managerial fee schedule $(\overline{\alpha^j}, \overline{\beta^j})$, $(\overline{x^j}, \overline{\phi_1^j}, \overline{y^j}, \overline{e^j}, \overline{l_D^j}, \overline{\mu^j})$ is a solution to

(MSE. P. 1) choose $(x^j, \phi_1^j, y^j, e^j, l_D^j, \mu^j)$ to maximize

$$EU^j = \sum_{s=1}^{S} p_s [c^j(x_s^j - e^j) - \frac{1}{2}(x_s^j - e^j)^2] \text{ subject to}$$

$$x_s^j = v(\phi_0 - \phi_1)\theta^{\cdot j} + \Pi_s \phi_1 \theta^{\cdot j} + \overline{\beta^j} v^j + \overline{\alpha^j} \tag{23}$$

$$y_s^j = f_s^j(e^j, l_D^j) \qquad \text{for } s=1, \cdots, S$$

where

$$\overline{\pi_s^k} \equiv \overline{y_s^k} - \overline{l_D^k}\,[(1-\overline{\mu^k})\overline{w} + \overline{\mu^k}\,\delta \overline{Y_s}] - \overline{\beta^k}\,\overline{v^k} - \overline{\alpha^k} \quad \text{for } s=1, \cdots, S,\ k=1, \cdots, M$$

(MSE. 2) for each non-manager $i=M+1, \cdots, N$, $(\overline{x^i}, \overline{\lambda^i})$ is a solution to

(MSE. P. 2) choose (x^i, λ^i) to maximize

$$EU^i \equiv p_s [c^i x_s^i - \frac{1}{2}(x_s^i)^2) \text{ subject to}$$

$$x_s^i = v(\phi_0 - \overline{\phi_1})\theta^{\cdot i} + \Pi_s \overline{\phi_1}\ \theta^{\cdot i} + (1-\lambda^i) + \lambda^i \delta Y_s \tag{24}$$

---

5 $\overline{x} \in R_+^{S\times N}$, $\overline{\phi_1} \in R^{M\times M}$, $\overline{\lambda} \in R^{N-M}$, $\overline{y} \in R^{S\times M}$, $\overline{e} \in R_+^M$, $\overline{l_D} \in R_+^M$, $\overline{\mu} \in R^M$, $\overline{\beta} \in R_+^M$, $\overline{\alpha} \in R_+^M$.

(MSE. 3) for each firm $j=1, \cdots, M$ $(\overline{y^j}, \overline{e^j}, \overline{l^j_D}, \overline{\mu^j}, \overline{\beta^j}, \overline{\alpha^j})$ is a solution to
(MSE. P. 3) choose $(y^j, e^j, l^j_D, \mu^j, \beta^j, \alpha^j)$ to maximize $v(y^j, e^j, l^j_D, \mu^j, \beta^j, \alpha^j)$
subject to $y^j_s = f^j_s(e^j, l^j_D)$

Individual Rationality (IR)
and Incentive Compatibility (IC)

and (MSE. 4) market clears, i. e.,

$$\sum_{i=1}^{N} \overline{x^i_s} = \sum_{j=1}^{M} \overline{y^j_s} \quad \text{for } s=1, \cdots, S \tag{25}$$

$$\sum_{j=1}^{M} \overline{\phi^{kj}_1} = 1 \qquad \text{for } k=1, \cdots, M \tag{26}$$

$$\sum_{j=1}^{M} \overline{l^j_D} = N - M \tag{27}$$

$$\sum_{j=1}^{M} \overline{\mu^j}\, \overline{l^j_D} = \sum_{i=M+1}^{N} \overline{\lambda^i} \tag{28}$$

4.1 Asset Price

In a market socialist economy, the process by which the value of the firm is determined is different from that in the capitalist stock market economy. In the latter, every individual participates in the trade of stocks, and the value of the firm reflects all consumers' aggregate risk tolerance. However, in the market socialist economy, only managers participate in the trade of stocks. Therefore the value of the firm is discounted by the manager group's consumption and risk tolerance.

Property IV. 1 (Value of the firm in MSE) In MSE for each firm $k=1, \cdots, M$, v^k, the value of the firm is determined by

$$v^k = E\overline{\pi^k} - \frac{Cov(\sum_{j=1}^{M} \overline{x^j}, \overline{\pi^k})}{\sum_{j=1}^{M} t^j} \tag{29}$$

Proof: Appendix 2.

4.2 Equilibrium Interfirm Share Holding

As with non-managers, every manager's consumption depends on the other managers' decisions. Thus, we need to look at the distribution of the managers' citizen shares to have equilibrium interfirm shareholding.

For some distribution of the managers' citizen shares, the equilibrium may not be unique. For example, suppose each manager $j=1, \cdots, M$ has the same citizen share in every firm. ($\theta^{kj}=\theta^{lj}$ for j, k, $l=1, \cdots, M$) Then, for every ϕ_1 that satisfies the market clearance condition $\sum_{k=1}^{M} \phi_1^{jk}=1$ for $j=1, \cdots, M$, the manager's dividend does not change, and any ϕ_1 that satisfies the market clearance condition can be equilibrium shareholding, and the non-manager's consumption may not be linear in aggregate output as in CSE. However, for a wide class of distribution of managers' citizen shares, the equilibrium interfirm shareholding is unique and the equilibrium allocation is as in capitalist stock market equilibrium.

Define

$\Theta^{M} \equiv (\Theta^{\cdot 1}, \cdots, \Theta^{\cdot M})$, $\Theta^{N-M} \equiv (\Theta^{\cdot M+1}, \cdots, \Theta^{\cdot N})$: the $M \times M$ and $M \times (N-M)$ matrix of the manager and non-manager groups citizen shares, respectively

$\Lambda \equiv (\lambda^{M+1}\delta, \cdots, \lambda^{N}\delta)$: $N-M$ element row vector of non-managers' fraction of state-dependent wage out of aggregate output

$T^{N} \equiv (t^{1}, \cdots, t^{M}, t^{M+1}, \cdots, t^{N})$: N element row vector of every consumer's absolute risk tolerance

$T^{M} \equiv (t^{1}, \cdots, t^{M})$, $T^{N-M} \equiv (t^{M+1}, \cdots, t^{N})$:the M and $N-M$ element row vector of managers and non-managers absolute risk tolerance, respectively

$\iota^{M} \equiv (1, \cdots, 1)$, $\iota^{N-M} \equiv (1, \cdots, 1)$: M and $N-M$ element column vectors of 1's, respectively, and

$I^{M} \equiv M \times M$ identity matrix.

Property IV. 2

(a) (Sufficient condition for unique interfirm shareholding) If the managers' citizen shares Θ^{M} are such that (i) there exists an inverse matrix Θ^{1} such that $\Theta^{1}\theta^{M} = \Theta^{M}\theta^{1}=I^{M}$ and (ii) $T^{M}\Theta^{1}\iota^{M}$ is non-zero, then the interfirm shareholding $\overline{\phi_1}$ is unique.

(b) (Equilibrium interfirm shareholding) If managers' citizen shares are as in (a), then equilibrium shareholding satisfies

$$\overline{\phi_1} = \frac{\iota^{M} T^{M} \Theta^{1}}{T^{M} \Theta^{1} \sigma^{M}} \tag{30}$$

Proof: Appendix 2.

4.3 Linear Risk Sharing Rule

Managers choose interfirm shareholding to maximize their utility. If Θ^M does not satisfy the above sufficient condition, the equilibrium interfirm shareholding is indeterminate, and it is not necessary but possible that $\overline{\phi}_1^{kj} = \overline{\phi}_1^{lj}$ and the linear risk sharing rule holds for all consumers. If the matrix of manager group's citizen share Θ^M satisfies the above condition, then managers' choice of equilibrium interfirm shareholding contribute to non-managers' optimal risk diversification.

Property IV. 3 (Linear risk sharing rule) If the distribution of manager group's citizen share Θ^M satisfies the condition in Property IV. 2, then at equilibrium, for each consumer $i=1, \cdots, M, M+1, \cdots, N$,

$$\overline{x}_s^i = E\overline{x}^i + (\overline{Y}_s - E\overline{Y}) \frac{t^i}{\sum_{h=1}^{N} t^h} \qquad s = 1, \cdots, S \tag{31}$$

Proof: Appendix 2.

Note that the firm's value depends on the absolute risk tolerance of the manager group. However, if the linear risk sharing rule holds for the manager group, then the functional form of the firm's value is identical to that in the capitalist stock market economy. (See (8) in capitalist stock market economy)

Corollary IV. 1 If the matrix of the manager group's citizen share Θ^M is as above, then in MSE,

(1) (Value of firm) for each firm $j=1, \cdots, M$ the value of firm v^j satisfies

$$v^j = E\overline{\pi}^j - \frac{Cov(\overline{Y}, \overline{\pi}^j)}{\sum_{i=1}^{N} t^i} \tag{32}$$

(2) (Wage Indexation) the index of fixed wage on aggregate output satisfies

$$\overline{w} = \delta \left(E\overline{Y} - \frac{Cov(\overline{Y}, \sum_{i=M+1}^{N} \overline{x}^i)}{\sum_{i=M+1}^{N} t^i} \right) \tag{33}$$

(3) (Competitive Price Perception) for every consumer $i=1, \cdots, N$, the change in the value of the firm is equal to the product of change in profit and marginal utility in each state.

$$\sum_{s=1}^{S} p_s \frac{\partial EU^i}{\partial x_s^i}(-\Delta v^j + \Delta \pi_s^j) = 0 \tag{34}$$

Proof : Appendix 2.

4.4 Principal-Agent Problem

As in the capitalist stock market economy, the choice of labor demand and ratio of state-dependent wage and fixed wage does not contradict the efficiency of equilibrium. However, the level of effort remains as the source of the principal-agent problem in the market socialist economy. And as we saw in Corollary IV. 1, for each firm $j=1,\cdots,M$, the functional form of the value of the firm is the same as that of the capitalist stock market economy.

As for the Individual Rationality condition, if we assume, as in the capitalist stock market economy, that each individual assumes the value of the firm and the price of wage indexation (δ) do not change with the change in managers, we have the same functional form of the Individual Rationality condition as in the capitalist stock market economy.

Property IV. 4 The Individual Rationality condition for each firm $j=1,\cdots,M$ is identical to $\alpha^j+\beta^j v^j-e^j \geq \overline{w}$ (35)

Proof: Appendix 2

The incentive compatibility condition is similar to that of CSE and can be expressed as $\Psi_e^j \dfrac{\phi_0^j\theta^{\prime j}+\beta^j}{1+\beta^j}=1$ and the principal-agent problem can be written as

(MSE.P.3)$'$ choose (e^j, β^j) to maximize $\dfrac{\Psi^j(e^j)-\alpha^j}{1+\beta^j}$

subject to $\Psi_e^j \dfrac{\phi_0^j\theta^{\prime j}+\beta^j}{1+\beta^j}=1$

$$\alpha^j+\frac{\beta^j}{1+\beta^j}(\Psi^j-\alpha^j)-e^j \geq \overline{w}$$

and $\alpha^j, \beta^j, e^j \geq 0$.

Since we have the same functional form of the objective function and constraints, we have the same solution as in CSE if $\theta_0^{\prime j}=\phi_0^j\theta^{\prime j}$.

V. COMPARISON OF CSE AND MSE

We have found that for a large class of distributions of consumer citizen shares, the MSE is such that the linear risk sharing rule holds, and the principal-agent problem in the market socialist economy is identical to that in capitalist stock market economy. Thus, we have an isomorphism between two economies.

Theorem 1 Given an MSE $(\overline{x}, \overline{y}, \overline{e}, \overline{l}_D, \overline{\phi}_1, \overline{\lambda}, \overline{\mu})$ relative to (ϕ_0, θ), if the matrix of the manager group's citizen share $\overline{\Theta}^M$ is non-singular and $T^M \Theta^1 \imath^M$ is non-zero, then there exists $(\theta_0, \overline{\theta}_1)$ such that $(\overline{x}, \overline{y}, \overline{e}, \overline{l}_D, \overline{\theta}_1, \overline{\lambda}, \overline{\mu})$ is a CSE relative to θ_0.

Proof: Appendix 3.

Theorem 2: Given a CSE $(\overline{x}, \overline{y}, \overline{e}, \overline{l}_D, \overline{\theta}_1, \overline{\lambda}, \overline{\mu})$ relative to θ_0, there exists $(\phi_0, \overline{\phi}_1, \theta)$ such that $(\overline{x}, \overline{y}, \overline{e}, \overline{l}_D, \overline{\phi}_1, \overline{\lambda}, \overline{\mu})$ is a MSE relative to (ϕ_0, θ).

VI. CONCLUSION

In this paper I have shown how a market socialist economy can be as efficient as a capitalist stock market economy. Each firm is given shares which it can trade with other firms and by which it can receive profits. Each consumer is given non-tradable citizen shares according to which second-hand profits are distributed. Besides the state dependent profits, each worker can choose a linear combination of a fixed wage and state-dependent wage which is indexed on aggregate output and can adjust his/her portfolio according to his/her attitude toward risk. Then, for a wide class of distribution of citizen shares, the equilibrium risk sharing is as efficient as in the capitalist stock market economy.

The lack of managerial incentive has been mentioned as one of the sources of inefficiency in the former socialist economies. Managers were given production input and ordered to produce a given amount of output. Managerial performances were not evaluated in an economic sense but through political negotiations with the Nomenklatura. In my model, the performance of a firm is evaluated through the trade of interfirm shares among firms, and can be used as an index to pay the managerial fee.

The distribution of citizen shares can be determined to satisfy a justice criterion such as the number of dependents in family, the necessities to support handicapped, or the number of labor hours each worker contributed to the national product, etc.

One of the inequalities in capitalist economies results from inheritance. If parents are rich, then their children are rich and given better opportunity to realize themselves. In my model, the citizen shares are not allowed to pass down to descendants and every new born baby can be equally treated, at least in terms of the distribution of wealth.

Each individual is denied the right to change and dispose of the shareholding. It may not be logical to deny the right to pass down citizen shares allowing change in shareholding. And Roemer (1992) has shown that concentration of shareholding would occur in a full private ownership economy and some virtues would be achieved by preventing the concentration.

APPENDIX

Define

$\overline{X_s^M} \equiv (\overline{x_s^1}, \cdots, \overline{x_s^M})$, $\overline{X_s^{N-M}} \equiv (\overline{x_s^{M+1}}, \cdots, \overline{x_s^N})$: the M element and $N-M$ element row vectors of consumption of manager group and non-manager group in each state $s=1, \cdots, S$ respectively

$\Theta_1^M \equiv (\theta_1^{\cdot 1}, \cdots, \theta_1^{\cdot M})$, $\Theta_1^{N-M} \equiv (\theta_1^{\cdot M+1}, \cdots, \theta_1^{\cdot N})$: the $M \times M$ and $M \times (N-M)$ matrix of the manager and non-manager groups share holding after trade, respectively

$\Lambda \equiv (\lambda^{M+1}\delta, \cdots, \lambda^N\delta)$: $N-M$ element row vector of non-managers' fraction of state-dependent wage out of aggregate output

$T^N \equiv (t^1, \cdots, t^M, t^{M+1}, \cdots, t^N)$: N-element row vector of every consumer's absolute risk tolerance

$T^M \equiv (t^1, \cdots, t^M)$, $T^{N-M} \equiv (t^{M+1}, \cdots, t^N)$:the M and $N-M$ element row vector of absolute risk tolerance of managers and non-managers, respectively

$\imath^M \equiv (1, \cdots, 1)$, $\imath^{N-M} \equiv (1, \cdots, 1)$: M and $N-M$ element column vectors of 1's, respectively.

APPENDIX 1.

Proof of Property III. 1 (Equilibrium Share Holding and Linear Risk Sharing Rule):

(1) For each consumer $i=1, \cdots, N$, the first order conditions with respect to Θ_1^{ki}, $k=1 \cdots, M$, are

$$\begin{bmatrix} \frac{\partial EU^1}{\partial \theta_1^{11}} & \cdots & \frac{\partial EU^N}{\partial \theta_1^{1N}} \\ \vdots & \vdots & \vdots \\ \frac{\partial EU^1}{\partial \theta_1^{M1}} & \cdots & \frac{\partial EU^N}{\partial \theta_1^{11}} \end{bmatrix} = 0$$

$$\Leftrightarrow (-v+E\Pi)\, T^N - Cov(\Pi, \overline{X^N}) = 0 \tag{A.1}$$

Then the post multiplication of (A.1) by $\imath^N$ yields

$$(-v+E\Pi) = \frac{Cov(\Pi, \overline{Y})}{T^N \imath^N} \tag{A.2}$$

(2) Since $\overline{\pi_s^j} - E\overline{\pi^j} = (\overline{y_s^j} - E\overline{y^j}) - \overline{l_D^j}\ \overline{\mu^j}\ \delta(\overline{Y_s} - E\overline{Y})$

$$(\Pi_s - E\Pi)\imath^M = (\overline{Y_s} - E\overline{Y}) - \sum_{j=1}^{M} \overline{l_D^j}\ \mu^j \delta(\overline{Y_s} - E\overline{Y})$$

$$= (\overline{Y_s} - E\overline{Y}) - \sum_{j=1}^{N} \overline{\lambda^j}\ \delta(\overline{Y_s} - E\overline{Y})$$

$$= (Y_s - E\overline{Y})(1 - \Lambda \imath^{N-M})$$

$$\Leftrightarrow (Y_s - E\overline{Y}) = (\Pi_s - E\Pi)\ \frac{\imath^M}{1-\Lambda\imath^{N-M}} \tag{A.3}$$

$$\text{and } (-v+E\Pi) = \frac{Cov(\overline{Y}, \Pi)}{T^N \imath^N} = \frac{Cov(\Pi, \Pi)}{(1-\Lambda\imath^{N-M})\, T^N \imath^N} \tag{A.4}$$

o For manager group, $\overline{X_s} - E\overline{X^M} = (\Pi_s - E\Pi)\theta_1^{\cdot M}$, (A.5)
and (A.1) becomes

$$(-v+E\Pi)T^M - Cov(\Pi, \overline{X^M}) = 0$$

$$= \frac{Cov(\Pi, \Pi)\imath^M T^M}{(1-\Lambda\imath^{N-M})\, T^N \imath^N} - Cov(\Pi, \Pi)\,\theta_1^{\cdot M} = 0 \qquad \text{by (A.4) and (A.5).}$$

$$\Leftrightarrow \overline{\theta_1^{\cdot M}} = \frac{T\imath^M \quad {}^M}{(1-\Lambda\imath^N T^M) \quad {}^N\imath^N} \tag{A.6}$$

o For non-manager group,

$$\overline{X_s^{N\cdot M}} - E\overline{X^{N\,M}} = (\Pi_s - E\Pi)\,\theta_1^{N\,M} + (\overline{Y_s} - E\overline{Y})\Lambda$$

$$= (\Pi_s - E\Pi)\left[\, \Theta_1^{N\cdot M} + \frac{\imath^M \Lambda}{(1-\Lambda \imath^{N-M})}\right] \tag{A.7}$$

and (A.1) becomes

$$(-v+E\Pi)\,T^{N\cdot M} - Cov(\Pi, \overline{X^{N\cdot M}}) = 0$$

$$\frac{Cov(\Pi, \Pi)\imath^M T^{N\cdot M}}{(1-\Lambda \imath^{N-M})\, T^N \imath^N} - Cov(\Pi, \Pi)\left[\, \Theta_1^{N\cdot M} + \frac{\imath^M \Lambda}{1-\Lambda \imath^{N-M}}\right] \text{ by (A.4) and (A.7)}$$

$$\Leftrightarrow \overline{\Theta_1^{N\cdot M}} = \frac{\imath^M}{1-\Lambda \imath^{N-M}}\left[\frac{T^{N-M}}{T^N \imath^N} - \Lambda\right] \tag{A.8}$$

(3) Linear Risk Sharing Rule

o For managers,

$$\begin{aligned}\overline{X_s^M} - E\overline{X^M} &= (\Pi_s - E\Pi)\,\overline{\Theta_1^M} \\ &= (\Pi_s - E\Pi)\,\frac{\imath^M T^M}{(1-\Lambda \imath^{N-M})} \\ &= (\overline{Y_s} - E\overline{Y})\,\frac{T^M}{T^N \imath^N} \qquad s=1, \cdots, S\end{aligned}$$

o For non-managers,

$$\begin{aligned}\overline{X_s^{N\cdot M}} - E\overline{X^{N\cdot M}} &= (\Pi_s - E\Pi)\left[\,\overline{\Theta_1^{N\cdot M}} + \frac{\imath^M \Lambda}{1-\Lambda \imath^{N\cdot M}}\right] \\ &= (\overline{Y_s} - E\overline{Y})\,\frac{T^{N\cdot M}}{T^N \imath^N} \qquad s=1, \cdots, S\end{aligned} \tag{A.9}$$ ■

Proof of Property III. 2 (Competitive Price Perception):

From Property III. 1,

$$\Delta v^j = \Delta E\overline{\pi^j} - \frac{Cov(\overline{Y}, \Delta\overline{\pi^j})}{\sum_{h=1}^{N} t^h} \tag{A.10}$$

$$\begin{aligned}\sum_{s=1}^{S} p_s \frac{\partial EU^i}{\partial x_s^i} \Delta\pi_s^j &= \sum_{s=1}^{S} p_s\,[t^i - (\overline{x_s^i} - E\overline{x^i})]\, \Delta\pi_s^j \\ &= t^i \Delta E\pi^j - Cov(\overline{x^i}, \Delta\pi^j) \qquad \text{by (A.9)} \\ &= t^i \Delta E\pi^j - Cov\left(\overline{Y}\,\frac{t^i}{\sum_{h=1}^{N} t^h}, \Delta\pi^j\right)\end{aligned}$$

Therefore, $\sum_{s=1}^{S} p_s \frac{\partial EU^j}{\partial x_s^j}(-\Delta v^j + \Delta\pi_s^j) = 0$ (A.11)■

Proof of Property III. 3 (Wage Indexation):

o For each manager $j=1, \cdots, M$, the first order condition with respect to μ^j is

$$\frac{\partial EU^j}{\partial \mu^j} = \sum_{s=1}^{S} p_s(c^j - \overline{x_s^j} + e^j)\left[\frac{\partial v^j}{\partial \mu^j}(\theta_0^{jj} + \overline{\beta^j}) + \left(-\frac{\partial v^j}{\partial \mu^j} + \frac{\partial \pi_s^j}{\partial \mu^j}\right)\overline{\theta_1^{jj}}\right] = 0$$

$$\Leftrightarrow \sum_{s=1}^{S} p_s[t^j - (\overline{x_s^j} - \overline{Ex^j})]\frac{\partial \overline{\pi_s^j}}{\partial \mu^j} = 0 \qquad \text{by (A.11)}$$

$$\Leftrightarrow t^j(-\overline{w} + \delta E\overline{Y}) - Cov(\overline{x^j}, \overline{Y})\delta = 0$$

$$\Leftrightarrow t^j(-\overline{w} + \delta E\overline{Y}) - Cov\left(\overline{Y}\frac{t^j}{\sum_{i=1}^{N} t^i}, \overline{Y}\right)\delta = 0 \qquad \text{by (A.9)}$$

$$\Leftrightarrow \overline{w} = \delta\left(\overline{Y} - \frac{Cov(\overline{Y}, \overline{Y})}{\sum_{i=1}^{N} t^i}\right) \qquad \text{(A.12)}$$

o For each non-manager $i = M+1, \cdots, N$, the first order condition with respect to λ^i is satisfied:

$$\frac{\partial EU^i}{\partial \lambda^i} = 0$$

$$\Leftrightarrow t^i(-\overline{w} + \delta E\overline{Y}) - Cov(\overline{x^i}, \overline{Y})\delta = 0 \qquad \text{(A.3)}$$

$$\Leftrightarrow \frac{Cov(\overline{Y}, \overline{Y})}{\sum_{h=1}^{N} t^h} t^i - Cov\left(\overline{Y}\frac{t^i}{\sum_{h=1}^{N} t^h}, \overline{Y}\right) = 0 \qquad \text{by (A.9)■}$$

Proof of Property III. 4 (Individual Rationality):

Suppose a managerial fee schedule in a firm j is such that $\alpha^j + \beta^j v^j - e^j = \overline{w}$. Denote $(\hat{x}, \widehat{Ex^j}, \hat{t}^j, \hat{\theta}_1^j, \hat{\lambda}^j)$, and $(\overline{x}, \overline{Ex^j}, \overline{t^j}, \overline{\theta_1^j})$ are the j^{th}'s decision as a non-manager and manager, respectively. Then, his/her expected consumption as a non-manager is

$$\widehat{Ex^j} = v\theta_0^{j} + (-v + E\overline{\Pi})\hat{\theta}_1^j + (-\overline{w} + \delta E\overline{Y})\hat{\lambda}^j + \overline{w}$$

$$= v\theta_0^{j} + \frac{Cov(\overline{Y}, \overline{\Pi})}{T^N \imath^N}\frac{\imath^M}{(1 - \Lambda\imath^{N\text{-}M})}\left(\frac{\hat{t}^j}{T^N\imath^N} - \hat{\lambda}^j\delta\right) + \frac{Cov(\overline{Y}, \overline{Y})}{T^N\imath^N}$$

$$\delta\hat{\lambda}^j + \overline{w}$$

by (A.2), (A.8), and (A.12)

$$= v\theta_0^{jj} + \frac{Cov(\overline{Y}, \overline{Y})}{(T^N \iota^N)^2} + \overline{w}$$

$$\Leftrightarrow \hat{t}^j \equiv c^j - E\hat{x}^j = \frac{c^j - v\theta_0^{jj} - \overline{w}}{1 + \frac{Cov(\overline{Y}, \overline{Y})}{(T^N \iota^N)^2}} \tag{A.14}$$

o The manager's expected consumption

$$E\overline{x}^j = v\theta_0^{jj} + (-v + E\Pi)\theta_1^{\overline{j}} + \alpha^j + \beta^j v^j$$

$$= v\theta_0^{jj} + \frac{Cov(\overline{Y}, \Pi)}{T^N \iota^N} \frac{\iota^M \overline{t}^j}{(1 - \Lambda \iota^{NM})} + \alpha^j + \beta^j v^j \qquad \text{by (A.2), and (A.6)}$$

$$= v\theta_0^{jj} + \frac{Cov(\overline{Y}, \overline{Y})}{(T^N \iota^N)^2} \overline{t}^j + \alpha^j + \beta^j v^j$$

$$\Leftrightarrow \overline{t}^j \equiv c^j - E\overline{x}^j - e^j = \frac{c^j - v\theta_0^{jj} - \alpha^j - \beta^j v^j + e^j}{1 + \frac{Cov(\overline{Y}, \overline{Y})}{(T^N \iota^N)^2}} \tag{A.15}$$

Therefore $t^j = \hat{t}^j$ if and only if $\alpha^j + \beta^j v^j - e^j = \overline{w}$.

$$\Leftrightarrow \overline{x}_s^j - e^j = E\overline{x}^j + e^j + (\overline{Y}_s - E\overline{Y}) \frac{\overline{t}^j}{T^N \iota^N}$$

$$= E\hat{x}^j + (\overline{Y}_s - E\overline{Y}) \frac{\hat{t}^j}{T^N \iota^N}$$

$$= \hat{x}_s^j$$

$$\Leftrightarrow EU^j(\overline{x}^j, e^j) = EU^j(\hat{x}^j)$$ ■

Proof of Property III. 5 (Solution to P-A problem):
Rearranging the Incentive Compatibility condition, we have

$$\beta^j = \frac{1 - \Psi_e^j \theta_0^{jj}}{\Psi_e^j - 1} \tag{A.16}$$

and the Principal-Agent problem can be rewritten as

(CSE. P. 3') choose (e^j, α^j, β^j) to maximize $\dfrac{(\Psi^j - \alpha^j)(\Psi_e^j - 1)}{(1 - \theta_0^{jj})\Psi_e^j}$

$$\text{subject to } \frac{\alpha^j(\Psi_e^j - 1) + (1 - \Psi_e^j \theta_0^{jj})\Psi^j}{\Psi_e^j(1-\theta_0^{jj})} - e^j \geq \overline{w}$$

$$\frac{1 - \Psi_e^j}{\Psi_e^j - 1} \geq 0 \text{ and}$$

$$e^j, \alpha^j \geq 0.$$

Then, the Lagrangian function is written as

$$\pounds(e^j, \alpha^j, \varsigma^j, \gamma^j) = \frac{(\Psi^j - \alpha^j)(\Psi_e^j - 1)}{(1-\theta_0^{jj})\Psi_e^j}$$

$$+ \varsigma^j \left[\frac{\alpha^j(\Psi_e^j - 1) + (1 - \Psi_e^j\theta_0^{jj})\Psi^j}{\Psi_e^j(1-\theta_0^{jj})} - e^j - \overline{w} \right]$$

$$+ \gamma^j \frac{1 - \Psi_e^j}{\Psi_e^j - 1}$$

o If there exists a solution to the above problem, then

$$1 < \Psi_e^j < \frac{1}{\theta_0^{jj}}, \text{ and } \mu^j = 0. \tag{A.17}$$

Suppose not, i. e. $\Psi_e^j\theta_0^{jj} = 1$. Then $\frac{\partial \pounds}{\partial \varsigma^j} = \alpha^j \frac{\Psi_e^j - 1}{\Psi_e^j(1-\theta_0^{jj})} - e^j - \overline{w} \geq 0$

$$\Leftrightarrow \alpha^j \geq (e^j + \overline{w}) \frac{\Psi_e^j(1-\theta_0^{jj})}{\Psi_e^j - 1} > 0, \text{ because } \Psi_e^j - 1 = \frac{1}{\theta_0^{jj}} - 1 > 0.$$

From

$$\overline{\alpha}^j \frac{\partial \pounds}{\partial \alpha^j} = \overline{\alpha}^j(-1 + \varsigma^j) \frac{\Psi_e^j - 1}{\Psi_e^j(1-\theta_0^{jj})} = 0, \ \varsigma^j = 1.$$

Then

$$\frac{\partial \pounds}{\partial e^j} = \Psi_e^j - 1 + \varsigma^j \frac{-\Psi_{ee}^j(1-\theta_0^{jj})}{(\Psi_e^j - 1)^2} > 0,$$

which does not satisfy the Kuhn-Tucker condition.

o From $\frac{\partial \pounds}{\partial e^j} = 0$, we have

$$\varsigma^j = \frac{(\Psi_e^j)^2(\Psi_e^j - \alpha^j) + (\Psi_e^j)^2(\Psi_e^j - 1)}{\Psi_{ee}^j(\Psi^j - \alpha^j) + (\Psi_e^j)^2(\Psi_e^j - 1)\theta_0^{jj}}. \tag{A.18}$$

From $\overline{\alpha^j}\dfrac{\partial \pounds}{\partial \alpha^j}=\overline{\alpha^j}(-1+\varsigma^j)=\overline{\alpha^j}\dfrac{(\Psi_e^j)^2(\Psi_e^j-1)(1-\theta_0^{jj})}{\Psi_{ee}^j(\Psi^j-\alpha^j)+(\Psi_e^j)^2(\Psi_e^j-1)\theta_0^{jj}}=0$, and

$$(\Psi_e^j)^2(\Psi_e^j-1)(1-\theta_0^{jj})\neq 0,\text{ we have }\overline{\alpha^j}=0. \tag{A.19}$$

Therefore, $(\overline{e^j}, \varsigma^j)$ is determined by

$$(\Psi_e^j)^2(\Psi_e^j-1)+\Psi^j\Psi_{ee}^j-\varsigma^j[(\Psi_e^j)^2(\Psi_e^j-1)\theta_0^{jj}+\Psi^j\Psi_{ee}^j]=0 \tag{A.20}$$

$$\varsigma^j\left[\frac{(1-\Psi_e^j\theta_0^{jj})}{\Psi_e^j(1-\theta_0^{jj})\Psi^j}-e^j-\overline{w}\right]=0,\text{ and}$$

$$\frac{(1-\Psi_e^j\theta_0^{jj})}{\Psi_e^j(1-\theta_0^{jj})}\Psi^j-e^j-\overline{w}\geq 0. \qquad \blacksquare$$

APPENDIX 2

Proof of Property IV. 1 (Value of Firm):
For managers $j=1,\cdots,M$, the first order conditions with respect to ϕ_1^{kj} $(k=1,\cdots,M)$ are

$$\begin{bmatrix} \dfrac{\partial EU^1}{\partial \phi_1^{11}} & \cdots & \dfrac{\partial EU^M}{\partial \phi_1^{1M}} \\ \vdots & \vdots & \vdots \\ \dfrac{\partial EU^1}{\partial \phi_1^{M1}} & \cdots & \dfrac{\partial EU^M}{\partial \phi_1^{MM}} \end{bmatrix}=0$$

$$\Leftrightarrow (-v+E\Pi)\,T^M-Cov(\Pi,\overline{X^M})=0 \tag{A.21}$$

and post-multiplication of Eq.A.2.1) by $\imath^M$ yields

$$v=E\Pi-\frac{Cov(\Pi,\overline{X^M})}{T^M\imath^M}=0 \tag{A.22}$$ $\blacksquare$

Proof of Property IV. 2:
(a) Using the (A.22), the (A.21) becomes

$$\begin{bmatrix} Cov(\overline{\pi^1},\overline{x^1}) & \cdots & Cov(\overline{\pi^1},\overline{x^M}) \\ \vdots & \vdots & \vdots \\ Cov(\overline{\pi^M},\overline{x^1}) & \cdots & Cov(\overline{\pi^M},\overline{x^M}) \end{bmatrix}\left(\frac{\imath^M T^M}{T^M\imath^M}-I^M\right)=0$$

$$\Leftrightarrow Cov(\Pi, \Pi)\overline{\phi}_1 \Theta^M \left(\frac{\imath^M T^M}{T^M \imath^M} - I^M \right) = 0 \tag{A.23}$$

Note that the $M\frac{th}{}$ column of coefficient matrix $\left(\frac{\imath^M T^M}{T^M \imath^M} - I^M \right)$ is a linear combination of the first $(M-1)$ columns. Thus, assuming $Cov(\Pi, \Pi)$ is non-singular, and combining (A.23) and the market clearance condition $\phi_1 \imath^M = \imath^M$, we have

$$\overline{\phi}_1 A = [0, \imath^M] \tag{A.24}$$

$$\text{where } A = \left[\begin{array}{cc} \Theta^M \begin{pmatrix} \frac{t^1}{T^M \imath^M} - 1 & \cdots & \frac{t^{M\text{-}1}}{T^M \imath^M} \\ \vdots & \vdots & \vdots \\ \frac{t^1}{T^M \imath^M} & \cdots & \frac{t^{M\text{-}1}}{T^M \imath^M} - 1 \\ \frac{t^1}{T^M \imath^M} & \cdots & \frac{t^{M\text{-}1}}{T^M \imath^M} \end{pmatrix} & \begin{array}{c} 1 \\ \vdots \\ 1 \\ 1 \end{array} \end{array} \right]$$

$$\text{o Determinant of } A = \frac{(-1)^{M\text{-}1} T^M \Theta^{-1} \imath^M}{T^M \imath^M Det(\Theta^{-1})} \tag{A.25}$$

$$\Leftrightarrow \text{Determinant of } \Theta^{-1} \cdot A = Det(\Theta^{-1}) \cdot Det(A)$$

$$= \begin{vmatrix} \frac{t^1}{T^M \imath^M} - 1 & \cdots & \frac{t^{M\text{-}1}}{T^M \imath^M} & (\Theta^{-1})^{1\cdot} \imath^M \\ \vdots & \vdots & \vdots & \vdots \\ \frac{t^1}{T^M \imath^M} & \cdots & \frac{t^{M\text{-}1}}{T^M \imath^M} - 1 & (\Theta^{-1})^{M\text{-}1\cdot} \imath^M \\ \frac{t^1}{T^M \imath^M} & \cdots & \frac{t^{M\text{-}1}}{T^M \imath^M} & (\Theta^{-1})^{M\cdot} \imath^M \end{vmatrix}$$

where $(\Theta^{-1})^{j\cdot}$ is the $j\frac{th}{}$ row of the inverse matrix Θ^{-1}

$$= \begin{vmatrix} -1 & \cdots & 0 & (\Theta^{-1})^{1\cdot} \imath^M - (\Theta^{-1})^{M\cdot} \imath^M \\ \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & -1 & (\Theta^{-1})^{M\text{-}1\cdot} \imath^M - (\Theta^{-1})^{M\cdot} \imath^M \\ \frac{t^1}{T^M \imath^M} & \cdots & \frac{t^{M\text{-}1}}{T^M \imath^M} & (\Theta^{-1})^{M\cdot} \imath^M \end{vmatrix}$$

$$=(-1)^{M-1}[(\Theta^{1})^{M\cdot}\imath^{M}+\frac{t^{1}}{T^{M}\imath^{M}}\{(\Theta^{1})^{1\cdot}\imath^{M}-(\Theta^{1})^{M\cdot}\imath^{M}\}+\cdots$$

$$\cdots+\frac{t^{M-1}}{T^{M}\imath^{M}}\{(\Theta^{1})^{M-1}\imath^{M}-(\Theta^{1})^{M\cdot}\imath^{M}\}]$$

$$=\frac{(-1)^{M-1}T^{M}\Theta^{-1}\imath^{M}}{T^{M}\imath^{M}}$$

o So, if and only if $(-1)^{M-1}T^{M}\Theta^{-1}\imath^{M}\neq 0$, the matrix $\left(\Theta^{M}\left(\frac{\imath^{M}T^{M}}{T^{M}\imath^{M}}-I^{M}\right),\imath^{M}\right)$ is of full row rank and solution $\overline{\phi_1}$ is uniquely determined.

(b) When Θ^{M} is non-singular and $T^{M}\Theta^{-1}\imath^{M}$ is non-zero, if

$$\overline{\phi_1}=\frac{\imath^{M}T^{M}\Theta^{-1}}{T^{M}\Theta^{-1}\imath^{M}} \tag{A.26}$$

then both (A.23) and the market clearance condition are satisfied.

$$\overline{\phi_1}\,\Theta^{M}\left(\frac{\imath^{M}T^{M}}{T^{M}\imath^{M}}-I^{M}\right)=\frac{\imath^{M}T^{M}\Theta^{-1}}{T^{M}\Theta^{-1}\imath^{M}}\Theta^{M}\left(\frac{\imath^{M}T^{M}}{T^{M}\imath^{M}}-I^{M}\right)$$

$$=\frac{1}{T^{M}\Theta^{-1}\imath^{M}}(\imath^{M}T^{M}-\imath^{M}T^{M})$$

$$=0$$

and

$$\overline{\phi_1}\,\imath^{M}=\frac{\imath^{M}T^{M}\Theta^{-1}}{T^{M}\Theta^{-1}\imath^{M}}\imath^{M}=\imath^{M}$$ ■

Proof of Property IV. 3:
Suppose Θ^{M} is non-singular and $T^{M}\Theta^{-1}\imath^{M}\neq 0$. Then,
o for each manager k, and firm j, $j,k=1,\cdots,M$

$$\sum_{s=1}^{S}\frac{\partial EU^{k}}{\partial x_s^{k}}(-\Delta v^{j}+\Delta\overline{\pi_s^{j}})=0 \tag{A.27}$$

$$\Leftrightarrow\sum_{s=1}^{S}p_s[t^{k}-(\overline{x_s^{k}}-E\overline{x^{k}})]\left[-\Delta E\overline{\pi^{j}}+\frac{Cov(\Delta\overline{\pi^{j}},\overline{x^{M}})\imath^{M}}{T^{M}\imath^{M}}+\Delta\overline{\pi_s^{j}}\right]=0 \quad \text{by (A.22)}$$

$$\Leftrightarrow t^{k}\frac{Cov(\Delta\overline{\pi^{j}},\overline{X^{M}})\imath^{M}}{T^{M}\imath^{M}}-Cov(\overline{x^{k}},\Delta\overline{\pi^{j}})=0$$

$$\because\ (\overline{X_s^{M}}-E\overline{X^{M}})\imath^{M}\frac{t^{k}}{t^{M}\imath^{M}}-(\overline{x_s^{k}}-E\overline{x^{k}})$$

$$=(\Pi_s - E\Pi)\overline{\phi}_1\, \Theta^{M}\iota^{M}\frac{t^k}{T^{M}\iota^{M}} - (\Pi_s - E\Pi)\overline{\phi}_1\theta^{\cdot k}$$

$$=(\Pi_s - E\Pi)\left[\frac{\iota^{M}\ T^{M}\Theta^{-1}\Theta^{M}\iota^{M}t^k}{T^{M}\Theta^{-1}\iota^{M}T^{M}\iota^{M}} - \frac{\iota^{M}\ T^{M}\Theta^{-1}}{T^{M}\Theta^{-1}\iota^{M}}\theta^{\cdot k}\right] \qquad \text{by (A.26)}$$

$$= 0$$

o For each manager $j=1, \cdots, M$,

$$\frac{\partial EU^j}{\partial \mu^j} = \sum_{s=1}^{S} p_s(c^j - \overline{x}_s^j + e^j)\left[\frac{\partial v^j}{\partial \mu^j}(\phi_0^j\theta^{\cdot j} + \overline{\beta}^j) + \left(-\frac{\partial v^j}{\partial \mu^j} + \frac{\partial \pi_s^j}{\partial \mu^j}\overline{\phi}_1\theta^{\cdot j}\right)\right]$$

$$= 0$$

$$\Leftrightarrow \frac{\partial v^j}{\partial \mu^j} = 0 \qquad \text{by (A.27)}$$

$$\Leftrightarrow -\overline{w} + \delta E\overline{Y} = \delta\frac{Cov(\overline{Y}, \overline{x}^{M})\iota^{M}}{T^{M}\iota^{M}} \qquad \text{(A.28)}$$

o For each non-manager $i = M+1, \cdots, N$,

$$\frac{\partial EU^i}{\partial \lambda^i} = (-\overline{w} + \delta E\overline{Y})t^i - Cov(\overline{Y}, \overline{x}^i)\delta = 0 \qquad \text{(A.29)}$$

and summation of (A.29) over $i = M+1, \cdots, N$ yields

$$(-\overline{w} + \delta E\overline{Y}) = \delta\frac{Cov(\overline{Y}, \overline{X}^{N\cdot M})}{T^{N\cdot M}\iota^{N\cdot M}} \qquad \text{(A.30)}$$

o From (A.29) and (A.30) we have

$$(-\overline{w} + \delta E\overline{Y}) = \delta\frac{Cov(\overline{Y}, \overline{X}^{N\cdot M})\iota^{N\cdot M}}{T^{N\cdot M}\iota^{N\cdot M}}$$

$$= \delta\frac{Cov(\overline{Y}, \overline{X}^{M})\iota^{M}}{T^{M}\iota^{M}} \qquad \text{(A.31)}$$

$$= \delta\frac{Cov(\overline{Y}, \overline{Y})}{T^{N}\iota^{N}}$$

and the (A.28) becomes

$$\frac{Cov(\overline{Y}, \overline{Y})}{T^{N}\iota^{N}} = \frac{Cov(\overline{Y}, \overline{X}^{M})\iota^{M}}{T^{M}\iota^{M}}$$

$$= \frac{Cov(\overline{Y}, \overline{\Pi})}{T^{N}\iota^{N}} \overline{\phi}_1 \Theta^{M}\iota^{M}$$

$$= \frac{Cov(\overline{Y}, \overline{Y})(1-\Lambda\iota^{M})}{T^{M}\Theta^{-1}\iota^{N}}$$

$$\Leftrightarrow \frac{(1-\Lambda\iota^{M})}{T^{M}\Theta^{-1}\iota^{M}} = \frac{1}{T^{N}\iota^{N}} \tag{A.32}$$

o Then for managers

$$\begin{aligned} \overline{X_s^{M}} - E\overline{X^{M}} &= (\Pi_s - E\Pi)\overline{\phi}_1 \Theta^{M} \\ &= (\overline{Y_s} - E\overline{Y}) \frac{(1-\Lambda\iota^{M})\, T^{M}}{T^{M}\Theta^{-1}\iota^{M}} \qquad \text{by (A.32)} \qquad (A.33) \\ &= (\overline{Y_s} - E\overline{Y}) \frac{T^{M}}{T^{N}\iota^{N}} \end{aligned}$$

$$\begin{aligned} \overline{X_s^{N\cdot M}} - E\overline{X^{N\cdot M}} &= (\Pi_s - E\Pi)\overline{\phi}_1 \Theta^{N\cdot M} + (\overline{Y_s} - E\overline{Y})\Lambda \\ &= (\overline{Y_s} - E\overline{Y}) \left(\frac{(1-\Lambda\iota^{M})\, T^{M}\Theta^{-1}\Theta^{N\cdot M}}{T^{M}\Theta^{-1}\iota^{M}} \right) \qquad \text{by (A.34)} \\ &= (\overline{Y_s} - E\overline{Y}) \frac{T^{N\cdot M}}{T^{N}\iota^{N}} \end{aligned}$$

$$\because \left[\frac{\partial EU^{M+1}}{\partial \lambda^{M+1}} \cdots \frac{\partial EU^{N}}{\partial \lambda^{N}} \right] = (-\overline{w} + \delta E\overline{Y})\, T^{N\cdot M} - Cov(\overline{Y}, \overline{X^{N\cdot M}})\delta$$

$$= Cov(\overline{Y}, \overline{Y})\, \delta \left[\frac{T^{N\cdot M}}{T^{N}\iota^{N}} - \frac{(1-\Lambda\iota^{M})\, T^{M}\Theta^{-1}\Theta^{N\cdot M}}{T^{M}\Theta^{-1}\iota^{M}} - \Lambda \right] = 0 \tag{A.35}$$ ■

Proof of Corollary IV. 1:

(1) From (A.22)

$$v^{k} = E\overline{\pi^{k}} - \frac{Cov(\overline{\pi^{k}}, \overline{X^{M}})}{T^{M}\iota^{M}} = E\overline{\pi^{k}} - \frac{Cov(\overline{\pi^{k}}, \overline{Y})}{T^{N}\iota^{N}}$$

because $\overline{X_s^{M}} - E\overline{X^{M}} = (\overline{Y_s} - E\overline{Y}) \dfrac{T^{M}}{T^{N}\iota^{N}}$ by (A.33)

(2) See Eq. (A.31)

(3) For each consumer $i=1, \cdots, N$, and firm j, $j=1, \cdots, M$,

$$\sum_{s=1}^{S} \frac{\partial EU^{i}}{\partial x_s^{i}} (-\Delta v^{j} + \Delta\overline{\pi_s^{j}}) = 0 \tag{A.36}$$

$$\Leftrightarrow \sum_{s=1}^{S} p_s[t^i-(\overline{x}_s^i-\overline{Ex}^i)]\left[-\Delta E\overline{\pi}^i+\frac{Cov(\Delta\overline{\pi}^j,\overline{X}^M)\iota^M}{T^M\iota^M}+\Delta\overline{\pi}_s^j\right]=0 \text{ by (A.22)}$$

$$\Leftrightarrow t^i\frac{Cov(\Delta\overline{\pi}^j,\overline{Y})}{T^N\iota^N} - Cov(\overline{x}^i,\Delta\overline{\pi}^j)=0$$

$$\because (\overline{Y}_s-E\overline{Y})\frac{t^i}{T^M\iota^M}-(\overline{x}_s^i-\overline{Ex}^i)=0 \qquad \text{by (A.33) and (A.34)}\blacksquare$$

Proof of Property Ⅳ. **4.** (Individual Rationality in MSE):
Suppose a managerial fee schedule in a firm j is such that $\alpha^j+\beta^j v^j-e^j=\overline{w}$. Denote $(\hat{x}^j, E\hat{x}^j, \hat{t}^j, \hat{\lambda}^j)$ $(\overline{x}^j, \overline{Ex}^j, \overline{t}^j, \overline{\phi}_1^j)$ are the j's decision as a non-manager and manager, respectively. Then, as a non-manager his/her expected consumption is

$$E\hat{x}^j = v\phi_0\theta^j+(-v+E\Pi)\hat{\phi}_1\theta^j+(-\overline{w}+\delta E\overline{Y})\hat{\lambda}^j+\overline{w}$$

$$= v\phi_0\theta^j+\frac{Cov(\overline{Y},\Pi)}{T^N\iota^N}\hat{\phi}_1\theta^j+\frac{Cov(\overline{Y},\overline{Y})}{T^N\iota^N}\delta\hat{\lambda}^j+\overline{w}$$

by Corollary IV. 1

$$= v\phi_0\theta^j+\frac{Cov(\overline{Y},\overline{Y})}{(T^N\iota^N)^2}\hat{t}^j+\overline{w}$$

because $\dfrac{\partial EU^j}{\partial\lambda^j}=0$

$$\Leftrightarrow (-\overline{w}+\delta E\overline{Y})\hat{t}^j-[Cov(\overline{Y},\Pi)\hat{\phi}_1\theta^j+Cov(\overline{Y},\overline{Y})\hat{\lambda}^j\delta]=0$$

therefore
$$\hat{t}^j\equiv c^j-E\hat{x}^j=\frac{c^j-v\theta_0^j-\overline{w}}{1+\dfrac{Cov(\overline{Y},\overline{Y})}{(T^N\iota^N)^2}} \tag{A.37}$$

· The manager's expected consumptions

$$\overline{Ex}^j = v\phi_0\theta^j+(-v+E\Pi)\overline{\phi}_1\theta^j+\alpha^j+\beta^j v^j$$

$$= v\phi_0\theta^j+\frac{Cov(\overline{Y},\Pi)}{T^N\iota^N}\frac{\iota^M T^M\Theta^{-1}\theta^j}{T^M\Theta^{-1}\iota^M}+\alpha^j+\beta^j v^j$$

by Corollary IV. 1

$$= v\phi_0\theta^j+\frac{Cov(\overline{Y},\overline{Y})}{T^N\iota^N}\frac{(1-\Lambda\iota^{NM})}{T^M\Theta^{-1}\iota^M}\overline{t}^j+\alpha^j+\beta^j v^j$$

$$= v\phi_0\theta^{\cdot j} + \frac{Cov(\overline{Y}, \overline{Y})}{(T^{N}\iota^{N})^2}\overline{t^j} + \alpha^j + \beta^j v^j \qquad \text{by (A.32)}$$

$$\Leftrightarrow \overline{t^j} \equiv c^j - E\overline{x^j} - e^j = \frac{c^j - v\theta_0^{\cdot j} - \alpha^j - \beta^j v^j + e^j}{1 + \frac{Cov(\overline{Y}, \overline{Y})}{(T^{N}\iota^{N})^2}} \qquad \text{(A.38)}$$

Therefore

$\overline{t^j} = \hat{t}^j$ if only if $\alpha^j + \beta^j v^j - e^j = \overline{w}$.

$$\Leftrightarrow \overline{x_s^j} - e^j = E\overline{x^j} + e^j + (\overline{Y_s} - E\overline{Y})\frac{\overline{t^j}}{T^{N}\iota^{N}}$$

$$= E\hat{x}^j + (\overline{Y_s} - E\overline{Y})\frac{\hat{t}^j}{T^{N}\iota^{N}}$$

$$= \hat{x}_s^j$$

$$\Leftrightarrow EU^j(\overline{x^j}, e^j) = EU^j(\hat{x}_s^j, 0)$$

■

APPENDIX 3.

Proof of Theorem 1:

If $(\overline{x}, \overline{y}, \overline{e}, \overline{l_D}, \overline{\phi_1}, \overline{\lambda}, \overline{\mu})$ is a MSE relative to (ϕ_0, θ), then there exist a vector of stock price v, fixed wage $\overline{w}$, and an index of state-dependent wage δ such that MSE 1)-4) holds.

Define $\theta_0 = \phi_0\theta$, and $\overline{\theta_1} = \overline{\phi_1}\,\theta$. Then

i) for each manager $j = 1, \cdots, M$ in capitalist stock market economy, $(\overline{x^j}, \overline{\theta_1^{\cdot j}}, \overline{y^j}, \overline{e^j}, \overline{l_D^j})$ is a solution to his/her utility maximization problem (CSE. P. 1).

o $\overline{\theta_1} = \overline{\phi_1}\,\theta$ is optimal for consumers.

$$\overline{\phi_1}\,\Theta^{M} = \frac{\iota^{M}\,T^{M}\Theta^{-1}}{T^{M}\Theta^{-1}\iota^{M}}\,\Theta^{M} \qquad \text{by (A.26)}$$

$$= \frac{\iota^{M}\,T^{M}}{(1 - \Lambda\iota^{N\cdot M})\,T^{N}\iota^{N}} \qquad \text{by (A.32)}$$

$$= \overline{\theta_1^{M}} \qquad \text{by (A.6)}$$

$$\overline{\phi_1}\,\Theta^{N\cdot M} = \frac{\iota^{M}\,T^{M}\Theta^{-1}}{T^{M}\Theta^{-1}\iota^{M}}\,\Theta^{N\cdot M} \qquad \text{by (A.26)}$$

$$= \frac{\iota^{M}}{1 - \Lambda\iota^{N\cdot M}}\left[\frac{T^{N\cdot M}}{T^{N}\iota^{N}} - \Lambda\right] \qquad \text{by (A.35)}$$

$$= \overline{\Theta_1^{N \cdot M}} \qquad \text{by (A.18)}$$

· If effort level is same as MSE manager's ($e^j = \overline{e^j}$), then his/her consumption in each state is the same as MSE manager's.

−Same expected consumption

$$Ex^j = v\theta_0^{ij} + (-v + E\Pi)\overline{\theta_1^{ij}} + \overline{\beta^j} v^j$$

$$= v\theta_0^{ij} + (-v + E\Pi)\overline{\phi_1}\theta_1^{ij} + \overline{\beta^j} v^j \qquad \text{by construction } \overline{\theta_1^{ij}} = \overline{\phi_1}\theta^{ij}$$

$$= \overline{Ex^j}$$

⇔ CSE manager has same absolute risk tolerance as MSE manager and same consumption in each state $s=1, \cdots, S$ by linear risk sharing rule.

· Given same managerial fee schedule, if $\theta_0^{ij} = \phi_0^j \theta^{ij}$, the first order conditions with respect to production plan (e^j, l_D^j) are the same and, effort level, labor demand, and production plan are the same.

ii) for each non-manager $i = M+1, \cdots, N$, x_s^i the consumption in each state $s=1, \cdots, S$ is the same as in CSE.

- identical to proof for manager.

iii) for each firm $j=1, \cdots, M$, the objective function, the Individual Rationality and Incentive Compatibility are the same, and managerial fee schedule is the same as in MSE.

iv) and market clears. ■

Proof of Theorem 2:

Define (ϕ_0, θ) such that $\theta_0^{ij} = \phi_0^j \theta^{ij}$ and $\theta_0 = \phi_0 \theta$. Then we have same IC, IR, and consumption in MSE as in CSE. The detailed proof is identical to that of Theorem 1. ■

REFERENCES

Bardhan, Pranab, 1990. "Some Reflections on Premature Obituaries of Socialism", *Economic and Political Weekly* 25. 5, 259-262.

_______, 1991. "Risktaking, Capital Market, and Socialism", UC Berkeley Working Paper 91-154.

Borch, Karl, 1968. "General Equilibrium in the Economics of Uncertainty", pp. 247-258 in *Risk and Uncertainty*, K. Borch and J. Mossin (ed.), London, MacMillan.

Borensztein, Eduardo, and M. S. Kumar, 1991. "Privatization in Eastern Europe", *IMF Staff Papers*, 38. 2, 300-326.

Brus, Wl̂odzimierz, and K. Laski, 1988. *From Marx to the Market: Socialism in Search of an Economic System*, Oxford, Clarendon Press.

Cohen, G. A., 1986 a. "Self-Ownership, World-Ownership, and Equality Part I", F. Lucas (ed) *Justice and Equality Here and Now*, Cornell University Press.

_______, 1986 b. "Ditto: Part II", *Social Philosophy and Policy*.

Drèze, Jacques H.,1974, "Investment Under Private Ownership: Optimality, Equilibrium and Stability", *Allocation under Uncertainty: Equilibrium and Optimality*, New York: Halsted Press, 129-166.

_______, 1987. "Decision Criteria for Business Firms", pp. 298-320 *in Essays on Economic Decisions under Uncertainty*, J. H. Drèze (ed.). Cambridge, Cambridge University Press.

_______, 1989 a. "The Role of Securities and Labour Contracts in the Optimal Allocation of Risk Sharing", *Risk, Information and Insurance: Essays in the Memory of Karl Borch*, H. Louverg (ed.), Kluwer.

_______, 1989 b. *Labor Management, Contract and Capital Market*, New York, Basil Blackwell.

Fama, Eugene F., 1970. "Efficient Capital markets: A Review of Theory and Empirical Work", *Journal of Finance XXV*, No. 2.

_______, 1980. "Agency Problems and the Theory of Firm", *Journal of Political Economy* 88. 2, 288-307.

Geanakoplos, John, 1990. "An Introduction to General Equilibrium with Incomplete Asset Markets", *Journal of Mathematical Economics* 19, 1-38.

Geanakoplos, J., M. Magill, M. Quinzii, and J. Drèze, 1990. "Generic Inefficiency of Stock Market Equilibrium when Markets are Incomplete", *Journal of Mathematical Economics* 19, 113-151.

Grossman, Sanford J. and O. D. Hart, 1979. "A Theory of Competitive Equilibrium in Stock Market Economies", *Econometrica* 47. 2, 293-329.

_______, 1983. "An analysis of the Principal-Agent Problem", *Econometrica* 51. 1, 7-45.

Harris, Milton and A. Raviv, 1979. "Optimal Incentive Contracts with Imperfect

Information", *Journal of Economic Theory* 20, 231-259.

Holmström, Bengt, 1979. "Moral Hazard and Observability", *Bell Journal of Economics* 10, 74-91.

Jensen, Michael C. and W. H. Meckling, 1976. "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure", *Journal of Financial Economics* 3, 305-360.

Lipton, David, and J. Sachs, 1990. "Privatization in Eastern Europe: The Case of Poland", *Brookings Papers on Economic Activity* 2, 293-341.

Makowski, Louis, 1983. a "Competitive Stock Markets", *Review of Economic Studies* L, 305-330.

_______, 1983. b "Competition and Unanimity Revisited", *American Economic Review* 73. 3, 329-339.

Manne, H. G., 1965. "Mergers and the Market for Corporate Control", *Journal of Political Economy* 73, 110-120.

Mossin, Jan, 1977. *The Economic Efficiency of Financial Markets*, Lexington, Toronto.

Nuti, D. M., 1987. "Financial Innovation under Market Socialism", *European University Institute Working Paper* No. 285

_______, 1988. "Competitive Valuation and Efficiency of Capital Investment in the Socialist Economy", *European Economic Review* 32, 459-464.

_______, 1989. "Remonetization and Capital Markets in the reform of Centrally Planned Economies", *European Economic Review* 33, 427-438

Ortuño-Ortin, I., J. E. Roemer, and J. Silvestre, 1990. "Market Socialism", UCD Working Paper #355 1990.

Roemer,J., February 1990. "The Possibility of Market Socialism", mimeo.

_______, July 1992. "A Proposal for Denationalizing the State Sector when Pollution is an Issue", mimeo.

Ross, Stephen, 1973. "The Economic Theory of Agency: The Principal's Problem", *American Economic Review* 63, 134-139.