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MEASUREMENT OF PRODUCTIVITY CHANGE REVISITED: TECHNICAL CHANGE VS PRODUCTIVITY GROWTH

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Recent developments in the methodological measurement of productivity change have contributed much to measuring the real productivity change. Productivity change is generally measured by either the rate of change an index of outputs divided by an index of inputs or the rate of shift in a production function. As Solow(1957) argued, total factor productivity(TFP) growth and technical change(TC) are the two sides of a same coin. One puzzing question is that two empirical measures, diverging from the theoretical identity between TC and TFP growth, show very different estimates, depending upon maintained assumptions and model specifications. The main purpose of the paper was to provide a systematic overview of technical change and productivity growth by reinterpreting the theoretical identity between TC and TFP growth and to investigate a theoretical background for adjusting the traditional mesures. We showed how traditional productivity mesures at a static equilibrium should be changed as the concepts of duality and dynamics are introduced into the underlying function, upon which parametric measurement of productivity is based.

1. INTRODUCTION

Since Slow(1957)[1] argued that technical change does not represent a movement along a production function, but a shift of production function, such a shift has been labeled as productivity growth, technical change, or technological progress in the literature. Productivity change may be used as a global definition. In his survey paper, Nadiri(1970) described a comprehensive notion of productivity change as follows;

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1) At the same time, Farrell(1957), using a stochastic production frontiers, presented productivity efficiency measure at a firm level which can be divided into technical and allocative efficiencies. This is another line of research area which has been currently developed in parallel with Solow's. Here, we will not trace Farrell's approach here because it is beyond a scope of this paper concerning an aggregate or industrial productivity.

> "*Productivity change is both the cause and the consequence of the evolution of dynamic forces operative in an economy-technical progress, accumulation of human and physical capital, enterprise, and institutional arrangements. Its measurement and the interpretation of its behavior at the microeconomic and macroeconomic levels require the untangling of many complex factors.*"

Based on the above definition, productivity change is generally measured by(i) the rate of change an index of outputs divided by an index of inputs(i.e., the unexplained productivity residual calculated by index number approach) or (ii) the rate of shift in a production function.

There have been efforts to measure productivity change by either *indirectly calculating* the index numbers or *directly estimating* the econometric model of standard time trend.[2] The earliest approach to *partial* productivity measurement used a ratio of an index of aggregate output divided by the observed quantity of a single input, typically labor. A more comprehensive index number approach to *total or multi* factor productivity measurement provided a clear improvement over partial or single measure. An alternative interpretation (ii) involves the explicit specification of a production function. We shall make a distinction between (i) and (ii) by interpreting (i) as the "productivity growth" and (ii) as the "technical change" in the next section II, but use these terminologies interchangeably in the section III. Generally, productivity change can be viewed as the composite outcome of technological and institutional change. Since institutional change can not be identified and measured in practice, the concept of productivity change is narrowly confined to either technical change or productivity growth.

Two excellent articles reviewed contemporary productivity issues over the last decades. Nadiri(1970) dealt with the theoretical and measurement issues, focusing on the aggregate productivity of a production function. A decade later, Cowing and Stevenson(1981) provided a clear overview of productivity measurement of industrial productivity of a cost function. They pointed out the the cost function model represents a powerful and flexible econometric tool for measuring technical change or productivity growth. Since Cowing and Stevenson(1981), another decade has passed. To examine more recent developments, we need to review the evolvement of measures of productivity change. In spite of the theoretical identities of productivity growh and technical change, there exists a divergence between primal productivity growth defined in the production side and its dual productivity growth defined in the cost side when constant returns to scale assumption is not maintained. Dynamic specification of a flexible cost function requires additional adjustments to scale economies effects due to the static duality. The main purpose of the paper is to review aspects of the theory and measurement of total factor productivity growth and clarify the differences between productivity growth and technical change by reinterpreting their dynamic and dual relationships between a long-run production func-

2) See Literature review in detail by Diewert (1981) and Baltagi and Griffin(1988).

tion and a short-run cost function.

2. MEASUREMENT OF PRODUCTIVITY CHANGE

As solow(1957) argued, total factor productivity(hereafter TFP) growth and technical change(hereafter TC) are the two sides of a same coin. One puzzling question is that two empirical measures, diverging from the stheoretical indentity between TC and TFP growth, show very different estimates, depending upon maintained assumptions and model specifications. We will implicitly associate TC with econometric estimation approach and TFP growth with index number approach in the literal *nua nce.* From the two approaches of measurement of (i) and (ii), we can derive the corresponding operational definitions of TC and TFP growth.

2.1 Measurement of Technical Change:Time Trend Approach

When productivity change is associated with a shift in the production function, technical change can be meausured, using a time trend representation of production function.

Definition: Technical change is any kind of shift in the production function or production possibilities set and is expressed as

$$TC \equiv \frac{\partial \ln Q(X, t)}{\partial t}$$

where $Q(X, t)$ is a production function of inputs X and time t.

2.1.1 Measurement.

Because standare time trend approach involves the explicit specification of a production function, econometric implementation directly yields parametric estimates of the production technology. Therefore, its parametric estimates depend on the parameters of the underlying production function. According to the order of time trend, we can classify several evolutionary measures of technical change. TC_1 through TC_7 in the following can be determined, respectively, by the specific functional forms that we are assuming.

[1] First-order time trend

Technical change is identified with a simple time trend. First order in time trend model yields a constant rate of technical change.

[1.1] Cobb-Douglas production technology.

One of the classical specifications of production function is the Cobb-Douglas production function whose input arguments X are usually capital input(K) and labor input(L).

$$Q(X, t)=F(K, L, t) = A(t)K^{\alpha}L^{1-\alpha} = e^{\lambda t}K^{\alpha}L^{1-\alpha} \tag{2a}$$

where $A(t)$ a measure of disembodied technical change and is an exponential function of time. λ is a shift parameter representing the constant rate of disembodied neutral technical change. Then, technical change is directly identified by λ.

$$TC_1 \equiv \frac{\partial \ln Q(X, t)}{\partial t} = \lambda \tag{2b}$$

[1.2] CES production technology

$$Q(X, t) = F(K, L, T) = A(t)[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-\mu/\rho} \tag{3a}$$

where $A(t) = e^{\lambda t}$ same as defined in (2a). In (3a), δ, ρ and μ represent the parameters of distribution, substitution- and degree of returns to scale, respectively. Similary, we can derive TC_2 as

$$TC_2 = \frac{\partial \ln Q(X, t)}{\partial t} = \lambda \tag{3b}$$

These two production functions exhibit Hicks-neutral technical change. There are alternative representations of neutral technical change. When Q (X, t) = $F(K, L, t) = F(K, A(t)\, L)$, the underlying production function represents Harrod−neutral (labor augmenting) technical change. When $Q(X, T) = F(K, L, t) = F(A(t)K, L)$, the underlying production function represents Solow-neutral (capital augmenting) technical change. In the case of disembodied technical change, three alternative neutral hypotheses suggested by Hicks, Harrod and Solow, result in the same value of constant shift parameter, λ.

[2] Second-order time trend:Translog production technology.

Quadratic terms in time trend are introduced in a translog[3] function whereas a linear time trend is involved in [1]. Unlike [1], *a priori* restrictions on the functional form are not necessary in the translog function which belongs to a family of flexible functional form. One advanced feature is that multiple inputs can be used as arguments of a production function, yielding no restrictions on the elasticity of factor substitutions. Following Berndt and Wood's(1975) KLEM model, assume that there exists a twic differentiable production function with four inputs : K(capital), L(labor), E(energy), M(intermediate materials) and t(state of technology).

$$Q(X, t) = F(K, L, E, M, t). \tag{4a}$$

Duality implies there exists a corresponding cost function C which is a function of factor prices P, output Q, and t.

$$C(P, Q, t) = C(P_K, P_L, P_E, P_M, Q, t) \tag{4b}$$

Where P_i represents a variable factor price for K, L, E, and M. According to Ohta(1974), technical change can be measured by the following relationship between primal technical change(upward shift) of production function and its dual technical change(downward shift) of cost function.

3) It takes a second-order approximation in logged arguments to an arbitrary function. Another flexible function form is a *Generalized Leontief* function in squared arguments, suggested by Diewert(1971).

$$TC \equiv \frac{\partial \ln Q(X, t)}{\partial t} \equiv -\epsilon_{CQ}^{-1} \cdot \frac{\partial \ln C(P, Q, t)}{\partial t} \tag{4.c}$$

Where ϵ_{CQ} represents the adjusting scale elasticity, $\partial \ln C/\partial \ln Q$. For the same cost function $C(P, Q, t)$ in (4.b), different forms of approximations, variants of Translog as well as Generalized Leontief, can be applied upon empirical implementation. When $C(P, Q, t)$ has an *ordinary* translog cost functional form, it is typically defined as

$$\ln C(Z) = \alpha_0 + \alpha' (\ln Z) + \frac{1}{2}(\ln Z')\beta(\ln Z).$$

Where α and β are parameter vector and matrix respectively, and Z represents an exogenous variable vector. Then, ln $C(P, Q, t)$ can be decomposed into two specific functional forms : $N1(P, Q)$ and $N(P, Q, t)$.

$$\ln C(P, Q, t) = N1(P, Q) + N2(P, Q, t) \tag{5.a}$$

where $N1(P, Q)$ is equal to a second-order Taylor expansion in the logged arguments excluding t

$$N1(P, Q) = \alpha_0 + \sum_i \alpha_i \ln P_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln P_i \ln P_j$$
$$+ \left[\alpha_Q \ln Q + \frac{1}{2}\beta_{QQ}(\ln Q)^2\right] + \sum_i \beta_{iQ} \ln P_i \ln Q \tag{5.b}$$

and $N2(P, Q, t)$ involves an argument of time t

$$N2(P, Q, t) = \left(\alpha_t t + \frac{1}{2}\beta_{tt} t^2 + \beta_{Qt} t \cdot \ln Q + \sum_i \beta_{it} t \cdot \ln P_i\right). \tag{5.c}$$

Then, we obtain a parametric equation for technical change.

$$TC_3 = -\frac{\partial \ln C(P, Q, t)}{\partial t} = -\frac{\partial N2(P, Q, t)}{\partial t}$$
$$= -(\alpha_t + \beta_{tt} t + \beta_{Qt} \ln Q \ \sum_i \beta_{it} \ln P_i). \tag{5.d}$$

The following model specifications [3], [4], and [5] are variants [4] of second-order translog functional form [2].

[3] Truncated third-order time trend : Stevenson(1980)

Stevenson (1980) proposed a truncated third-order Taylor expansion in time, the logged input price, output, and state-of-nature variables. The non-time second-order coefficients are implicitly assumed to be constant over time under ordinary translog form as in (5.a). If we approximate (4.b) in this manner, the pro-

4) In addition, Baltagi and Griffin(1988) proposed a general index of technical change ($A(t)$), both nonneutral and scale augmenting, by replacing t and t^2 terms with $A(t)$ in the standard time trend model.

posed model can be written as

$$\ln C(P, Q, t) = N1(P, Q) + N2(P, Q, t) + N3(P, Q, t) \tag{6.a}$$

where $N3(P, Q, t)$ captures a truncated third-order Taylor expansion.

$$N3(P, Q, t) = N1(P, Q) \cdot t \tag{6.b}$$

We can reconstruct a new parameter set by rearranging $N2(P, Q, t)$ and $N3(P, Q, t)$. For instance, $\sum_i(\alpha_i + \beta_{it})t\ln P$ in (6.a) is equal to $\sum_i(\gamma_{it}t\ln P)$ in (6.c). (6.a) can be rewritten as

$$\begin{aligned}\ln C = {} & N1(P, Q) + \gamma_t t + \frac{1}{2}\gamma_{tt}t^2 + \sum_i \gamma_{it}t\ln P_i \\ & + \frac{1}{2}\sum_i\sum_j \gamma_{ijt}t\ln P_i\ln P_j \\ & + \gamma_{Qt}t\ln Q + \frac{1}{2}\beta_{QQt}(\ln Q)^2 + \sum_i \gamma_{iQt}t\ln P_i\ln Q.\end{aligned} \tag{6.c}$$

Technical change rate from equation(6.c) is written as

$$\begin{aligned}TC_4 = {} & -\frac{\partial \ln C(P, Q, t)}{\partial t} \\ = {} & -\left(\gamma_t + \gamma_{tt}t + \gamma_{Qt}\ln Q + \sum_i \gamma_{it}\ln P_i\right) \\ & -\left(\frac{1}{2}\sum_i\sum_j \gamma_{ijt}\ln P_i\ln P_j + \frac{1}{2}\gamma_{QQt}(\ln Q)^2 + \sum_i \gamma_{iQt}t\ln P_i\ln Q\right).\end{aligned} \tag{6.d}$$

As a result of truncated third-order expansion, the second parenthesis in (6.d) is added to the normal equation(5.d).

[4] Stochastic time trend : Slade(1989)

Considering the state of technology t an unobserved or latent variable, Slade (1989) modeled it as a stochastic trend upon estimating the rate of direction of technical change. She specified the translog cost function as

$$\begin{aligned}\ln C(P, Q, t) = {} & F(t, Q) + \sum_i \alpha_i\ln P_i \\ & + \frac{1}{2}\sum_i\sum_j \beta_{ij}\ln P_i\ln P_j + \sum_i \beta_{it}t\ln P_i\end{aligned} \tag{7.a}$$

Where $F(t, Q)$ is the Hicks-neutral portion of the productivity term. Thus, TC_5 is defined as

$$TC_5 = -\frac{\partial \ln C(P, Q, t)}{\partial t} = -\left(\frac{\partial F(t, Q)}{\partial t} + \sum_i \beta_{it} \ln P_i\right) \tag{7.b}$$

$$\text{where} \quad \frac{\partial F}{\partial t} \equiv f(t, Q) = \phi\frac{\dot{Q}}{Q} + \omega_t. \tag{7.c}$$

The first term in (7.c) is the cyclical component, and the second term represents the stochastic trend. Unlike other approaches[5], TC_5 is replaced with the observed Tornqvist TFP index in the left hand side of (7.b) while an additive disturbance terw ε_t^N is added to the right side of (7.b). Let $\tilde{F}(t, Q)$ denote productivity terms which are obtained from equation (5.a). $\tilde{F}(t, Q)$ can be expressed as

$$\tilde{F}(t, Q) = (\alpha_0 + \alpha_t t + \frac{1}{2}\beta_{tt}t^2) + \beta_{Qt} t \ln Q$$

$$+ \left[\alpha_Q \ln Q + \frac{1}{2}\beta_{QQ}(\ln Q)^2 + \sum_i \beta_{iQ} \ln P_i \ln Q\right]$$

Thus, we can derive a partial derivative of $\tilde{F}(t, Q)$, say $\tilde{f}(t, Q)$, similar to (7.c)

$$\tilde{f}(t, Q) \equiv \frac{\partial \tilde{F}(t, Q)}{\partial t} = \beta_{Qt} \ln Q + (\alpha_t + \beta_{tt} t) \tag{7.d}$$

It follows that Slade's (1989) equation(7.c) can be viewed as a variant of $\tilde{f}(t, Q)$ with time-varying parameters α_t and β_{tt} in the ordinary translog model. Thus, $N-1$ share equations from(7.a) and one technical change equation from(7.b) (with (7.c) substituted into (7.b)) constitute N *measurement equations* with multivariate normal distribution ($\varepsilon \sim$ *niid*$(0, \Omega)$). Slade (1989) considered the two time-varying parameters which are derived from(7.c)

$$\omega_t = \omega_{t-1} + \nu_{t-1} + \eta_t^1 \tag{7.e}$$
$$\nu_t = \nu_{t-1} + \eta_t^2 \tag{7.f}$$

where $\eta_t^i \sim$ *niid*$(0, \sigma_{\eta i}^2)$, $i = 1, 2$. To estimate the time-varying parameters for technical change coefficients, ω_t and ν_t[6], the two *transition equations* are added to the N *measurement equations.* Slade (1989) showed two special cases. Combining(7.e) and (7.f) yields

$$\omega_t = \omega_{t-1} + \nu_{t-1} + \eta_t^1 = (\omega_{t-2} + \upsilon_{t-2} + \eta_{t-1}^1) + \upsilon_{t-1} + \eta_t^1$$

$$= \omega_0 + \sum_{i=0}^{t-1} \nu_i + \text{sum of error terms}$$

(i) When $\sigma_{\eta i}^2 = 0$, a stochastic time trend reduces to a deterministic trend. Since ν_t

5) Other TC's except TC_5 is calculated parametrically from the obtained parameter estimates.

6) These correspond to α_t and β_{tt}

$= \nu_{t-1} = \cdots = \nu_0 \, \omega_t = \omega_0 + \sum_{i=0}^{t-1} \nu_i = \omega_0 + \nu_0 t$. Under this circumstance, (7.c) can be rewritten as

$$f(t, Q) = \phi \frac{\dot{Q}}{q} + (\omega_0 + \nu_0 t) \tag{7.c'}$$

Note that (7.c') is very similar to (7.d).

(ii) When $\phi = 0$ in (7.c')(correspondingly $\beta_{Qt} = 0$ in (7.d) and $\sigma^2_{\eta 1} = 0$, we get the identical results from either (7.d) or (7.c') : $\omega_0 = \alpha_t$ and $\nu_0 = \beta_{tt}$ as long as $\beta_{Qt} = 0$. It is evident that a deterministic equation (5.d) is a special case of stochastic equation (7.b).

[5] Exponential time trend : Gollop and Roberts (1981), Berndt and Wood (1982)[7]

This approach combines [1] and [2] by assuming that factor i augments at a constant rate of λ_i and a factor expressed in effective unit enters the translog cost function.

$$C^*(t) = C(P_K^*(t), P_L^*(t), P_E^*(t), P_M^*(t), Q) \tag{8.a}$$

$$P_i^*(t) = P_i \exp_i^{\lambda_i t} = K, L, E, M. \tag{8.b}$$

Gollop and Roberts (1981) considered the second-order exponential time trend, but applied the first-order factor augmentation because of an econometric identification problem. Berndt and Woold(1982) expressed the multi-factor productivity growth as a weighted average of cost shares and augmentation rates.

$$TC_6 = -\frac{\partial \ln C^*(t)}{\partial t} = -\sum_i S_i \lambda_i, \; \sum_i S_i = 1 \tag{8.c}$$

The technical change equation of multiple factor augmentation can be regarded as an extension to single factor augmenting technical change can be expressed as a single constant parameter, say λ as an equivalent rate of factor augmentation.

$$TC_7 = \sum_i S_i \lambda_i = \bar{\lambda} \sum_i \bar{S}_i = \lambda \tag{8.d}$$

2.2 Measurement of Productivity Growth : Index Number Approach.

The pioneering work of Solow(1957) demonstrated the Divisia index of productivity growth could be identified with the rate of Hicks-neutral technical change under some restrictive assumptions.[8]

Definition : TFP growth$(\dot{TFP})$ is a difference in the percentage change in output $\dot{Q}$ less percentage change inputs$(\dot{X})$ weighted by revenue or cost shares (S_i) whose

7) See Lee(1992a), Berndt, Kolstad and Lee(1993), Lee and Kolstad(1994) for recent empirical studies.

8) In particular, these assumptions require (i) disembodied neutral technical change, (ii) constant returns to scale, (iii) static (long-run) equilibrium, and (iv) perfect competition in both output an factor input markets.

sum is unity.

$$T\dot{F}P \equiv \frac{\dot{A}(t)}{A(t)} = \frac{\dot{Q}}{Q} - \sum_{i=1}^{n} S_i \frac{\dot{X}_i}{X_i} \tag{9}$$

where $A(t)$ is a general index of technical change. We outline Solow's derivation of Divisia index. Totally and logarithmically differentiating the production function in Solow (1957, P312), $Q = F(K, L, t) = A(t)\, f(K, L)$, with respect to time yields

$$\frac{d\ln Q}{dt} = \frac{\partial \ln K(t)}{\partial \ln K(t)} \cdot \frac{d\ln K(t)}{dt} + \frac{\partial \ln Q(t)}{\partial \ln L(t)} \cdot \frac{d\ln L(t)}{dt} + \left[\frac{\partial \ln Q(X, t)}{\partial t}\right]$$

$$= A\frac{\partial f}{\partial K}\frac{\dot{K}}{Q} + A\frac{\partial f}{\partial L}\frac{\dot{L}}{Q} + \left[\frac{d\ln A(t)}{dt}\right]. \tag{10}$$

Note that both the last terms in brackets of the above right hand side are equivalent to each other.

$$\left[\frac{\partial \ln Q}{\partial t}\right] \equiv \left[\frac{d\ln A}{dt}\right]$$

Thus, the left hand side of (11) describes TC in (1) whereas the right hand side of (11) stands for TFP growth in (9).

2.2.1 Measurement.

An advantage of this method of measurement is that it does not require any assumptions about the functional form of the production function. Productivity growth is measured by the indirect calculation of productivity residual using equation(9). This includes Divisia(1926) index, Solow's general index of TC ($A(t)$), Tornqvist index (a discrete approximation to the Divisia index), and Diewert's(1976) Exact index, all of which can be characterized by the *productivity residual.*

2.3. Productivity Growth vs. Technical Change.

From 2.1 and 2.2, we can identify a theoretical relationship between productivity growth and technical change in terms of cost function. If we assume $KLEM$ model under $CRTS$,

$$C = C(P_K, P_L, P_E, P_M, t) \tag{4.b'}$$

Totally and logarithmically differentiating this cost function with respect to time yields

$$\frac{d\ln C}{dt} \equiv \left(\sum_{i=K,L,E,M} \frac{\partial \ln C}{\partial \ln P_i}\frac{d\ln P_i}{dt}\right) + \frac{\partial \ln C}{\partial t}. \tag{12}$$

Because $\partial \ln C/\partial \ln P_i = S_i$ by shephard's lemma, we can derive two definitons of productivity change.

$$-\left[\frac{d\ln C}{dt} - \sum_{i=K,L,E,M} S_i \frac{d\ln P_i}{dt}\right] \equiv -\left[\frac{\partial \ln C}{\partial t}\right] \tag{13}$$

The left hand side in (13) describes a cost diminution rate similar to a productivity growth residual defined in equation(9) while the right hand side indicates a rate of technical change defined in (4.c). Equation(13) will be considered a basis for understanding adjustment procedures in the next section.

3. ADJUSTMENTS OF PRODUCTIVITY MEASUREMENT

As the application of duality theory brought about the development of cost function dual to the underlying production function in the 1970s, interests of measurement moved from aggregate productivity toward industrial productivity. While the former was measured using the aggregate production function, the latter was generally measured utilizing the sectoral cost function where competitive equilibrium is assumed to be sustainable. Since mid-1970s, technical change has been viewed as a downward shift of cost function instead of an upward shift of production function, and measured by way of a cost diminution rate. Duality concept dichotomizes the primal productivity growth and dual productivity growth. When duality is linked to issues of static and dynamic specifications of functional forms, observed productivity measurement should be adjusted to identify the genuine productivity measurement by correcting the bias that is generated from the dynamic specification[9]. Three versions of adjusting elasticities were proposed by Ohta(1974), Caves, Christensen, and Swanson(1981), and Morrision(1986). Each dealt with the case of nonconstant returns to scale(hereafter NCRTS), quasi-fixity and adjustment costs. Above all, introduction of dynamic equilibrium concept had an influence on the previous ways of measuring technical change.

3.1. Economies of Scale.

There are two ways to consider the economies of scale effect under a static specification of the cost function. According to Ohta(1974), the dual rate of technical progress is equal to the primal rate only in the case of constant returns to scale. When Berndt and Khaled(1979) developed a parametric approach to TFP measurement, which did not rely on CRTS assumption, they interpreted Ohta(1974) as follows. The primal rate of total factor productivity can be expressed using its dual relationships.

$$\varepsilon_{Qt} = \varepsilon_{CQ}^{-1} \varepsilon_{Ct} \tag{4.c'}$$

where

9) This bias occurred owing to the breakdown of basic assumptions which Solow(1957) considered.

ε_{Qt} is the *primal* rate of total factor productivity ($= \frac{\partial \ln Q}{\partial t}$ with quantities fixed),

ε^{-1}_{CQ} is the *dual* rate of returns to scale, ($\varepsilon_{CQ} = \frac{\partial \ln C}{\partial t}$ with input prices fixed),

ε_{Ct} is the *dual* rate of total cost diminution, ($= -\frac{\partial \ln C}{\partial t}$ with input prices and output quantity fixed).

Equation(4.c') implies that primal TFP growth is identical to the TFP growth via scale elasticity(ε_{CQ}). Furthermore, since $\varepsilon^{-1}_{CQ} = 1$ under CRTS, CRTS technology results in identity between primal and dual TFP growth.

On the other hand, Caves, Christensen, and Swanson(1981) developed a slightly different definition of productivity growth in the case of a general structure of production with multiple inputs and multiple outputs, employing the generalized translog multi-product-cost function. Assume F is the transformation function for a general structure of production function,

$$F(\ln Q_1, \cdots, \ln Q_m, \ln X_1, \cdots, \ln X_n, t) = 1 \tag{14}$$

Where $Q_i(X_j)$ represents an output i (input j). They derived the three definitions from the total differential of (14), similar to Ohta(1974).

$$\sum_i^m F_{Q_i} \mathrm{d}\ln Q_i + \sum_j^n F_{X_j} \mathrm{d}\ln X_j + F_t \mathrm{d}t = 0 \tag{15.a}$$

$$PGY \equiv \frac{\mathrm{d}\ln Q_{i1}}{\mathrm{d}t} = \frac{\mathrm{d}\ln Q_{i2}}{\mathrm{d}t} = \frac{\mathrm{d}\ln \bar{Q}}{\mathrm{d}t} \text{ with } \mathrm{d}\ln X_j = 0 \ \forall_j, \tag{15.b}$$

$$PGX \equiv -\frac{\mathrm{d}\ln X_{j1}}{\mathrm{d}t} = -\frac{\mathrm{d}\ln X_{j2}}{\mathrm{d}t} = \frac{\mathrm{d}\ln \bar{X}}{\mathrm{d}t} \text{ with } \mathrm{d}\ln Q_i = 0 \ \forall_i, \tag{15.c}$$

$$RTS \equiv \frac{\mathrm{d}\ln Q_i}{\mathrm{d}\ln X_j} \text{ with } \mathrm{d}t = 0, \tag{15.d}$$

Where PGY(PGX) represents the common rate at which all outputs(inputs) can grow over time with inputs(outputs) held fixed, and RTS, the degree of returns to scale, is the proportional increase in all outputs resulting from a proportional increase in all inputs with time fixed. (15.b), (15.c) and (15.d) imply $PGY = -F_t/\sum_i F_{Q_i}$, $PGX = F_t/\sum_j F_{X_j}$ and $RTS = -\sum_j F_{X_j}/\sum_i F_{Q_i}$. Caves et al. (1981) derived a total cost function dual to (14)

$$\ln C = C(\ln Q_1, \cdots, \ln Q_m, \ln P_1, \cdots, \ln P_n, t) \tag{14'}$$

where C is a total cost ($\sum P_j X_j$) and P_j represents price of input X_j. Totally differentiating (14') yields

$$0 = \sum_i \frac{\partial \ln C}{\partial \ln Q_i} \mathrm{d}\ln Q_i + \sum_j \frac{\partial \ln C}{\partial \ln P_j} \mathrm{d}\ln P_j + \frac{\partial \ln C}{\partial t} \mathrm{d}t \tag{16.a}$$

From(16.a), we can derive the corresponding definitions to (15.b), (15.c), and (15.d). Because we concern the total costs, a superscript T below represents the total cost function.

[1] PGY^{T}(when $\mathrm{d}\ln P_i = 0$)

$$PGY^{T} \equiv -\frac{F_t}{\sum F_{Q_i}} = \frac{\mathrm{d}\ln \bar{Q}}{\mathrm{d}t} = -\frac{\partial \ln C/\partial t}{\sum \partial \ln C/\partial \ln Q_i} \tag{16.b}$$

[2] PGX^{T}(when $\mathrm{d}\ln Q_i = 0$)

Within the structure of cost function, a dual version for PGX in (15.c) can be defined as[10]

$$PGX^{T} \equiv \frac{F_t}{\sum F_{X_j}} = -\frac{\partial \ln C}{\partial t} \tag{16.c}$$

[3] RTS^{T}(when $\mathrm{d}t = 0$)

By the definition of RTS, it is obtained by

$$RTS^{T} \equiv -\frac{\mathrm{d}\ln \bar{Q}}{\mathrm{d}\ln \bar{P}} = \left(\frac{\sum \partial \ln C}{\partial \ln Q_i}\right)^{-1} \tag{16.d}$$

When it comes to the total cost function of a single product ($m = 1$), $\ln Q$ will be substituted for $\sum \ln Q_i$. Once again, a dual relationship between two alternative definitions of productivity growth is obtained.

$$PGY^{T} = RTS^{T} \cdot PGX^{T} \tag{17}$$

PGY^{T} is equal to PGX^{T} when $RTS^{T} = 1$, just as ε_{CQ} is equal to 1 under CRTS. Thus, two approaches lead to the same results. An advantage of these definitions, developed by Caves et al. (1981), is that it provides a flexible framework when it is applied to the dynamic specification of cost function.

3.2 Quasi-fixity.

As the existence of quasi-fixed factors in the cost function has been widely recognized, the traditional method for TFP measurement, which is valid only at the long-run static equilibrium, was improved by refining the functional forms. We can consider quasi-fixity effect in terms of either total or variable cost function in the short-run.

The first approach is based on the implicit assumption that there exists a short-run total cost function[11] within a temporary equilibrium context. When the firm minimizes the variable costs (VC) over the subset of total factors, conditional on the

10) Here, $\sum(\partial \ln C/\partial \ln P) = 1$ and $-\mathrm{d}\ln X_i/\mathrm{d}t = -\mathrm{d}\ln X_j/\mathrm{d}t = \mathrm{d}\ln P_i/\mathrm{d}t = \mathrm{d}\ln P_j/\mathrm{d}t$.

11) Slade(1986) considered four different cost functions; the long-run total cost function, the restricted or variable cost function, the disequilibrium total cost function, and the shadow total cost function.

given quantity of quasi-fixed factors of X_j, $j \in FX$[12], short-run total cost function(STC) at a temporary equilibrium can be expressed as

$$STC(P_i, \mu_j, Q, t) = VC(P_i, X_j, Q, t) + \sum_{j \in FX} \mu_j X_j(P_i, \mu_j, Q, t) \quad (18)$$

Where $VC = VC(P_i, X_j, Q, t)$ is a restricted variable cost function, P_i represents an equilibrium price of variable factor in the short-run, and $\mu_j = -\partial VC/\partial X_j$ represents a shadow price. Berndt and Fuss(1981) first introduced the concept of a shadow cost function to explain the existance of excess capacity associated with capacity under-utilization. Later, Berndt and Fuss (1986) defined temporary equilibrium as a state in which the shadow value of any input and/or output deviates from its equilibrium market price. Thus, the current quasi-fixed stock X_j is not necessarily a long-run equilibrium stock X_j^*.

When the shadow prices μ_j obtained from the current temporal equilibrium, are equal to the long-run equilibrium prices P^*_j, a subequilibrium coincides with the long-run equilibrium where a minimum tangential point between long-run cost curve and short-run cost curve is obtained. Then, the long-run cost function(LTC) is written as

$$LTC(P_i, P^*_j, Q, t) = VC(P_i, X^*_j, Q, t) + \sum_{j \in FX} P_j^*, X_j^*(P_i, P_j^*, Q, t) \quad (19)$$

On the other hand, the second approach focused on the short-run variable cost function. Unlike either $\partial \ln C/\partial t$[13] or $dSTC/dt$[14], the $\partial \ln VC/\partial t$ from(18) concerns only the shift in the variable cost function regardless of a shift in the total cost function. Under this circumstance, we have two sources of bias : scale effect and quasi-fixity effect. Short-run total costs(STC) can be decomposed into the sum of variable costs (VC) plus sum of quasi-fixed costs(FC), i.e., $STC = VC + FC$. Caves, Christensen, and Swanson(1981) proposed a revised formula for (16.b)-(16.d) to isolate a genuine technical change in the short-run variable cost function by emphasizing that there exists a relationship between variable costs and variable cost function, i.e., $VC = VC(P_i, X_j, Q, t)$. Analogue to equations(16), the following equations(20), derived by Caves et al.(1981), hold for short-run variable cost function of a single output.

$$PGY^V = \frac{\partial \ln Q}{\partial t} = -\left(\frac{\partial \ln VC}{\partial t}\right) \Big/ \left(\frac{\partial \ln VC}{\partial \ln Q}\right), \quad (20.a)$$

$$PGX^V = \left(\frac{\partial \ln VC}{\partial t}\right) \Big/ \left(1 - \sum_{j \in FX} \frac{\partial \ln VC}{\partial \ln X_j}\right), \quad (20.b)$$

12) FX implies index for quasi-fixed factors such as capital stock, which are assumed to be fixed in the short-run. On the other hand, V denotes index for variable factors such as labor and intermediate materials.

13) This represents the technical change in the full static and long-run equilibrium total cost function.

14) This implies the overall technical change of the short-run total cost function.

$$RTS^V = \frac{(1-\Sigma(\partial \ln VC/\partial \ln X_j))}{\Sigma(\partial \ln VC/\partial \ln Q_i)}, \tag{20.c}$$

$$PGY^V \equiv RTS^V \cdot PGX^V, \tag{20.d}$$

where superscript ^{V} denotes a variable cost function.

Because RTS^V in (20.c) becomes RTS^T in (16.c) in the long-run, the relationship between long-run (ε^l_{CQ}) and short-run adjusting scale elasticity (ε_{VCQ}) is obtained.[15]

$$\varepsilon^{l}_{CQ} = \left(1-\sum_{j\in FX}\frac{\partial \ln VC}{\partial \ln X_j}\right)^{-1} \varepsilon^{V}_{VCQ} \tag{21}$$

where $\varepsilon^{l}_{CQ} = \partial \ln C/\partial \ln Q = \mathrm{d}\ln C/\mathrm{d}\ln Q$ and $\varepsilon^{V}_{VCQ} = \partial \ln VC/\partial \ln Q$. It is worth while to note two special cases.

(i) if and only if market rental prices of the quasi-fixed factor are equal to their shadow prices, does equation (21) reduce to

$$\frac{\partial \ln C}{\partial \ln Q} = \frac{VC}{C}\frac{\partial \ln VC}{\partial \ln Q}. \tag{22}$$

(ii) $PGY^V = PGX^V$ when $RTS^V = 1$(i.e., under short-run CRTS assumption).

$$RTS^V = 1 \rightleftarrows \varepsilon_{VCQ} \equiv \frac{\partial \ln VC}{\partial \ln Q} = \left(1-\sum_{j\in FX}\frac{\partial \ln VC}{\partial \ln X_j}\right) \tag{23}$$

There may be some differences between ε_{CQ} used in Berndt and Khaled (1979) and RTS in Caves et al. (1981) in the temporary equilibrium. The former, however, is still consistent with the latter. Short-run CRTS requires that $(\varepsilon_{VCQ} + \sum_j \varepsilon_{VCX_j})$ be equal to 1 while ε_{CQ} is equal to 1 by the long-run CRTS restriction.[16] It follows that

$$RTS^V = \frac{1-\Sigma\,\varepsilon_{VCX}}{\varepsilon_{VCQ}} \tag{24}$$

RTS^V can be thought of as a relative ratio which may be greater or less than 1. When short-run CRTS is assumed, RTS^V will be equal to 1.

Now, we can reconcile the two seemingly different expressions : Ohta(1974) and Caves et al. (1981).

$$\varepsilon_{Qt} = -\varepsilon^{-1}_{VCQ}\varepsilon_{VCt}, \tag{25.a}$$

$$\varepsilon_{Ct} = -\frac{\varepsilon_{VCt}}{(1-\Sigma\partial \ln VC/\partial \ln X_j} \tag{25.b}$$

15) This cost elasticity relationship is described in Schankerman and Nadiri(1986). See also Callan's(1987) empirical study of fossil-fueled electric utility industry.

16) Short-run CRTS does not necessarily require that ε_{VCQ} should be equal to 1.

Equation(25.a) describes the dual relationship of TFP between the production and the costs by considering the scale effect while equation(25.b) explains how the long-run elasticity of total cost with respect to time(ε_{Ct}) is retrieved from the elasticity of short-run variable cost function (ε_{VCt}) by considering the quasi-fixed effect due to temporary equilibrium. By combining (25.a) and (25.b), the equation (4.c'), used in Berndt and Khaled (1979), can be rewritten for short-run variable cost functions as

$$E_{Qt} = RTS^{V} \varepsilon_{Ct}. \tag{25.c}$$

3.3 Adjustment Costs.

In the special issue dealing with *temporary equilibrium*,[17] Berndt and Fuss(1986) developed a modified productivity growth accounting equation. Hulten(1986) argued that the growth rate of real short-run average cost can be decomposed into multifactor productivity (*MFP*) growth rate in the *ex post* sense of Jorgenson and Griliches(1967) and capacity utilization effect in Berndt and Fuss(1986). Furthermore, Morrison(1986) extended the temporary equilibrium model to an explicit dynamic adjustment framework and provided a more general approach to productivity growth by allowing NCRTS, nonstatic expectations, and internal adjustments costs.

We will develop the case of *NCRTS* and combined adjustment costs. Combined adjustment costs framework[18] comprises internal and external adjustment costs. Here, internal adjustment cost affect production possiblilities in the short-run, whereas external adjustment costs affects the external costs of acquiring a quasi-fixed factor without affecting production possibilities. Consider the total costs which comprise variable costs with internal adjustment costs, external adjustment costs, external adjustment costs $h\dot{X}_j$),and quasi-fixed costs $\sum P_j X_j$

$$C(P_i, P_j, X_j, \dot{X}_j, Q, t) = G(P_i, X_j, \dot{X}_j, Q, t) \sum_{j\in FX} h(\dot{X}_j) + \sum_{j\in FX} P_j X_j \tag{26}$$

3.3.1 Caves, Christensen, and Swanson(1981) Approach.

If we focus on only the 'short-run variable cost function with internal adjustments' (G), we can derive the augmented RTS by extending the framework of Caves et al. (1981).[19] A generalized formula in the presence of adjustment costs can be derived by modifying the *primal* static production function in (14) as follows:

$$F(\ln Q_m, \ln X_i, \ln X_j, \ln \dot{X}_j, t) = 1 \tag{27.a}$$

By definition, $\widetilde{RTS}$ is written as

17) See the special edition of Journal of Econometrics (Vol. 33) published in 1986.

18) See Lee (1992b) for details.

19) They did not consider the effect of the adjustment costs in the original framework.

$$\tilde{RTS} \equiv \frac{-\sum F_{Xl}}{\sum F_{Qm}} = \frac{-[\sum_{i\in X} F_{X_i} + \sum_{j\in FX}(F_{X_j} + F_{\dot{X}_j})]}{\sum F_{Qm}}$$

$$= -\frac{\{\sum_{i\in X} F_{X_i} + \sum_{j\in FX}(F_{X_j} + F_{\dot{X}_j})\}/\sum_{i\in X} F_{X_i}}{\sum F_{Qm}/\sum_{i\in X} F_{X_i}} \tag{27.b}$$

where l represent the indices of factors, X_i, X_j, and $\dot{X}$

Then, a *dual* cost function to (27.a) is written as

$$\ln G = G(\ln Q_m, \ln P_i, \ln X_j, \ln \dot{X}_j, t) \tag{27.a'}$$

where G implies short-run variable costs and $G(\ln Q_m, \ln P_i, \ln X_j, \ln \dot{X}, t)$ represents a restricted variable cost function. Totally differentiating (27.a') yields

$$0 = \sum \frac{\partial \ln G}{\partial \ln Q_m} \mathrm{d}\ln Q_m + \sum \frac{\partial \ln G}{\partial \ln P_i} \mathrm{d}\ln P_i + \sum \frac{\partial \ln G}{\partial \ln X_j} \mathrm{d}\ln X_j$$

$$+ \sum \frac{\partial \ln G}{\partial \ln \dot{X}_j} \mathrm{d}\ln \dot{X}_j + \frac{\partial \ln G}{\partial t} \mathrm{d}t. \tag{28.a}$$

Similar to (16.b), (16.c), and (16.d), we can define the short-run idea of productivity growth or technical change of the "restricted variable cost function with internal adjustment costs". In the following, a superscript G denotes this extended form of cost function which is distinguished from either C or VC.

[1] $PGY^G(\mathrm{d}\ln P = \mathrm{d}\ln X_j = \mathrm{d}\dot{X}_j = 0)$

Equation (28.a) yields

$$PGY = -\frac{(\partial \ln G/\partial t)}{\sum_m \partial \ln G/\partial \ln Q_m}. \tag{28.b}$$

[2] $PGX^G(\mathrm{d}\ln Q = 0)$

If we extend the definition of PGX as the common rate at which all inputs and prices can grow over time with outputs held fixed, we get $PGX^G \equiv \mathrm{d}\ln P/\mathrm{d}t = -\mathrm{d}\ln X/\mathrm{d}t = -\mathrm{d}\ln \dot{X}/\mathrm{d}t \ \forall \ i$ and j. Then,

$$0 = \sum \frac{\partial \ln G}{\partial \ln P_i} \cdot \frac{\mathrm{d}\ln P_i}{\mathrm{d}t} + \sum \frac{\partial \ln G}{\partial \ln X_j} \cdot \frac{\mathrm{d}\ln X_j}{\mathrm{d}t} + \sum \frac{\partial \ln G}{\partial \ln \dot{X}_j} \cdot \frac{\mathrm{d}\ln \dot{X}_j}{\mathrm{d}t} + \frac{\partial \ln G}{\partial t}$$

$$= \left(\sum \frac{\partial \ln G}{P_i} - \sum \frac{\partial \ln G}{\partial \ln X_j} - \sum \frac{\partial \ln G}{\partial \ln \dot{X}_j}\right) \cdot PGX^G + \frac{\partial \ln G}{\partial t}$$

Since $\sum \partial \ln G/\partial \ln P_i = 1$,

$$PGX_G = \frac{-\partial \ln G/\partial t}{1 - \sum \partial \ln G/\partial \ln X_j - \sum \partial \ln G/\partial \ln \dot{X}_j}. \tag{28.c}$$

[3] $RTS^G(\mathrm{dt} = 0)$

Since RTS is the ratio of PGY to PGX, we can derive RTS^G as follows:

$$RTS^G = \frac{PGY^G}{PGX^G} = \frac{1 - \sum \partial \ln G/\ln X_j - \sum \partial \ln G/\partial \ln \dot{X}_j}{\sum_m \partial \ln G/\partial \ln Q_m}. \tag{28.d}$$

This approach, however, is limited in its applicability to the external adjustment costs ($h(\dot{X}_j)$) because it concerns only the short-run variable cost function $G(P_i, X_j,$

$\dot{X}_i$, Q, t) which corresponds to short-run variable costs ($\sum P_i X_i$). External adjustment costs and quasi-fixed costs are implicitly assumed to be fixed in the short-run or non-relevant to the functional relationship between variable costs and variable cost function. When total costs ($\sum P_i X_i + \sum P_j X_j$) are considered, it is possible to analyze a more general case. Morrison (1986) provided a more flexible framework[20] by extending Berndt and Fuss(1986).

3.3.2 Morrison(1986) Approach.

To deal with the most general case asin 926), we will extend the framework developed by Morrison(1986), based on the relationship between short-run and long-run total costs. When we focus on the total costs, we can implicitly assume the short-run total cost function for a single product.

$$C = C(P_i, P_j, X_j, \dot{X}, Q, t) \tag{26'}$$

where C represents the total costs in the short-run. Totally and logarithmically differentiating (26') yields

$$\begin{aligned} \mathrm{d}\ln C = & \sum \frac{\partial \ln C}{\partial \ln P_i} \mathrm{d}\ln P_i + \sum \frac{\partial \ln C}{\partial \ln P_j} \mathrm{d}\ln P_j + \sum \frac{\partial \ln C}{\partial \ln X_j} \mathrm{d}\ln X_j \\ & + \sum \frac{\partial \ln C}{\partial \ln \dot{X}_j} \mathrm{d}\ln \dot{X}_j + \frac{\partial \ln C}{\partial \ln Q} \mathrm{d}\ln Q + \frac{\partial \ln C}{\partial t} \mathrm{d}t. \end{aligned} \tag{29.a}$$

Since scale effects are related to the stock variables, X_j and $\dot{X}_j$, we can derive a long-run scale elasticity, η[21], by assuming that $\mathrm{d}\ln P_i = \mathrm{d}\ln P_j = \mathrm{d}t = 0$ in(29.a).

$$\begin{aligned} \eta = \frac{\mathrm{d}\ln C}{\mathrm{d}\ln Q} &= \frac{\partial \ln C}{\partial \ln Q} + \sum \frac{\partial \ln C}{\partial \ln X_J} \frac{\mathrm{d}\ln X_J}{\mathrm{d}\ln Q} + \sum \frac{\partial \ln C}{\partial \ln \dot{X}_J} \frac{\mathrm{d}\ln \dot{X}_J}{\mathrm{d}\ln Q} \\ &= \frac{Q}{C}\left[\frac{\partial C}{\partial Q} + \frac{Q}{C}\sum \frac{\partial C}{C} \cdot \frac{X_J}{\partial X_J} \cdot \frac{\mathrm{d}X_j}{X_j} \cdot \frac{Q}{\mathrm{d}Q} + \frac{C}{Q}\sum \frac{\partial C}{C} \cdot \frac{\dot{X}_j}{\partial \dot{X}_j} \cdot \frac{\mathrm{d}\dot{X}_j}{\dot{X}^j} \frac{Q}{\mathrm{d}Q}\right] \\ &= \frac{Q}{C}\left[\frac{\partial C}{\partial Q} + \sum_{j\in FX} \frac{\partial C}{\partial Xj} \frac{\mathrm{d}X_j}{\mathrm{d}Q} + \sum_{j\in FX} \frac{\partial C}{\partial \dot{X}^J} \frac{\mathrm{d}\dot{X}_J}{\mathrm{d}Q}\right]. \end{aligned} \tag{29.b}$$

Morrison(1986) interpreted the first term in (29.b) as the short-run scale elasticity of total costs.

$$\tilde{\varepsilon}_{CQ} = \frac{\partial \ln C}{\partial \ln Q} = \frac{Q}{C} \cdot \frac{\partial Q}{\partial C} \tag{29.c}$$

Just as Berndt, Fuss, and Waverman(1979) defined the long-run elasticity(η), it can be expressed as the short-run elasticity($\tilde{\varepsilon}_{CQ}$) and the additional effect of quasi-fixed

20) She did not consider the case of external adjustment costs, but that of internal adjustment costs.

21) Note that η^{-1} reflect the long-run returns to scale.

factors, including adjustment costs. If we define

$$\eta_i = \frac{\mathrm{d}\ln X_i}{\mathrm{d}\ln Q} = \frac{\mathrm{d}\ln \dot{X}^i}{\mathrm{d}\ln Q}, \tag{30}$$

then equation (29.b) can be rewritten as

$$\eta = \tilde{\varepsilon}_{CQ} + \sum_{j\in FX} \eta_j(\varepsilon_{CX_j} + \varepsilon_{C\dot{X}_j})$$

$$\text{or}\quad \varepsilon_{CQ} \doteq \eta[1 - \sum_{j\in FX} \frac{\eta_j}{\eta}(\varepsilon_{CX_j} + \varepsilon_{C\dot{X}_j})] \tag{31}$$

$$\text{where } \tilde{\varepsilon}_{CQ} = \frac{\partial \ln C}{\partial \ln Q},\ \varepsilon_{CX_i} = \frac{\partial \ln C}{\partial \ln X_i},\ \varepsilon_{C\dot{X}_i} = \frac{\partial \ln C}{\partial \ln \dot{X}_i}.$$

$\varepsilon_{C\dot{X}_j}$ can be further decomposed into a convex combination of elasticities associated with internal and external adjustment costs under the framework of the combined adjustment costs in (26).

$$\varepsilon_{C\dot{X}_j} = \frac{\partial \ln C}{\partial \ln \dot{X}} = \frac{\dot{X}_j}{C}(G_{\dot{X}_j} + h_{\dot{X}_j})$$

$$= \frac{1}{C}\left(G\frac{\dot{X}_j}{G}\frac{\partial G}{\partial \dot{X}} + h\frac{\dot{X}_j}{h}\frac{\partial h}{\partial \dot{X}_j}\right) = \frac{G}{C}\varepsilon_{G\dot{X}_j} + \frac{h}{C}\varepsilon_{h\dot{X}_j}, \tag{32}$$

where $\varepsilon_{G\dot{X}_i} = \partial \ln G/\partial \ln \dot{X}_j$ and $\varepsilon_{h\dot{X}_j} = \partial \ln h/\partial \ln \dot{X}_j$ Substitution of (32) into (31) yields a general expression for short-run scale elasticity of total costs

$$\tilde{\varepsilon}_{CQ} = \eta\left[1 - \sum_{j\in FX}\frac{\eta_j}{\eta}(\varepsilon_{CX_j} + \frac{G}{C}\varepsilon_{G\dot{X}_j} + \frac{h}{C}\varepsilon_{h\dot{X}_j})\right]. \tag{33}$$

We can also derive a long-run cost elasticity with respect to time, analogue to a long-run scale elasticity which can be decomposed into the short-run elasticity and the additional effects of quasi-fixed factors.

$$\frac{\mathrm{d}\ln C}{\mathrm{d}t} = \frac{1}{C}\left[\frac{\mathrm{d}C}{\mathrm{d}t}\Big|_{X_i} + \sum_{j\in FX}\frac{\partial C}{\partial X_i}\frac{\mathrm{d}X_i}{\mathrm{d}t}\right] \tag{34}$$

where $\mathrm{d}\ln C/\mathrm{d}t$ represents the change in total costs from technical progress toward long-run equilibrium level. Expanding (34) gives

$$\frac{\mathrm{d}\ln C}{\mathrm{d}t} = \frac{1}{C}\left[\frac{\partial C}{\partial t} + \frac{\partial C}{\partial Q}\frac{\mathrm{d}Q}{\mathrm{d}t} + \sum_{i\in V}\frac{\partial C}{\partial P_i}\frac{\mathrm{d}P_i}{\mathrm{d}t} + \sum_{j\in FX}\frac{\partial C}{\partial P_j}\frac{\mathrm{d}P_j}{\mathrm{d}t} + \sum_{j\in FX}\frac{\partial C}{\partial \dot{X}_j}\frac{\mathrm{d}\dot{X}^j}{\mathrm{d}t}\right]$$

$$+ \left(\frac{1}{C}\sum_{j\in FX}\frac{\partial C}{\partial X_j}\frac{\mathrm{d}X_j}{\mathrm{d}t}\right).$$

$$= \tilde{\varepsilon}_{Ct} + \tilde{\varepsilon}_{CQ}\frac{\dot{Q}}{Q} + \sum_{i\in V}\frac{P_iX_i}{C}\frac{\dot{P}_i}{P_i} + \sum_{j\in FX}\frac{P_jX_j}{C}\frac{\dot{P}_j}{P_j} + \sum_{j\in FX}\frac{(P+G_{X_j})X_j}{C}\frac{\dot{X}_j}{X_j}$$
$$+ \sum_{j\in FX}\frac{(G_{\dot{X}_j}+h_{\dot{X}_j})\dot{X}_j}{C}\frac{\ddot{X}_j}{\dot{X}_j}$$

where $\dot{X}_j$ is the first total derivative with respect to time, $\frac{dX_j}{dt}$; $\ddot{X}_j$ is the second total derivative with respect to time, $\frac{d\dot{X}_j}{dt}$; G_{X_j}, $G_{\dot{X}_j}$, and $h_{\dot{X}_j}$ represent the first partial derivative of G and h with respect to their arguments in subscript; $\tilde{\varepsilon}_{Ct}$ is technical change of short-run total cost function, $\frac{\partial \ln c}{\partial t}$.

Alternatively, equation(35) can be also obtained from(29a). Just as Ohta(1974) linked the total costs to the total cost function, Morrison(1986) matched the variable plus quasi-fixed costs with the total cost function. Similarly, if we defined the long-run $\dot{C}/C$ from the total cost dquation[22] $C = \sum_{i\in V}P_iX_i + \sum_{j\in FX}P_jX_j$

$$\frac{\dot{C}}{C} = \sum_{i\in V}[\frac{X_iP_iP_i}{C}\frac{\dot{P}_i}{P_i} + \frac{P_jX_j}{C}\frac{\dot{X}_j}{X_j}]\sum_{j\in FX}[\frac{X_jP_j}{C}\frac{\dot{P}_j}{P_j} + \frac{P_jX_j}{C}\frac{\dot{X}_j}{X_j}] \quad (36)$$

Because two equation (35) and (36) are assumed to be identical to each other, a new relationship is obtained by rearanging the both sides.

$$-\tilde{\varepsilon}_{Ct} = \tilde{\varepsilon}_{CQ}\frac{\dot{Q}}{Q} - \sum_{i\in V}\frac{P_iX_i}{C}\frac{\dot{X}_i}{X_i} - \sum_{k\in FX}\frac{\mu_{X_j}X_j}{C}\frac{\dot{X}_j}{X_j} - \sum_{j\in FX}\frac{\mu_{\dot{X}_j}\dot{X}_j}{C}X \quad (37)$$

Where $\mu_{X_j} = -G_{X_j}$ is a shadow value for X_j and $\mu_{\dot{X}_j} = -(G_{\dot{X}_j} + h_{\dot{X}_j})$ is a shadow value for net investment, $\dot{X}_j$.[23]

Equation(37), an extension of equation (13), shows again the relationship between technical change ($-\varepsilon_{Ct}$) in the left hand side and productivity growth in the right hand side. Either productivity measurement will deviate from the genuine measure because scale effect(Q), quasi-fixity(X), and adjustment costs ($\dot{X}$) in the underlying cost function (26) will bias it. Since an interpretation of technical change is in parallel with that of productivity growth, we will use either terminology interchangeably in the following. For convenience's sake, ε_{Ct} is denoted as the observed or unadjusted TFP growth or technical change. To adjust the observed TFP growth, substituting ε_{CQ}

22) In the long-run equilibrium, $h(\dot{X})$ in the total cost will vanish since $\dot{X}_j = 0$ at a steady state.

23) When adjustment costs are involved, first-order Euler condition, $-G_{X_j} = P_j + r(G_{\dot{X}_j} + h_{\dot{X}_j})$, for intertemporal minimization of total costs requires that $\mu_{X_j} + \gamma\mu_{\dot{X}_j} = P_j$

in (33) into (37) and dividing both sides by $\tilde{\varepsilon}_{CQ}$ yields the corrected TFP growth, say ε_{Ct}.

$$\varepsilon_{Qt} \equiv -(\tilde{\varepsilon}_{CQ})^{-1}\tilde{\varepsilon}_{Ct} = \frac{\dot{Q}}{Q} - \frac{1}{\eta\left[1-\sum_{j\in FX}\frac{\eta_j}{\eta}(\varepsilon_{CX_j} + \frac{G}{C}\varepsilon_{G\dot{X}_j} + \frac{h}{C}\varepsilon_{h\dot{X}_j})\right]}$$

$$\left(\sum_{i\in X}\frac{P_iX_i}{C}\frac{\dot{X}_i}{X_i} + \sum_{j\in FX}\frac{\mu_{X_j}X_j}{C}\frac{\dot{X}_j}{X_j} + \sum_{j\in FX}\frac{\mu_{\dot{X}_j}\dot{X}_j}{C}\frac{\ddot{X}_j}{\dot{X}_j}\right) \tag{38}$$

Several implication emerge from the equation (38) which provides the correct productivity measurement.

Adjusted TFP Growth Weighted by Shoadow Values. The left hand side of (38) represent the primal rate of technical change, ε_{Qt} reconfirming the dual relationship (4. c'), and the right hand side of (38) shows the adjusted productivity residual which is usually seen in the index number approach. Note that the sum of weighted shares by shadow prices ($\mu_{X_j}$and $\mu_{\dot{X}_j}$) is equal to 1 because adjustment by $\tilde{\varepsilon}_{CQ}$ will purge the effect of temporary equilibrium since $\tilde{\varepsilon}_{CQ}$ converts the term C (sum of total equilibrium costs) in the denominator in the second part of (38) into the term C^* (sum of total shadow costs), the adjusted share of costs weighted by shadow costsf will sum to unity.

Generalized Adjusting Elasticity. Previous equations for ε_{CQ} or RTS, which are derived under the assumption of NCRTS and quasi-fixity, are special cases of (33). Morrison (1986) showed that if cost function is homothetic, $\eta = \eta$. Equation (33) reduces to

$$\tilde{\varepsilon}_{CQ} = \left[1-\sum_{j\in FX}(\varepsilon_{CX_j} + \frac{G}{C}\varepsilon_{G\dot{X}_j} + \frac{h}{C}\varepsilon_{h\dot{X}_j})\right]. \tag{39.a}$$

When a long-run CRTS[24]($\eta = 1$) is further imposed on (39.a),

$$\tilde{\varepsilon}_{CQ} = \left[1-\sum_{j\in FX}(\varepsilon_{CX_j} + \frac{G}{C}\varepsilon_{G\dot{X}_j} + \frac{h}{C}\varepsilon_{h\dot{X}_j})\right]. \tag{39.b}$$

Combined adjustment costs framework comprises both internal (when $\varepsilon_{h\dot{X}_j} = 0$) and external (when $\varepsilon_{G\dot{X}_i} = 0$) adjustment costs cases. When $\varepsilon_{C\dot{X}}(= \varepsilon_{G\dot{X}_j} = \varepsilon_{h\dot{X}}) = 0$, $\tilde{\varepsilon}_{CQ}$ will represent the case of restricted variable cost function. Then, equation (39.b) is more simplified to

$$\tilde{\varepsilon}_{CQ} = \left[1-\sum_{j\in FX}\varepsilon_{CX_j}\right]. \tag{39.c}$$

Equation (39.c) turns out to be consistent with that of Caves et al. (1981) who paid the restricted attention to the short-run variable cost function G. Let ε_{GQ} denote $\partial \ln G/\partial \ln$

24) A stronger short-run CRTS assumption can be also imposed on (39a)

Q. First, we get the relationship between $\tilde{\varepsilon}_{CQ}$ and ε_{GQ}. The left hand side of (39.c) is written as

$$\tilde{\varepsilon}_{CQ} = \frac{\partial \ln C}{\partial \ln Q} = \frac{G}{C} \cdot \varepsilon_{GQ}$$

The right hand side of (39.c) can be expressed as

$$\tilde{\varepsilon}_{CQ} = 1 - \Sigma\left(\frac{G}{C}\frac{\partial \ln G}{\partial \ln X_j} + \frac{P_j X_j}{C}\right) = 1 - \Sigma\left(\frac{\partial G}{\partial X_j} + P_j\right) \cdot \frac{X_j}{C}$$

$$= \frac{C - \Sigma P_j X_j}{C} - \frac{G}{C}\Sigma\frac{\partial G}{\partial X_j} \cdot \frac{X_j}{C} \cdot \frac{C}{G} = \frac{G}{C}\left(1 - \Sigma\frac{\partial \ln G}{\partial \ln X_j}\right)$$

Thus, we can derive ε_{GQ} as

$$\varepsilon_{GQ} = \left(1 - \Sigma\frac{\partial \ln G}{\partial \ln X_j}\right). \tag{39.d}$$

Equation (39.d) defined by Caves et al. (1981), can be directly obtained from (39.c) which is a special case of Morrison's (1986) framework. If $\varepsilon_{C\dot{X}_j} = \varepsilon_{CX_j} = 0$ in the long-run where $\dot{X}_j = 0$ and market rental prices for the quasi-fixed factors, P_j, are equal to the corresponding shadow values, μ_{X_j} for all j,[25] then we get Ohta's (1974) version of long-run scale elasticity.

$$\tilde{\varepsilon}_{CQ} = \eta = \frac{\partial \ln C}{\partial \ln Q}. \tag{39.e}$$

Therefore, it can be confirmed that the three generations of adjusting elasticities, (39.c), (39.d), and (39.e), are consistent each other, and previous versions of (39.d) [Caves et al. (1981)] and (39.e)[Ohta (1974)] are special cases of generalized adjusting elasticity, (33) [extended version of Morrison (1986)].

Capacity Utilization Ratios. Berndt and Fuss (1986) associated Tobin's q[26] with capacity utilization (hereafter CU). Under CRTS ($\eta = \eta_j = 1$), equation (33) reduces to equation (39.b).

$$\dot{\varepsilon}_{CQ} = 1 - \sum_{j \in FX}\left(\varepsilon_{CX_j} + \frac{G}{C}\varepsilon_{G\dot{X}_j} + \frac{h}{C}\varepsilon_{h\dot{X}_j}\right)$$

$$= \frac{C - \sum_{j \in FX}(P_j + G_{X_j})X_j - \sum_{j \in FX}(G_{\dot{X}_j} + h_{\dot{X}_j})\dot{X}_j}{C}$$

$$= \frac{C - \sum_{j \in FX}(P_j - \mu_{X_j})X_j + \sum_{j \in FX}\mu_{\dot{X}_j}\dot{X}_j}{C} = \frac{G + \sum \mu_{X_j}X_j + \sum \mu_{\dot{X}_j}\dot{X}_j}{C}$$

25) This implies that $E_{CX_j} = (X/C)(P_j - \mu_{X_j}) = 0$.

26) Tobin's q is defined as the market value of the firm divided by the replacement cost of its physical capital stock.($q \equiv \mu_K / P_K$).

$$= \frac{\text{sum of total shadow costs at temporary equilibrium}}{\text{sum of total costs at lonqg-run equilibrium}}$$
$$= \frac{C^*}{C} = \frac{Q}{Q^*} = CU. \tag{40}$$

Furthermore, Morrison (1986) interpreted CU as *multi-input Tobin's q* and argued that CU adjustment is a clear measure in the case of multiple quasi-fixed factors. If a temporary equilibrium (or subequilibrium) coincides with the long-run equilibrium ($\mu_{X_j} = P_j$ and $\mu_{\dot{X}_j} = 0$), $\tilde{\varepsilon}_{CQ} = CU = 1$ in (40). Since C is equal to C^*, (38) is rewritten as

$$\begin{aligned}
\varepsilon_{Qt} &= -(\varepsilon_{CQ})^{-1}\tilde{\varepsilon}_{Ct} \\
&= \frac{\dot{Q}}{Q} - \frac{C}{C^*} \cdot \left(\sum_{i \in V} \frac{P_i X_i}{C} \frac{\dot{X}_i}{X_i} + \sum_{j \in FX} \frac{\mu_{X_j} X_j}{C} \frac{\dot{X}_j}{X_j} + \sum_{j \in FX} \frac{\mu_{\dot{X}_j} \dot{X}_j}{C} \frac{\ddot{X}_j}{\dot{X}_j} \right) \\
&= \frac{\dot{Q}}{Q} - \left(\sum_{i \in V} \frac{P_i X_i}{C^*} \frac{\dot{X}_i}{X_i} + \sum_{j \in FX} \frac{\mu_{X_j} X_j}{C^*} \frac{\dot{X}_j}{X_j} + \sum_{j \in FX} \frac{\mu_{\dot{X}_j} \dot{X}_j}{C^*} \frac{\ddot{X}_j}{X_j} \right) \\
&= \frac{\dot{Q}}{Q} - \left(\sum_{i \in V} \frac{P_i X_i}{C} \frac{\dot{X}_i}{X_i} + \sum_{j \in FX} \frac{P_j X_j}{C} \frac{\dot{X}_j}{X_j} \right) \\
&= \frac{\dot{Q}}{Q} - \sum_{i \in V, F} S_i \frac{\dot{X}_i}{X_i}
\end{aligned} \tag{38'}$$

When $\tilde{\varepsilon}_{Ct}$ is adjusted by $\tilde{\varepsilon}_{CQ}$ as in (38'), this verifies the primitive relationship defined in (9). Under NCRTS($\eta \neq 1$), $\tilde{\varepsilon}_{CQ}$ captures both scale and CU effects so that $\varepsilon_{CQ} = \eta \cdot CU$. Table 1 provides relationships of functional forms with regard to the adjusting elasticity $\tilde{\varepsilon}_{CQ}$. Just as combined funcrtional form provides the general structure of the total cost function, $\tilde{\varepsilon}_{CQ}$ under combined adjustment costs also describes the most general case. Berndt and Fuss(1989) also considered various measures [27] of capacity utilization rates in the multiproduct context ($Q = \sum_m Q_m$, for $m = 1, \cdots, M$). In the case of multiple inputs and outputs, shadow valuation measure of CU like(40) seems to provide a clearer interpretation of capacity utilization than any other alternative.

To sum up the three main approaches developed by Ohta (1974), Caves et al. (1981), Morrison (1986), the last two approaches differ in one respect. Caves et al. (1981) tried to capture a shift in the short-run variable cost function from the concept of short-run total costs, while Morrison (1986) tried to adjust the oberved TFP

27) They suggested alternative CU measures : $CU_Q = Q/Q^*$ for output-specific CU rates ; $CU_X = X^*/X$ for quasi-fixed input-specific CU rates ; $CU_{MC} = SRMC/LRMC$ for short-run and long-run marginal costs ; $CU_q = \mu_X/P$ for Tobin's q ; CU_{TC} = Shadow Total Costs/Total Costs ; $CU_V = X/X^*$ for variable input-specific CU rates. Lee and Siegel(1992) and Lee(1994) also presented empirical comparisons for various CU measures in US disaggregate manuacturing sectors.

growth from the short-srun total costs (SRTC) by the adjusting scale elasticity $(\tilde{\varepsilon}_{CQ})$[28] obtained from the relationship between short-run and long-run total costs.

4. CONCLUDING REMARKS

Over the last decades, much contribution has been made to the standard time trend approach by relaxing the restrictive assumptions on the production technology such as disembodied, neutral TC, and constand returns to scale, and by introducing several concepts of duality, dynamics, factor augmentation and capital vintage into the traditional production function.

Recent developments in empirical studies can be characterized by two distinct measurement issues. One concerns adjusting traditional measures of productivity in terms of scale economies and temporary equilibrium. The other concerns measuring embodied *vs.* disembodied technical change. The first issue was further developed in this paper, using ther *third generation* dynamic factor demand model. The second issue can be developed and applied to embodied factor demand model. In this paper we focused on the duality and dynamic property of technical change, drawn from a short-run cost function, by adjusting the traditional measures of TFP growth, derived from a long-run cost function, by adjusting the traditional measures of TFP growth, derived from a long-run production function.

The main purpose of this paper was to provide a systematic overview of technical change and productivity growth by reinterpreting the theoretical identity between TC and TFP growth and to investigate a theoretical background for adjusting the traditional measures. Focusing on the time-trend model for a cost function which is dual to a production function, we reviewed and compared three main approaches developed by Ohta(1974), Caves et al. (1981), Morrison (1986), and to isolate the genuine TC from observed TFP growth. Furthermore, we reviewed and compared three main approaches develped by Ohta (1974), Caves et al. (1981), Morrison (1986), and to isolate the genuine TC from observed TFP growth. Furthermore, we extended Morrison's (1986) framework by including the external adjustment costs as well as the internal adjustment costs and showed how traditional productivity measures at a static equilibrium should be changed as the concepts of duality and dynamics are introduced into the underlying function, upon which parametric measurement of productivity is based.

28) This adjusts the scale effect in the narrow sense, quasi-fixity effect, net change effect of quasi-fixed factor stocks. Since $\tilde{\varepsilon}_{CQ} = \eta \cdot CU$, this measure represents the capacity utilization ratios under CRTS. when $\eta = 1$, $\tilde{\varepsilon}_{CQ} = CU$.

Table 1. Relationship Between Functional Form and Adjusting Elasticity

	Functional Forms	ε_{CQ}
Full Static	$G(P_i, Q, t)$	η
Partial Static	$G^S(P_i, X_j, Q, t) + \sum P_j X_j$	$\eta[1-\sum\frac{\eta_j}{\eta}(\varepsilon_{CX_j})]$
Dynamic Internal Adj.	$\tilde{G}^S(P_i, X_j, \dot{X}_j, Q, t) + \sum P_j X_j$	$\eta[1-\sum\frac{\eta_j}{\eta}(\varepsilon_{CX_j} + \frac{G}{C}\varepsilon_{G\dot{X}_j})]$
External Adj.	$\tilde{G}^S(P_i, X_j, Q, t) + h(\dot{X}) + \sum P_j X_j$	$\eta[1-\sum\frac{\eta_j}{\eta}(\varepsilon_{CX_j} + \frac{h}{C}\varepsilon_{h\dot{X}_j})]$
Combined Adj.	$\tilde{G}^S(P_i, X_j, \dot{X}_j, Q, T) + h(\dot{X}) + \sum P_j X_j$	$\eta[1-\sum\frac{\eta_j}{\eta}(\varepsilon_{CX_j} + \frac{G}{C}\varepsilon_{G\dot{X}_j} + \frac{h}{C}\varepsilon_{h\dot{X}_j})]$
Mixed Adj.	$\tilde{G}^S(P_i, X_j, \dot{X}_j^n, Q, t) + h(\dot{X}^x) + \sum P_j X_j$	$\eta[1-\sum\frac{\eta_j}{\eta}(\varepsilon_{C\dot{X}_j} + \frac{G}{C}\varepsilon_{G\dot{X}^n}\frac{h}{C}\varepsilon_{h\dot{X}_j})]$

Note ;

In case of mixed adjustment cost technology, a vector of quasi-fixed factor can be spilited into X^n and X^x, where $X^n(X^x)$ is associated with a quasi-fixed factor with internal (external) adjustment costs.

ε_{CQ} : adjusting elasticity of total costs

C : a total cost function as described in the column of functional forms

G : a full static long-run cost function

G^S : a restricted cost function

$\tilde{G}^S$: a restricted cost function with internal adjustment costs

h : an external adjustment cost function

$\eta = \frac{d\ln C}{d\ln Q}$, long-run scale elasticity

$\eta_j = \frac{d\ln X_j}{d\ln Q}$, $\varepsilon_{ij} = \frac{\partial \ln i}{\partial \ln j}$

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