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ON THE EMPIRICAL RELEVANCE OF THE AGGREGATION PROBLEM OF THE INPUT OF CAPITAL STOCK IN PRODUCTION FUNCTION*

CHEOL-HO CHA**

1. INTRODUCTION : PURPOSE OF THE STUSY

Most empirical studies of production relationships have been based on aggregate indexes of capital and labor inputs. One of the commonly maintained hypotheses in empirical production function studies has been that the aggregate inputs of labor and capital are homogeneous. The progressive sophistication of the theory of production has inevitably generated dissatisfaction with the treatment of labor and capital as homogeneous inputs and has stimulated attempts to disaggregate them.

There are two problems in dealing with aggregation. One is to reduce by aggregation various types of inputs(or commodities) to a set of inputs with distinctive characteristics in a production function (or utility function). The other problem is the case where a single macro-relation is derived by aggregation from a set of micro-relations (e.g., derivation of an aggregate production function from individual firm's production function, and derivation of the welfare function from individual consumer's utility function). It should be noted that we discuss only the first problem; i. e., the aggregation problem of inputs in the context of an aggregate production function. The discussion of the problem of aggregation of microrelations into a macro-relation is beyond the scope of this study.

Capitial stock as an aggregate consists of various kinds of machines and bulidings at different stages in their life cycles.

The problem of disaggregation of a single(composite) factor into distinct types of inputs in a production function has been a controversial issue in current research about the interrelations among different types of production inputs.[1]

Examination of the literatire on functional relationships among economic varia-

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** Prof. of Economics, College of Social Science, Chung-Ang University.

1) For further discussions of the aggregation problem of micro-relations into a macro-relation, see R. G. D. Allen, *Mathematical Economics*, Second ed.(London : MacMillan Co., 1964), pp. 694~724; L. R. Klein, "Remarks on the Theroy of Aggregation," *Econometrica*, Vol. 14(1946), pp. 302~12; and H. Theil, *Linear Aggregation of Economic Relations* (Amsterdam : North-Holl and Publishing Co., 1954)

bles reveals that the "functional separability" condition must be satisfied in order to permit a disaggregation of the aggregate inputs(labor and capital) into a set of distinct inputs in a production function.[2] It has been further show that functional separability condition can be expressed equivalently in terms of the equality condition of the partial elasticities of substitution(PES) between the inputs in question(i. e., $\sigma_{ik}=\sigma_{jk}$ where i, j and k are the production inputs).[3]

The purpose of this study is to test the functional separability condition for the aggregate input of capital stock in order to provide empirical evidence about whether or not the aggregat input of capital stock adequately represent the sum of homogeneous inputs.[4]

2. REVIEW OF THE LITERATURE ON THE CAPITAL INPUT DISAGGREGATION AND SUBSTITUTABILITY

Sato[5] estimated the substitution possibilities between structures(K_i) and equipment(K_j) in the U. S. manufacturing industry(total) from 1929 to 1963, and obtained a $\sigma_{K_iK_j}$ estimate of 1.643. He concluded that

"··· the elasticity of substitution between structures and equipment is much leless than infinity.··· Substitutabity between different types of capital goods is not perfect. This fact makes it necessary to examine the conventional measure of capital stock which is obtained by adding up the stocks of heterogeneous capital goods' evaluated at some base-year prices.[6]

On the basis of this conclusion Sato funther emphasized the necessity of disaggregation the aggregate measure of capital stock into the two components, structures and equipment. In relation to the functional separability condition his conclusion implies that $\sigma_{K_iL} \neq \sigma_{K_jL}$.

In assesing shifts in the composition of capital goods, Boddy and Gort, Sato, dis-

2) As Denny and Fuss clearly stated : "the use of aggregate inputs for postulating a production relationship between output and inputs requires the assumption that the prodiction function is(functionally) separable in these aggregates. Yet until recently, (functional) separability and the existence of aggregate inputs were assumed a priori in virtually in all production function studies." See M. Denny and M. Fuss, "The Use of Approximation Analysis to Test for Separability and the Existence of Consistent Aggregates," *American Economic Review*, Vol. 67, No. 3(June 1977), p.404.

3) W. W. Leontief, "Introduction to a Theory of the Internal Structure of Functional Relationships," *Ecomnometrica*, Vol. 15, No. 4(October 1947), pp. 361~73; and W. W. Leontief, "A Note on the Interrelation of Subsets of Independent Variables of a Continuous Function with Continuous First Derivatives," *Bulletin of the American Mathematical Society*, Vol. 53, No. 4(1947), pp. 343~50.

4) E. R. Berndt and L. R. Christensen, "The Internal Structure of Functional Relationships : Separability, Substitutability and Aggregation," *Review of Economic Studies*, Vol. 40(July 1973), pp. 403~10; and C. Blackorby and R. R. Russell, "Functional Structure and the Allen Partial Elasticities of Substitution : An Application of Duality Theory," *Review of Economic Studies*, Vol. 43(June 1976), pp. 285~91.

5) K. Sato, "A Two-Level Constant Elasticity of Substitution Production Function," *Review of Economic Studies*, Vol. 34, No. 98(August 1967), pp. 201~18.

6) Ibid., p.207 and p.213.

cussed the necessity of disaggregation of capital stock into structures(K_i) and equipment(K_j).[7] They presented the estimated of 1. 72 for the elasticity of substitution between structures and equipment. They concluded that

"On empirical and theoretical grounds, we reject the assumption that all capital is one class vis-a-vis labor and find, instead, that the relative costs of labor and equipment influence the equipment-structures proportion."[8]

It should be pointed out, however, that Boddy and Gort have not performed an empirical test of the functional separability condition regarding disaggregation of the aggregate input of capital stock into structures and equipment. Instead they contended **a priori** that equipment is more eaily substituted for labor than when structures are substituted for labor. In other words they implicitly argued that the possibility of substituting equipment(K_j) for labor(L) seems to be greater than the possibility of substituting structures(K_i) for labor(L);[9] i. e. $\sigma_{K_iL} < \sigma_{K_jL}$.

Berndt and Christensen tested the functional separability condition of capital input regarding the disaggregation of the aggregate input of capital stock into structures and equipment in the U. S. manufacturing industry.[10] Assuming constant returns to scale and Hicks-neutral technical change for the U. S manufacturing industry(total) time-series data(1929-1968), they derived the following three semilogarithmic input demand equations from the translog function(TL) :

(2-1) $M_{K_i}=b_{K_i}+c_{K_iK_i}\ln K_i+c_{K_iK_j}\ln K_j+c_{K_iL}\ln L$

(2-2) $M_{K_j}=b_{K_j}+c_{K_iK_j}\ln K_i+c_{K_jK_j}\ln K_j+c_{K_jL}L$

(2-3) $M_L=b_L+c_{K_iL}\ln K_i+c_{K_jL}\ln K_j+c_{LL}\ln L$

where M_i is the cost share of the factor(i$=K_i$, K_j and L) in the total cost of producing the given level of output, Y, (i, e., $\partial\ln Y/\partial\ln K_i=M_{K_i}$), and b and c are parameters.

Berndt and Christensen then computed the PES which is not constrained to be constant but may vary with the values of the cost shares of the inpurs in question(e. g., M_i);[11] i. e., $\sigma_{ij}=\frac{c_{ij}}{M_iM_j}+1$ where i, j$=K_i$, K_j and L, i$\neq$j,

7) R. Boddy and M. Gort, "The Substitutuin of Capital for Labor," *Review of Economics and Statistics*, Vol. 53(May 1971), pp. 179~88.

8) Ibid., p. 185.

9) For an empirical estimation of the elasticity of substitution between equipment and labor, see C. W. Bischoff, "Hypothesis Testing and the Demand for Capital Goods," *Review of Economics and Statistics*, August 1969, pp. 354~68. Bischoff performed a two-input study of equipment and labor for the U. S. private economy based on the quarterly data of 1949~62. He estimated an investment function with distributed lags based on the CES production function. He then found that the elasticity of substitution between equipmenr and labor equals 1.023. He did not, however, provide an estimate of the elasticity of substitution between structures and labor.

10) E. R. Berndt and L. R. Christensen, "The Translog Function and the Substitution of Equipment, Structures and Labor in U. S. Manufacturing 1929~1968," *Journal of Econometrics*, Vol.1(March 1973), pp. 81~114.

11) For a proof of the derivation of the PES in the TL Function see H. P. Binswanger, "A Cost Function Approach to the Measurement of Elasticities of Factor Demand and Elasticities of Substitution," *American Journal of Agricultural Economics*, May 1974, pp. 379~80.

and c_{ij} is the parameter estimated from the three input demand equations (2-1), (2-2) and (2-3).

As shown Table 1 Berndt and Christensen's estimated PES were greater than unity. They concluded that

"··· Equipment and structures are more highly substitutable for each other than for labor. Our estimates of the equipment-sturcture of substitution range from 4.39 to 8.38(the mean value is 6.554) while the equipment-labor elasticities of substitution range from 1.22 to 1.79(the mean value is 1.43). We are unable to reject the hypothesis that the equipment-labor and structures-labor elasticaties are equal."[12]

The conclusion reached by Berndt and Christensen thus implies an aggregation of structures and equipment into a single measure of capital stock because their conclusions support the functional separability hypothesis that $\sigma_{K_iL} = \sigma_{K_jL}$.

Table 1. Berndt and Christensen's Estimates of the PES Among Structures, Equipment and Labor in U. S. Manufacturing Industry 1950~68 : Selected Years

PES	1950	1955	1960	1965	1968
σ_{K_iL}	1.494	1.619	1.657	1.727	1.789
σ_{K_jL}	1.305	1.278	1.270	1.252	1.228
$\sigma_{K_iK_j}$	5.616	6.828	7.189	7.704	7.751

Note : σ_{K_iL} = PES between structures and labor
σ_{K_jL} = PES between equipment and labor
$\sigma_{K_ik_j}$ = PES between structures and equipment

Sourece : Quoted from E. R. Berndt and L. R. Christensen, "The Translog Function and the Substitution of Equipment, Structures and Labor in U. S. Manufacturing 1929-1968," *Journal of Econometrics*, Vol. 1 (March 1973), p. 111, Statistical Appendix, Table A4.

In Table 2 we summarize briefly the results of the above mentioned studies undertaken by Sato, Boddy and Gort, Berndt and Christensen. We are particularly concerned with the implication of their conclusions for the functional separability condition.

First, Sato, and Boddy and Gort confirmed that their estimated elasticities of substitution between

structures and equipment are not infinite and therefore the substitutability between structures and equipment is not perfect. As a result, they argued for the necessity of disaggregation of the aggregate measure of capital stock into structures and equip ment. In terms of the functional separability condition this conclusion implies that $\sigma_{K_iL} \neq \sigma_{K_jL}$.

Second, Berndt and Christensen found the estimated elasticity of substitution between structures and equipment to be 6.554(a mean value), thereby indicating that structures and equipment are highly substituiable for each other. They concluded

12) E. R. Berndt and L. R. Christensen, "The Translog Function," p.83.

that the PES between structures(K_i)and labor(L) is equal to the PES between equipment(K_j) and labor(L); i. e., $\sigma_{K_iL}=\sigma_{K_jL}$

Third, the previous studies have provided results only for total manufacturing industry, rather than for individual manufacturing industries(e. g., two-digit manufacturing industries).

Consequently, it seems worthwhile to develop additional empirical evidence concerning the disaggregation of the aggregate input of capital stock and the substitutability of its components.

Table 2. Comparison of Estimations of the PES Among Structures, Equipment and Labor

Author, Year and Forms of Equation	Method	Data base	σ_{ij}	Implication of Results for the Separability Condition
Sato, 1967 : $\log(K_i/K_j)=\log(d_{K_i}/d_{k_j})+\sigma_K \log(P_{K_i}/P_{K_j})+u$	Multiple log linear regression : CES production function	U. S. Manufacturing industry (total), Time series 1929-63	$\sigma_{K_iK_j}=1.643$	$\sigma_{LK_i}\neq\sigma_{LK_j}$
Boddy and Gort, 1971 $\log(L/K_i)=a+b_1\log_{q_{k_i}} b_2\log_{q_{k_j}}+b_3\log w+u$	Multiple log linear regression	U. S. private business sector (total), time-series 1927-68	$\sigma_{K_iK_j}=1.720$	$\sigma_{LK_i}\neq\sigma_{LK_j}$
Berndt and Christensen, $M_{k_i}=b_{k_i}+c_{k_ik_i}\ln K_i+c_{k_ik_j}\ln K_j+c_{k_il}\ln L+u$ $M_l=b_l+c_{k_il}\ln K_i+c_{K_jl}\ln K_j+c_{ll}\ln L+u$	1973: Multiple semilog linear regression:3-input demand equations,(Translog production function)	U.S manufacturing industry (total), time-series 1929-68	$\sigma_{K_iK_j}=6.544$ $\sigma_{LK_i}=1.518$ $\sigma_{LK_j}=1.342$	$\sigma_{LK_i}=\sigma_{LK_j}$

Notes : 1) Definition of the variables for Sato's equation : K_i=dollar value of structures, K_j=dollar value of equipment, d_{k_i}=distribution parameter of K_i, P_{K_i} =price of capital, σ_K=elasticity of substitution between K_i and K_j, u=disturbance term

2) Definitions of the variables for Boddy and Gort's equation;
K_i=expenditure on structures, K_j=expenditure on equipment, w=money wage rate, P_{K_i}=unit service cost of new K_i, u =disturbance term

3) Definition of the variables for Berndt and Christensen's equations
K_i=quantity of structures, K_j=quantity of equipment, b, c =parameters L=amount of labor(man-hours), M_i=cost share of the ith factor, u=disturbance term

3. METHOD

3.1 Hypothesis and the Model

For testing the functional separability condition of an aggregate input of capital stock(K) with two categories of capital inputs, structures(K_1) and equipmenr(K_2), the hypothesis to be tested is:

$$H_0 : \sigma_{K1L} = \sigma_{K2L} \qquad H_1 : \sigma_{K1L} \neq \sigma_{K2L}$$

where σ_{ij} is the PES between the two inputs i and j.

For testing the hypothesis of the functional separability condition for the capital input with two categories of capital inputs(K_1 and K_2), We used the CRESH(constant ratios of elasticity of substitution and homothetic) production function. Then the following two equations(input demand equations) derived from the CRESH function will be used :

(3-1) $\log(L/K_1)=b_0+b_1 \log(r_1/w) + b_2 \log(V/K_1)+u_1$

(3-2) $\log(L/K_2)=b_0+b_3 \log(r_2/w) + b_4 \log(V/K_2)+u_2$

where V=value added
- K_1=capital structures
- K_2=capital equipment
- L=man-hours of all employees
- w=average wage rate per man-hour of L
- r_1=rate of return on capital structures
- r_2=rate of return on capital equipment
- b=parameters
- u=random disturbance term

3.2. Estimation Method

It seems reasonable to consider on a priori grounds that the disturbance terms(e. g., u_1 and u_2 in equation(3-1) and (3-2) are mutually correlated. [13] That is, since the two input demand equations will be estimated jointly in the same industry, the random deviations of the input demand equations from profit maximization(cost minimization) would affect both the labor and the capital inputs markets in the given industry [14]. Put differently, each input demand function can be explained by a separate equation, but since the two input demand functions are in the same industry, there may be random shock which affect both the labor and the capital inputs markets. [15]

13) The system of equations to be estimated are seemingly unrelated but are disturbance-related equations. See H. Theil, *Principle of Econometrics*(N. Y. : John Wiley and Sons, Inc., 1971), p. 298.

14) For a discussion of this problem, see D. B. humphrey and J. R. Moroney, "Substitution Among Capital, Labor and Naturnal Resource product in American Manufacturing," *Journal of Political Economy*, Vol. 83(February 1975), p. 65.

15) R. W. Bacon, "A Simplified Exposition of Seemingly Unrelated Regressions And Three Stage Least Squares," *Oxford Bulletin of Economics And Statistics*, Vol. 39(August 1974), p. 229.

In order to alleviate the effect of heteroscedastic disturbances in the linear model the GLS estimation method has generally been suggested.[16] An obvious practical difficulty in the application of the GLS estimator is that the true disturbance are not known. More specifically, if the variance-covariance matrix is unknown, the GLS estimator could not be employed. If the variance-covariance matrix of residuals is misspecified, the resulting estimates are not efficient. Moreover, the estimator of the variance-covariance matrix of the estimates of the regression coefficients is biased.[17] Consequently, as noted in the literature, the variance-covariance matrix can be calculated by using the almost unbiased estimator(hereafter called AUE) method introduced by Horn, Horn and Duncan.[18]. These authors showed that the AUE is consistent, has smaller mean square error than other methods,[19] and nonnegative estimates in the calculation of the variance-covariance matrix. The AUE is not unbiased, but its bias does vanish when the weights of sample variances are corrected and in this respect the method, sccording to the 'literature, is an almost unbiased estimator. For these reasons, we apply the AUE method for calculating the unknown variance-covariance matrix of the GLS estimator.[20]

Suppose, for example, that we consider a general linear model :

$$Y = X\beta + \varepsilon \qquad (1)$$

under the assumptions

$$E(\varepsilon) = 0$$
$$E(\varepsilon \varepsilon') = \sigma^2 V$$

where Y is the n × 1 vector of dependent variable, X is the n×m matrix of known independent variable, β is the m × 1 vector of unknown parameters, ε is the n × 1 vector of unknown residuals, and $\sigma^2 V$ is the n × m positive definite dispersion(variance-covariance) matrix of residuals. Then the GLS estimator is generally given by

$$b=(XV^{-1}X)^{-1}X'V^{-1}Y \qquad (2)$$

An obvious practical difficulty in the application of GLS estimator is that the true disturbances are not known. If V is unknown, the GLS estimator could not be employed.

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16) 16) A. S. Goldberger, "Best Linear Unbiased Prediction in the Generalized Linear Regression Model," Journal of the American Statistical Association, Vol. 57, No. 298(June 1962), pp. 369~77; and T. Amemiya, "GLS with an Estimated Auto-covariance Matrix," *Econometrica*, Vol. 41, No. 4(July 1973), pp. 723~32.

17) T. Amemiya, "Specification Analysis in the Estimation of Parameters of a Simultancous Equation Model with Autoregressive Residuals," *Econometrica*, Vol. 34, No. 2 (April 1966), pp. 283~306.

18) S. D. Horn, R. A. Horn and D. B. Duncan, "Estimating Heteroscedastic Variances in Linear Model," *Journal of the American Statistical Association*, Vol. 70, No. 350(June 1975), pp. 380~85.

19) For a Discussion of the Properties of Alternative Estimators of Heteroscedastic Variances, See S. D. Horn and R. A. Horn, "Comparison of Estimators of Heteroscedastic Variances in Linear Model," *Journal of the American Statistical Association*, Vol. 70, No. 352(Dec. 1975), pp. 872~79.

20) A computer program for the AUE estimation method is found in the RAL(Research Analysis of Language), IMF, Data Processing Division, 1985, V5, ML4, pp. 9. 52~9. 53. 4.

In this study, the variance-covariance matrix is calculated by using the method of the almost unbiased estimator (AUE) proposed by Horn, Horn and Duncan. Let us show how the AUE is calculated from the given linear model, (1),

$$U_i = Y_i - X_i' b = Y_i - \hat{Y}_i, \qquad i = 1, 2, \cdots n. \tag{3}$$

where $\hat{Y}_i = X_i' b$
$b = (X' X)^{-1} X' Y_i$
U_i = estimated residuals

Then, $Y_i = \hat{Y}_i + U_i$ and $(Yi - X_i'\beta) = (\hat{Y}_i - X_i'\beta) + U_i$
+ For simplicity this can be rewritten as

$$\varepsilon_i = e_i + U_i \tag{4}$$

where $e_i = \hat{Y}_i - X_i' \beta$
The variance of both sides of equation (4) is then

$$\sigma_i^2 = Var(e_i) + Var(U_i)$$

or Var $U_i = \sigma_i^2 - Var(e_i) = (1 - var(e_i) / \sigma_i^2)\, \sigma_i^2$
Thus $\sigma_i^2 = (1 - Var(e_i) / \sigma_i^2)^{-1} Var(U_i)$
Replacement of $Var(U_i)$ by its unbiased estimator U_i^2 an obtain the estimator

$$\hat{\sigma}_i^2 = (1 - h_i)^{-1} U_i^2 \tag{5}$$

as an estimator for σ_i^2, where h_i is the ratio of the variance of the its fitted value, $\hat{Y}_i$, to the variance of the its observed value, Y_i; $Var(e_i)/\sigma_i^2$.
Var(e_i) ccan be expressed as follows;

$$\begin{aligned} Var(e_i) &= Var(\hat{Y}_i - X_i'\beta) = var(X' b - X_i' \beta) \\ &= Var(X_i'(b - \beta)) \\ &= X_i' Var(b - \beta) X_i \end{aligned} \tag{6}$$

From the definition of b in the GLS estimator and form the assumption that the weights are correct, we obtain

$$Var(b - \beta) = (X' V^{-1} X)^{-1}$$

and hence,

$$Var(e_i) = X_i' (X' V^{-1} X)^{-1} X_i \tag{7}$$

Using W, a positive diagonal matrix to approximate V^{-1}

$$\hat{\sigma}_i^2 = (1 - X_i'(X' WX)^{-1} X_i w_i)^{-1} U_i^2 \tag{8}$$

where $W \equiv diag(w_1, \cdots, w_n)$; weighting matrix, and w_i are positive numbers.
The diagonal matrix of AUE, (8), is then used to form the GLS estimates. A computer program for this estimation method of the AUE is available in the RAL (Research anlysis of Language), IMF.

4. EMPIRICAL RESULTS

4. 1. Two-Digit Manufacturing Industries

From the results presented in Table 3, three points can be made :

First, based on the results of the t-test, the null hypothesis, $\sigma_{K1L}=\sigma_{K2L}$, rejected at the .01 significance level in total manufacturing industry and in all two-digit manufacturing industries, except for three industries-textile(# 22), clothing(# 23), and stone, clay and glass(# 32). In these three industries the null hypothesis of no significant difference between the two PES is accepted.

Table 3. Time-Series Estimates of the PES:Two-Digit Manufacturing Industries

Industry Code	σ_{K1L}	σ_{K2L}	t
Mfg. Total	.056	1.519	48.77
# 20	.183	.945	121.74
# 22	2.482	2.588	2.09*
# 23	5.481	5.613	1.01*
# 28	.063	1.657	204.62
# 29	1.109	.193	67.16
# 30	.026	-14.583	68.04
# 31	-.577	.221	53.92
# 32	.622	.632	2.35*
# 33	.261	.751	17.56
# 34	.477	.812	83.75
# 35	.115	.553	86.73
# 36	.276	-2.521	34.57
# 37	1.781	.943	30.04
# 38	.551	-2.027	82.46
# 24	.017	-1.083	174.18
# 25	-6.074	.918	73.60
# 26	-.222	3.466	73.17
# 27	-.357	1.415	23.98

-Notes : The industries under the dotted line violate the concavity condition
* Indicates that $\sigma_{K1L}=\sigma_{K2L}$ at the .01 significance level.

-Data sources : *Annual Survey of Manufactures*, Bureau of the Census, U. S. Department of Commerce, 1957-86;
Census of Manufactures, 1958, 1963, 1967, 1972, 1977, 1982. Bureau of the Census, U. S. Department of Commerce.

Second, the elasticity of substitutio substitution between equipment and labor is relatively higher than that between structures and labor(i. e., $\sigma_{K1L} < \sigma_{K2L}$) in total manufacturing industry and in most of the two-digit manufacturing industries, except for the petroleum and coal(# 29), rubber and plastics(# 30), electrical equipment(# 36), transportation equipment (# 37) and instruments(# 38) industries.

Third, the estimates of σ_{K1L} in one group of industries-e. g., textile(# 22), clothing

(# 23)-appear to be greater than unity (i. e., $\sigma_{K11} > 1$), while those in other groups of industries-e. g., primary metal(# 33), fabricated metal(# 34), and machinery(# 35)-appear to be less than unity(i. e., $\sigma_{K1L} < 1$). Similar results can also be found in the estimated of σ_{K2L}, and a very large and negative value for σ_{K2l} in the rubber and plastics industry(# 30) remains to be explained.

Based on the empirical results, three findings are discussed : First, we have found that the difference between the two PES, σ_{K1L} and σ_{K2L}, is statistically significant at the .01 significance level(i. e., $\sigma_{K1L} \neq \sigma_{K2L}$) in the total manufacturing industry and in most of two-digit manufacturing industries, except for the textile(# 22), clothing(# 23), and stone, clay and glass(# 32) industries. This test result seems to indicate that the two categories of capital inputs, structures and equipment, serve distinctive functions in the production process. Therefore, the internal structure of a production function involving two functional groups, labor and capital, should be expressed as:

$$Y = F(K_1, K_2 L)$$

where Y is output, K_1, and K_2 are structures and equipment, respectively, and L is assumed to be an aggregate input of labor.

The test results, on the other hand, show that the dufference between the two PES, σ_{K1L} and σ_{K2L}, is not significant(i. e., $\sigma_{K1L} = \sigma_{K2L}$) in the three industries mentioned above(# 22, # 23 and # 32). This implies, in terms of the functional separbility condition, that the composite measure of capital stock does represent the homogeneous characteristics of two different types of capital inputs, structures and equipment; i. e., $K=f(K_1, K_2)$.[21] No significant difference between σ_{K1L} and σ_{K2L} means that structures and equipment are highly substitutable for each other(i. e., $\sigma_{K1K2} > 1$). In this context, it may be interesting to observe the characteristics of these industries(# 22, # 23 and # 32) with reference to their relative values of σ_{K1K2}. Those industries, in which the null hypothesis, $\sigma_{K1L}=\sigma_{K2L}$, is accepted, appear to have relatively high etimates of the σ_{K1K2} except for the stone, clay and glass industry(# 32). That is, the estimates of $_{K1K2}$ are 1,343 for the etxtile(# 22) 2.260 for the clothing(# 23), and 0.226 for the stone, clay and glass(# 32) industies, respectively.

Seound, it was expected a priori that $\sigma_{K1L} < \sigma_{K2L}$. Higher substitutability between equipment and labor than that between structures and labor is not only theoretically plausible, but also intuitively applicable to the real world.

We have found that, with several exceptions, $\sigma_{K1L} < \sigma_{K2L}$ in the total manufacturing industry and in most of the manufacturing industires. This result may be due to the fact that equipment could be substituted for labor, in its motive and control functions of labor, more easily than structures. This result confirms Boddy-Gort's contention that "equipment is more easily substituted for labor than when structures are substituted for labor.[22]

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21) The compostite measure of capital stock may be referred to as the conventional measure of capital stock which is obtained by adding up the stocks of heterogeneous capital goods evaluated at some base-year prices.

22) R. Boddy and M. Gort, "The Substitution of Capital for Labor," p. 183.

Furthermore, a shift in the composition of capital stock between structures and equipment [23] can be explained by reference to the relative values of the PES, σ_{K1L} and σ_{K2L}. That is, in the industries in which the σ_{K2L} is relatively gigher than the σ_{K1L} the ratio of equipment(K_2) to structures(K_1) would rise(i. e., the capital equipment-intensity would increase) when the wage rate increases relative to the capital cost. [24] However, if different types of capital stock are treated as an aggregate input in the industrial production relations, there is no way to investigate the determinants of the shifts in the composition of capital stock between structures and equipment. [25]

Third, based on the values of the PES, the patterns of factor substitution can be categorized in two groups. One shows that $\sigma_{K1L} > 1$ in some of the consumer-oriented industries--e. g., textile(# 22) and Clothing (# 23). The other group of industries shows that $\sigma_{K1L} < 1$ in some of the producer-oriented industries--e. g., primary metals(# 33), fabricated metal (# 34) and machinery(# 35). However, in other consumer-oriented industries--e. g., food(# 20), rubber and plastics (# 30), the estimated of σ_{K1L} are less than unity, while they are greater than unity in other producer-oriented industries--e. g., petroleum and coal(# 29) and transportation equipment(# 37), (Similar variation among industries can also be found in the estimates of σ_{K2L}). No common pattern can be found in the two groups of industries. Therefore, no generalized conclusion can be derived from the variable factor substitution behavior among industries.

4.2 Comparison of the Results

It is of interest to examine the patterns of possible discrepancies in the values of PES over a variety of the functional forms. We compare our results with the results obtained from the Griliches model, the translog model(Berndt-Christensen's and Denny-Fuss' results), and others(Bischoff, Boddy-Gort, and Sato).

Table 5 shows a comparison of our results obtained from the CRESH model and the results presented by Berndt-Christensen, Bischoff, Boddy-Gort, and Sato.

Berndt and Christensen have concluded that structures and equipment are highly substitutable for each other and that the σ_{K1L} is equal to the σ_{K2L} in the U. S. total manufacturing industry 1929~68. The Berndt-Christensen's findings imply that structures and equipment are functionally separable from aggregate labor. On the other hand, Boddy-Gort, and Sato have focused on the esitmated values of the σ_{K1K2}, which

23) Kendrick pointed up that "there have been significant changes in the composition of capital stock in the U. S. economy. Such changes have been in favor of equipment at the expense of structures since 1937. The stock of equipment increased at rates comparable to domestic output, but the stock of structures lagged considerably behind output." See J. W. Kendrick, *Productivity Trends in the U. S.*, p. 93.

24) Boddy and Gort contended that "this will occur even though the prices and the service costs of structures and equipment move symmetrically." Thus, they ascribe the chages in composition of capital stock to variations in the relative costs of capital and labor. See R. Boddy and M. Gort, "The Substitution of Capital for Labor," p. 183.

25) In an attempt to explain changes in the composition of capital stock, Sato, Boddy and Gort disaggregated the capital stock into structures and equipment an estimated elasticity of substitution between them.

are much less than infinity(i. e., the substitutability between structures and equipment is not perfect), and thereby maintain the necessity of disaggregation of a composite measure of capital stock into structures and equipment.

Table 5. Comparison of the Time-series Estimates of the PES Among Structures, Equipment and Lobor in the U. S. Manufacturing Total Industry

Author	σ_{K1L}	σ_{K2L}	σ_{K1K2}	Time period	N
Ours(CRESH)	.056	1.519	.785	1957~86	30
Berndt-Christensen	1.518	1.342	6.554	1929~68	39
Bischoff	–	1.023	–	1927~62*	27
Boddy-Gort	–	–	1.720	1927~68	40
Sato	–	–	1.643	1929~63	34

Notes : σ_{K1L} = PES between structures and labor.
σ_{K2L} = PES between equipment and labor.
σ_{K1K2} = PES between structures and equipment.
* The years from 1940 to 1947 were excluded.

Thus, the problem of disaggregation of the aggregate input of capitial stock and its substitution behaviors have been controversial issue in the literature. Our empirical results presented in the previous section seem to add another contrasting point to the issue they turn out to be quite divergent from the others. As shown in Table 5, our estimate of the σ_{K1L} conflicts with that of berndt-Christensen's. Structures and labor are not easily sustitutable for each other in our result($\sigma_{K1L} < 1$), while the two factors are highly substitutable for each other in Berndt-Christensen's($\sigma_{K1L} > 1$).

An interesting and contradictory result found through this comparison is that the absolute value of the σ_{K1K2} in our result is less than unity, implying that structures and equipment can not be easily substitutable for each other, while those in Berndt-Christensen, Boddy-Gort, and Sato are greater than unity. Further research is clearly required on this matter. Our empirical results confirm the complementary character of structures and equipment in the production process. The complementary relationship between structures and equipment is not only theoretically plausible, but also intuitively applicable to the real world. Only in the estimate of the PES between equipment and labor is our result consistent with all the others'; i. e., all the results of the σ_{K2L} were shown to be greater than unity.

Our results conflict with berndt-Christensen's findings, while our results tend to support Boddy-Gort and Sato's argument that : (i) equipment is more easily substituted for labor than are structures; and that(ii) the composite measure of capital stock should be disaggregated into structures and equipment.

The comparisons which have been done in this section suggest that not only do the estimated values of the PES differ considerably, but the test results are also very sensitive to the choice of the functional form fitted and to the data base. In this respect we concur with Nerlove who finds, in an extensive survey, that "even slight variations in the period or concepts tend to produce drastically different estimates of the elasticity." [26] It seems reasonable to expect some differences in absolute values and/or in sign(positive or negative values) for the PES estimated from different specification of the models

5. CONCLUDING REMARKS

The purpose of this study has been to analyze and test the functional separability condition for the aggregate input of capital in order to provide empirical evidence about whether or not aggregate input of capital does represent well the sum of heterogeneous components of the input. Capital input of production have generally been assumed to be homogeneous in most empirical studies of production functios.

We have applied the CRESH production function which possesses the property of variable PES for different factor proportions. The parameters of the CRESH model have been estimated by using Zellner's two-stage GLS method in order to obtain efficient(minimum variance)estimates of the PES.

In order to investigate the possibility of the disaggregation of the capital aggregate into structures and equipment, the null hypothesis of no significant difference between two PES, $\sigma_{K1L} = \sigma_{K2L}$, has been tested against the alternative hypothesis of significant difference between them. The test results show that in the time-series data (1957~1986), the estimates of the σ_{K1L} are significantly different from those of the σ_{K2L} in most U. S. manufactuquring industries, except for the textile(# 22), clothing(# 23) and stone, clay and glass(# 32) industries which appear to have relatively high elasticities of substitution between structures and equpment. This can be interpreted to mean that in most manufacturing industries, structures and equipment are not highly substitutable for each other since they serve distincive functions for the production process. Therefore, for specification of the capital stock in a production function, it is necessary to disaggregate the capital aggregate into structure and equipment, rather than adding up the stock of heterogeneous capital goods into a composite measure of capital aggregate. The conclusion of capital disaggregation presented here contradicts Berndt-Christensen's findings, while it lends supports to the views of Sato, and Boddy-Gort.

As a consequence of the test of the functional separability hypothesis for the aggregate input of capital, we have also made some interesting observations.

The estimates of the σ_{K2L} from the CRESH model appear to be relatively higher than those of the σ_{K1L} in a majority of the industries. Our estimates of the σ_{K1L} and σ_{K2L} are in contrast to Berndt-Christensen's estimates, while they tend to confirm Boddy-Gort's contention that equipment can be more easily substituted for labor than can structures.

From the results of the estimated PES, we have recognized that the degrees and the signs of the estimated PES(positive or negative values of the PES) between factors of production vary widely from industry to industry. We have found little evi-

26) M. Nerlove, "Recent Empirical Studies of the CES and Related Production Function," p. 58.

dence to support the argument that consumer-oriented industries tend to be characterized by high elasticities of substitution between factors of production than to producer-oriented industries. We have failed to find any generally vaild pattern of the factor substitution behavior among industries. This will require further research to analyze the inter-industry patterns of the estimates of the PES in relation to, for example, the skill-intensity, the productivity of factors and the capital-labor ratio for the various manufacturing industries.

We have also recognized that the estimated values(absolute size and/or sign) of the PES are very sensitive to the differences in the aggregation levels of industries. This seem to reflect the existence of aggregation bias in our data. On an attempt to alleviate the possible aggregation bias, one could apply the present model to individual firms for a more clear representation of its factor substitution behavior.

From the comparison of our estimates of the PES obtained from the CRESH model with those obtained by other authors, we have learned that our estimations were not always consistent with the results of others in some of the manufacturing industries. This also requires further research to confirm the validity of our findings. It is, however, difficult to derive any conclusion from the comparisons, because of the differences in the specifications of the models and the fact that there is no a priori bais for preferring one specific model over the other.

In conclusion, the emprical evidence presented in this study lends support to the argument that the capital aggregate does not represent well the sum of heterogeneous characteristics of the various components of capital inputs. This empirical result implies for production studies that structures and equipment should be explicitly treated as distinct factors of production, rather than aggregating them into a composite measure of capital input.

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APPENDIX. List of Two-digit Manufacturing Industries

Industry Code	Industry Groups
Mfg. Total	Manufacturing total
# 20	Food Products
# 22	Textile Mill Products
# 23	Clothing Products
# 24	Lumber & Wood Products
# 25	Furniture & Fixtures
# 26	Paper Products
# 27	Printing & Publishing
# 28	Chemicals & Products
# 29	Petroleum and Coal Products
# 30	Rubber & Plastics Products
# 31	Leather Products
# 32	Stone, Clay & Glass Products
# 33	Primary Metal Products
# 34	Fabricated Metal Products
# 35	Machinery
# 36	Electrical Equipment
# 37	Transportation Equipment
# 38	Instruments Products

Source : *Census of Manufactures* 1992, Bureau of the Census, U.S. Dept. of Commerce; and *Standard Industrial Classification Manual*, Bureau of the Budget, Executive Office of the President, 1987.