

INTERNATIONAL TRADE IN ASSETS UNDER UNCERTAINTY; A THEORETICAL ANALYSIS

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In the recent article Svensson (1988) developed a theory of the determinants of international trade in real assets under uncertainty. This paper examines how differences between countries with regard to preference, and perceived returns to savings determine autarky asset price differences, and consequently the trade pattern in risky assets. In this paper we concentrate on the effect of differences in the degree of relative risk aversion, the elasticity of substitution and the rate of time preference on the trade in risky asset among countries. To analyze this, we extend Svensson's model by using a generalized utility function developed by Epstein (1987). Some clear and general interpretations not expected from a conventional additive preference structure can be made.

1. INTRODUCTION

The analysis of international trade in assets provides a standard to understand the prospects for a country's present and future international asset holdings. Svensson (1988) developed a simple theory of the determinants of international capital flow in the absence of money. It may thus be regarded as a counterpart to the real determinants of international trade. His results were derived within the law of comparative advantage by interpreting commodities as time commodities, such as, real assets. He suggested the following form: $(q_i - q_i^*) Z_i \geq 0$, where i = risk free asset or risky asset, q_i is an asset price, and Z_i denotes net import of asset from abroad. It states that, like the principle of comparative advantage, the home country will import assets whose autarky prices are higher than the foreign country.

Svensson then introduced the general asset-pricing theory developed by Lucas (1978), and compared relative asset prices between countries based upon

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the above form. However, Svensson used the expected utility theory and Selden's preference. It is well known that Selden's preference is intertemporally inconsistent. To avoid these problems, we employ a recursive utility function that is of recent origin. Some clear interpretation not expected from a conventional additive preference structure will be presented.

In section 2, the Svensson result with respect to the effect of the difference in the degree of risk aversion on trade in asset is to be generalized using the conventional expected utility function. To examine fully the role of the factor on international trade patterns, we will use the generalized recursive utility structure in section 3. Its structure has the advantage of separating the two key parameters: degree of risk aversion and elasticity of substitution.

2. DIFFERENCE IN RISK AVERSION AND EXPECTED UTILITY FUNCTION

We would like to consider differences in risk aversion across countries. The economy described here is a variant of the Mehra and Prescott's (1985) pure exchange model. We assume that the growth rate of consumption is independently and identically distributed (henceforth i.i.d.).

Suppose that the additively separable utility function for a representative agent in the two countries is of the constant relative risk aversion class,

$$U(c) = \frac{c^{1-\alpha}}{1-\alpha} \quad \alpha > 0$$

which produces a homothetic indifference mapping. The parameter α is the coefficient of relative risk aversion. The behavior of an agent allocates his resources in each period between consumption, (c), and saving in the form of stocks. A share of stock pays a dividend (y) in period t and has a price of (p) at the end of period t . Let (s) be the number of shares of stock the consumer holds at the end of period t : we assume that all of the agent's disposable resources come from his stock holdings. Then the budget constraint of the agent is

$$(p + y)s = c + ps_{t+1}$$

Here we assume that the growth rate in perishable output y , i.e., x , is i.i.d.; that is,

$$\frac{\tilde{y}_{t+1}}{y_t} = \tilde{x}_{t+1} \sim i.i.d.$$

The random variable x is observed at the beginning of the period t , at which dividend payments are made. Because of the assumption of i.i.d. for x 's, knowledge of x is enough to fix completely the stochastic evolution of all future out-

put. Since the utility function is homogeneous and the growth rate of output is i.i.d., the equity price may be written as

$$p_t = p(x_t, y_t) = wy \quad (1)$$

where w is a constant coefficient. If we substitute (1) into the first order condition for the maximization problem, we have, using the fact that in an equilibrium period (t) perishable output should be consumed during that period ($c_t = y_t \forall t$),

$$p(y_t) = \beta E \left[\left(\frac{\tilde{y}_{t+1}}{y_t} \right)^{1-\alpha} (p(\tilde{y}_{t+1}) + \tilde{y}_{t+1}) \right]$$

$$\text{and } w = \beta E[(\tilde{x}_{t+1})^{1-\alpha} (1+w)], \quad (2)$$

The perceived gross return to savings may be written as

$$\tilde{r} = \frac{\tilde{p}_{t+1} + \tilde{y}_{t+1}}{y_t} = \frac{w+1}{w} \tilde{x}$$

Since the growth rate of return is i.i.d., this equation can be expressed without subscripts,

$$\tilde{r} = \frac{w+1}{w} \tilde{x}$$

and, after taking the expectation,

$$E(\tilde{r}) = \frac{w+1}{w} E(\tilde{x})$$

This is a new version of rate of return of risky asset (r). Substituting this into (2) yields

$$E(\tilde{r}) = \frac{E(\tilde{x})}{\beta E(\tilde{x}^{1-\alpha})}$$

Our next step is to derive a gross rate of return for the risk free asset r_f . From

$$\text{equation (2), it is clear that } r_f = \frac{1}{\beta E(\tilde{x}^{1-\alpha})}.$$

Since p_{t+1} is any random process that obeys $E p_{t+1} = p_t$ (that is a "martingale"), and the dividend is i.i.d., clearly the asset price has an inverse relation to the gross rate of return. $q \equiv 1/R$. Now we are able to examine trade patterns by looking at the differences in autarky prices of risky asset and risk free asset.

The differences are given by

$$q - q^* = \frac{\beta E(\bar{x}^{1-\alpha})}{E(\bar{x})} - \frac{\beta^* E(\bar{x}^{*1-\alpha^*})}{E(\bar{x}^*)} \quad (3)$$

$$q_i - q_i^* = \beta E(\bar{x}^{-\alpha}) - \beta^* E(\bar{x}^{*- \alpha^*}) \quad (4)$$

We would like to know under what conditions these differences are positive or negative. We now assume that the two countries differ only with respect to the measure of a constant relative risk aversion.

Let us take the Taylor expansion of $E(\bar{x}^{-\alpha})$, around the mean of random variable (x). Next we take the Taylor expansion around α since it holds that $\alpha = \alpha^* + \Delta\alpha$. Then (for details, see APPENDIX)

$$q - q^* = \beta \{ -\bar{x}^{-\alpha^*} \log \bar{x} + \frac{1}{2} (2\alpha^* - 1) \bar{x}^{-\alpha^* - 2} \text{var}(x) \} \Delta\alpha \leq 0 \quad (5)$$

$$q_i - q_i^* = \beta \{ -\bar{x}^{-\alpha^*} \log \bar{x} + \frac{1}{2} (2\alpha^* - 1) \bar{x}^{-\alpha^* - 2} \text{var}(x) \} \Delta\alpha \leq 0 \quad (6)$$

Equation (5) and (6) can be simplified as

$$q - q^* = H(\alpha^*) \Delta\alpha$$

$$q_i - q_i^* = I(\alpha^*) \Delta\alpha$$

If $H > 0$ and $\Delta\alpha > 0$, then $q - q^* > 0$. Since $\Delta\alpha > 0$ implies that the home country is more risk averse, the home country imports risky assets. And if $I > 0$ and $\Delta\alpha > 0$, in the case of the risk free asset, the home country imports the risk free asset. In order for H and I to simultaneously have a positive and a negative sign, respectively, (or vice versa) at the same time, which means that the home country imports risky asset and exports risk free asset (or vice versa), it must hold that¹⁾

$$-\frac{1}{2} \text{var}(x) < -\log \bar{x} + \alpha \text{var}(x) < \frac{1}{2} \text{var}(x)$$

Since all parameters in the above equations except for risk aversion can be observable, it would be easy to analyze the trade pattern of assets by using plausible values of the risk aversion parameters. This result is an extension of Svensson's (1988) result in which risky assets are exported when the home country is more risk averse. Svensson's result holds when $H < 0$ and $\Delta\alpha > 0$ as

1) From equation(5) and (6), let A be $(\log \bar{x} - \alpha^* \bar{x}^{-2} \text{var}(x))$ and B be $(\frac{1}{2} \bar{x}^{-2} \text{var}(x))$. When $A+B > 0$ in equation(5), then it follows that $A-B < 0$ in equation(6). Therefore, to get a different sign, it should be $-B < A < B$.

in our results. This case hold when the average growth rate of output(\bar{x}) is high and the variance of it is small. Then the home country will export risk assets because $q - q^* < 0$.

We can also derive results from the effect of differences in (i) growth rate (x), (ii) rate of time preference (β), (iii) variance of growth rate, etc. For these effects, you can refer to Svensson (1988).

3. GENERAL STRUCTURE OF UTILITY FUNCTION AND ASSET PRICES

In this section a more general form of preference function developed by Epstein (1987) is discussed. It is important to note that there are two reasons regarding our motivations for investigating this kind of preference function.

One reason is that representative agent optimizing models based on expected utility preferences have not performed well empirically (Mehra and Prescott (1985), etc.). One explanation often suggested for this poor performance is that the maintained specification of preference is too rigid. Indeed, in the case of the common homogeneous specification, the elasticity of substitution and the risk aversion parameters are reciprocals of one another.

A second reason is that because the elasticity of substitution is the crucial factor in determining the effect of risk aversion on savings under uncertainty, we should look at its effect by introducing the general structure. However, it is known that estimates of the elasticity of substitution using expected utility would not be efficient.

The economic environment in which a representative agent decides his consumption program over time is the same as before except for the structure of the utility function with one risky asset for simplicity. To derive the Euler equation using such a different type of utility function, additional techniques are needed. For a formal treatment, you can refer to Epstein and Zin(1990).

Two key assumptions underlying the specification of utility are that the agent (i) computes a certainty equivalent of this random future utility, and (ii) combines it with current consumption (C) via an aggregator function W , to compute utility at $t=0$. The functional structure of the utility function is, therefore, defined by W and μ μ is a certainty equivalent of random future utility (V_1). Consequently, this structure contains two parameters: W defines an ordering of deterministic path of consumption and μ embodies a risk aversion. The recursive structure is

$$\text{Max } W(C_0, \mu(V_1)) \quad (7)$$

If W is CES specification, then

$$W(c, z) = \{c^\rho + \beta z^\rho\}^{\frac{1}{\rho}} \quad c, z \geq 0, 0 \neq \rho < 1, 0 < \beta < 1$$

which is an intertemporal CES utility function with elasticity of substitution $\sigma = (1-\rho)^{-1}$. Thus we interpret ρ as reflecting a substitution. We also make an assumption regarding the shape of μ . Epstein(1987) suggests that the certainty equivalent is α mean $\mu^{\frac{1}{\alpha}}$

$$\mu(\tilde{m}) \equiv (\tilde{E}m)^{\frac{1}{\alpha}}.$$

Here α may be interpreted as a relative risk aversion parameter with the degree of risk aversion increasing as α falls. Epstein defined the relative degree of risk aversion under equation (7) as $(1-\alpha)$. However, we can intuitively see the effect of risk aversion on the determination of the asset price with only the new parameter ρ , without reference to any specific structure of the certainty equivalent.

The 'new' Euler equation is given by

$$\beta^{\frac{1}{\rho}} \mu \left[\left(\frac{C_{t+1}}{C_t} \right)^{\frac{\rho-1}{\rho}} \tilde{R}_t^{\frac{1}{\rho}} \right] = \beta^{\frac{\alpha}{\rho}} E \left[\left(\frac{C_{t+1}}{C_t} \right)^{\frac{\alpha(\rho-1)}{\rho}} \tilde{R}_t^{\frac{\alpha}{\rho}} \right] = 1 \quad (8)$$

This equation can be generally expressed in the familiar form by employing the fact that in equilibrium $C_t = y_t$ and $\tilde{r}_t = \frac{p_{t+1}}{p_t} + y_t$. We assume again that the growth rate in y_t , i.e., x_t , is taken to be i.i.d. Taking the same way that we obtained equation (2), we get the difference in equilibrium price equations for risky asset as follows

$$\begin{aligned} q - q^* &= \frac{\beta \mu^{\rho}(\tilde{x})}{E(\tilde{x})} - \frac{\beta^* \mu^{\rho}(\tilde{x}^*)}{E(\tilde{x}^*)} \\ &= \frac{\beta E(\tilde{x}^{\frac{\rho}{\alpha}})^{\frac{\rho}{\alpha}}}{E(\tilde{x})} - \frac{\beta^* E(\tilde{x}^{*\frac{\rho}{\alpha}})^{\frac{\rho}{\alpha}}}{E(\tilde{x}^*)} \end{aligned} \quad (9)$$

We would like to know again under what conditions this difference is positive or negative. We now assume that the two countries differ only with respect

2) Therefore, the generalized recursive structure becomes

$$W(C_t, \tilde{C}) = \{C_t^{\frac{\alpha}{\rho}} + \beta(EU_t^{\frac{\alpha}{\rho}})^{\frac{1}{\alpha}}\}^{\frac{\rho}{\alpha}}$$

When $\alpha = \rho$, the structure specializes to the familiar expected utility specification

$$U = [E \sum_{t=0}^{\infty} \beta^t C_t^{\frac{\rho}{\alpha}}]^{\frac{1}{\rho}}$$

In this case, we are indifferent to the way in which uncertainty about consumption is resolved over time in the sense of Kreps and Porteus(1978). An early(late) resolution of uncertainty is preferred if $\alpha < (>) \rho$.

to the measure of constant relative risk aversion and elasticity of substitution.³

Let us assume that the μ in the home country and μ^* in the foreign country have different cumulative distribution functions F and F^* , respectively. We assume that F dominates F^* , $F >_1 F^*$, according to the first order stochastic dominance (the same expected value but less noise in the home country). It implies that the home country has more optimistic probability beliefs about output than the foreign country. We also assume that the elasticity of substitution in the home country is larger than the foreign country, $\frac{1}{1-\rho} > \frac{1}{1-\rho^*}$ ($= \rho > \rho^*$). Then possible cases are,

1. $q > q^*$ if $\sigma > \sigma^* > 1$
2. $q > q^*$ if $\sigma > 1 > \sigma^*$
3. $q > q^*$ if $1 > \sigma > \sigma^*$

In what follows, we shall concentrate on the first case (1); the other examples can be analyzed in a similar fashion. Since $F >_1 F^*$, optimism regarding future dividends in the home country will increase the perceived return to savings. The substitution effect dominates more in the foreign country because σ is larger than σ^* , and both are greater than one. (If $\sigma < 1$, income effect dominates.) Since there is no transfer, this effect increases the asset price in the home country in order to restore equilibrium since it enhances demand for securities. Therefore, the home country tends to import the risky asset.

Next we can consider the effect on equilibrium prices of an increase in the degree of risk aversion. We also assume that $F >_1 F^*$, $\rho > \rho^*$, and the home country is less risk averse than the foreign country, $(1-\alpha) < (1-\alpha^*)$.

The intuitive results can be obtained without reference to the specific form of the certainty equivalent. Above arguments regarding the substitution effect and income effect are used for deriving the results again. For a given equilibrium asset price equation, a decrease in risk aversion i.e., a decrease in $(1-\alpha)$ acts to increase the certainty equivalent return to savings. The effect on behavior is similar to the consequence of a higher rate of return in a deterministic model.

We can then list several cases.

1. $q > q^*$ if $\alpha > \alpha^*$ and $\sigma > \sigma^* > 1$
2. $q > q^*$ if $\alpha > \alpha^*$ and $\sigma > 1 > \sigma^*$
3. $q > q^*$ if $\alpha > \alpha^*$ and $1 > \sigma > \sigma^*$

In case (1), the home country has a more favorable situation and is less risk

3) Epstein (1988) analyzed the sign of equation (9) in a partial approach between individuals.

averse. Also the elasticity of substitution in the home country is higher than the foreign country. The dominant substitution effect through a higher rate of perceived return implies enhanced future consumption in the home country and more demand for securities than in the foreign country. Thus, asset prices are forced to rise. It follows that the home country imports the risky assets.

These results are an extension of the expected utility model; but the limited flexibility of the expected model precludes as clear and sharp an interpretation as the above model. We should note that the restrictive assumptions i.e., a homogeneous utility function and i.i.d. are used for obtaining such extended results.

4. CONCLUSION

We have analyzed a theory of the determinants of trade pattern in assets focusing on the difference in the major parameters such as the degree of risk aversion and elasticity of substitution. For this, this paper adopted a generalized utility function. We realize from our results that the new preference structure can be easily applied to analyze the trade pattern in assets. Obviously, the approach and its results can also be interpreted in terms of trade in risky assets between individuals.

Without the use of money, this approach has been restricted to a real asset trade. If money is included in this approach, exchange risk and monetary policy will affect the trade pattern in assets as described in economics textbooks.

For the completion of our analysis, our next research is to explore, and to estimate key preference parameters by the Generalized Method of Moment (GMM), whether the model and its results derived above have implication for time series data from several countries, and then test our predictions about the trade pattern in asset among countries using the cross-sectional evidence .

APPENDIX

Let us approximate $E(\tilde{x}^{-\alpha})$, by a Taylor series expansion around the mean of the random variable. The formulation is,

$$\begin{aligned} f(x) &\approx f(E(\tilde{x})) + f'(E(\tilde{x}))(x - E(\tilde{x})) + \frac{1}{2}f''(E(\tilde{x}))(x - E(\tilde{x}))^2 \\ \Rightarrow E(f(\tilde{x})) &= f(E(\tilde{x})) + \frac{1}{2}Var(\tilde{x})f''(E(\tilde{x})) \end{aligned}$$

The first term of (3) can be approximately expressed by

$$\begin{aligned} q - q^* &= \beta \left[\{ \bar{x}^{-\alpha} + \frac{1}{2}\alpha(\alpha-1)\bar{x}^{-\alpha-2}Var(x) \} - \{ \bar{x}^{-\alpha^*} + \frac{1}{2}\alpha^*(\alpha^*-1)\bar{x}^{-\alpha^*-2} \right. \\ &\quad \left. Var(x) \} \right] \end{aligned}$$

We assume that the home country is more risk averse, i. e., $\alpha = \alpha^* + \Delta\alpha$, $\Delta\alpha > 0$. Next we take a Taylor expansion around α^* , that is

$$\begin{aligned} q - q^* &= \beta \left[-(\log \bar{x})\bar{x}^{-\alpha^*}\Delta\alpha + \frac{1}{2} \{ (\alpha^*(\alpha^*-1) + (2\alpha-1)\Delta\alpha \} \right. \\ &\quad \left. \{ \bar{x}^{-\alpha^*-2} - (\log \bar{x})\bar{x}^{-\alpha^*-2}\Delta\alpha \} Var(x) - \{ \frac{1}{2}\alpha^*(\alpha^*-1)\bar{x}^{-\alpha^*-2} Var(x) \} \right] \\ q - q^* &= \beta \left[-(\log \bar{x})\bar{x}^{-\alpha^*}\Delta\alpha + \frac{1}{2} \{ (\alpha^*(\alpha^*-1) + (2\alpha-1)\Delta\alpha \} \right. \\ &\quad \left. \{ \bar{x}^{-\alpha^*-2} - (\log \bar{x})\bar{x}^{-\alpha^*-2}\Delta\alpha \} Var(x) - \{ \frac{1}{2}\alpha^*(\alpha^*-1)\bar{x}^{-\alpha^*-2} Var(x) \} \right] \\ &\approx \{ -(\log \bar{x})\bar{x}^{-\alpha^*} + \{ \frac{1}{2}(2\alpha^*-1)Var(x)\bar{x}^{-\alpha^*-2} \} \Delta\alpha \end{aligned}$$

In addition, we can derive equation(4) in the same way.

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