

IS INCREASED WAGE FLEXIBILITY STABILIZING?

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1. INTRODUCTION

There has been a long debate about the relationship between price flexibility and output stability. It is widely believed that output in the U.S. has been more stable since World War II than it was before World War II while wages and prices have been less flexible in the postwar period than in the prewar period¹⁾. Driskill and Sheffrin(1986) examine whether increased price flexibility can be destabilizing in a version of Taylor's(1979) contracting model extended to include real interest rate effects and the presence of white noise wage shock. Taylor's model includes both backward and forward elements in wage-setting behavior and thus permits some rationality in the wage-setting process. Driskill and Sheffrin(1986) find that increased wage flexibility leads to a decrease in the variance of output.

De Long and Summers(1986) also use Taylor's model of overlapping contracts to show that increased wage and price flexibility can easily be destabilizing. They conclude that, although increased price flexibility is stabilizing with respect to supply-side shocks(specifically a shock to the level of wages), it may have relatively small stabilizing effects and may even amplify the output fluctuations induced by demand shocks.

In the analysis of De Long and Summers, the definition of increased flexibility is an increased responsiveness of wages to expected output. King(1988) uses a different definition. He defines increased flexibility as an increase in the proportion of workers whose wages are determined in spot markets rather than contract markets. With this definition, he argues that increased flexibility unambiguously decreases the variance of output resulting from both demand and supply disturbances.

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1) See Romer(1986).

In the above studies, most researchers define the aggregate price level as an average of the two different wages in force at that moment—the contract which begins in period $t-1$ and the contract which begins in period t . The contract wage at time t is set as the linear combination of the average of the contracted wages and the average of output levels. The De Long and Summers' papers (1986, 1988), along with the other papers in this series of studies, treat the degree of flexibility of the price system as exogenous. In contrast, Gray and Kandil (1991) examine the relationship between price flexibility and output variability in a framework in which both are endogenously determined and conclude that output stability and flexibility are not causally related. Both prices and output are endogenously determined and the relationship between the two depends on the exogenous factors affecting both. They demonstrate that a smaller variance for aggregate demand shocks in the postwar period compared to the prewar period can explain the apparently contradictory observation of reduced output volatility in the postwar period. In their model increased stability of aggregate demand reduces output variability. Increased contract length, in turn, has a destabilizing effect on output. However the effect does not eliminate the decline in output volatility produced by a reduction in aggregate demand variability.

The objective of this paper is to examine the relationship between the flexibility of the price system and cyclical variability of output in the presence of both supply and demand shocks. Using a framework in which both are endogenously determined as in Gray and Kandil, we employ the stochastic continuous time model and extend the contracting models developed by Fischer (1977), Gray (1978), Canzoneri (1980), and Gray and Kandil (1991). We first add supply shocks and demand shocks which are assumed to follow Brownian Motion (or Wiener process), the continuous time counterpart of a random walk. We allow workers to index their wages to price innovation. The major issues we address in this essay are: If there is a mixed distribution of shocks, how does each shock affect output fluctuations? What is the implication of contract length on the relationship between shocks and output variability? How are the results changed if workers can choose the optimal indexation parameter. Additionally, we would like to determine if a smaller variance of aggregate demand shocks can still explain the observations of reduced output volatility in the postwar period in the presence of supply shocks and wage indexation.

2. THE MODEL

2.1 The Case of Exogenous Contract Length

The stochastic continuous time model described in this section and the next synthesized and extends the contracting models developed by Fischer (1977), Gray (1978), and Canzoneri (1980). The model is log-linear and unless otherwise noted all variables are expressed in logarithmic form.

The supply side of the economy consists of a continuum of identical firms that

are indexed by j , where j is distributed uniformly between zero and one. Each firm faces a log-linear production technology of the form

$$y_{jt} = a l_{jt} + S_{jt}, \quad 0 < a < 1, \quad (1)$$

where y_{jt} denotes the output of the firm j at time t and l_{jt} the associated level of labor input. S_{jt} is assumed to follow a Brownian (or Wiener) process :

$$S_{jt} = \int_{x=0}^t u_{jt-x} dx. \quad (2)$$

The unit innovations, u_{jt-x} , have mean zero and variance σ_u^2 . Labor demand and supply functions are as follows :

$$l_{jt}^d = b[(w_{jt} - p_t) - \ln a - S_{jt}], \quad -b = 1/(1-a) > 1, \quad (3)$$

$$l_{jt}^s = d, \quad d > 0. \quad (4)$$

The labor demand schedule (3) is elastic while, without loss of generality, the labor supply in (4) is assumed inelastic to make calculation simple. Once contract wages are determined, the labor supply function becomes irrelevant to the determination of employment levels. The nominal wage at time t is denoted by w_{jt} , and p_t is the price of output. Equations (3) and (4) may be solved jointly to obtain the Walrasian market clearing wage level, given by

$$w_{jt}^* = d/b + \ln a + p_t + S_{jt}. \quad (5)$$

Firms and workers specify a contract length, T , and a path of nominal wages, w_{jt}^* . The contractual wage path is determined by solving (3) and (4) for the market clearing wage and taking expectations based on the information available at the time of the last negotiated contract. We allow workers to index their wages to changes in the price level. That is,

$$\bar{w}_{jt} = w_{jt/t-s}^* + \theta (p_t - p_{t/(t-s)}), \quad (6)$$

Where s denotes the time elapsed since firm j negotiated its last contract ; s is distributed uniformly between zero and T . θ measures the adjustment in the contracted wage to an unexpected change in the price level. Substituting equations (2), (3) and (6) into equation (1) we have firm j 's supply function :

$$y_{jt} = ad + ab(\theta - 1)(p_t - p_{t/(t-s)}) - ab \int_{x=0}^t u_{jt-x} dx + \int_{x=0}^t u_{jt-x} dx. \quad (7)$$

Economy-wide output, y_t , is obtained by integrating over the outputs of individual firms. Assuming that j is uniformly distributed between 0 and 1 we have :

$$y_t = \int_{j=0}^1 y_{jt} dj = y_{jt} \quad (8)$$

Ordering firms by contract negotiation date, substituting equation (7) into (8), and changing the variable of integration from j to s produces the following expression for aggregate supply²⁾:

$$y_t^s = ad + \frac{1}{T} \left\{ ab(\theta - 1) \int_{s=0}^1 (p_t - p_{t-s}) ds - ab \int_{s=0}^1 \int_{x=0}^s u_{t-s} dx ds \right\} + \int_{x=0}^1 u_{t-x} dx \quad (9)$$

The aggregate demand schedule is represented by a single equation,

$$y_t^d = \alpha - p_t + D_t \quad (10)$$

The term D_t is intended to capture a variety of possible disturbances to aggregate demand, including changes in the money supply, velocity, government spending, and private spending. D_t is assumed to follow another Brownian motion:

$$D_t = \int_{x=0}^1 v_{t-x} dx. \quad (11)$$

The v_{t-x} denotes unit innovation in the demand process that occurs between the time history begins (time zero) and the present (time t). The demand side innovations are distributed with mean zero and variance σ_v^2 , and the integral of their values from time zero to time t defines D_t . Now, equating aggregate demand and aggregate supply, and using the method of undetermined coefficients, we have the following expression for the price level (see Appendix 1):

$$p_t = \alpha - ad - \int_{x=0}^1 u_{t-x} dx + \int_{x=0}^1 v_{t-x} dx + \int_{x=0}^1 \frac{ab\theta(T-x)}{T + ab(\theta-1)(T-x)} u_{t-x} dx - \int_{x=0}^T \frac{ab(\theta-1)(T-x)}{T + ab(\theta-1)(T-x)} v_{t-x} dx \quad (12)$$

Using (9) and (12) we have the following aggregate output schedule:

$$y_t = ad + S_t - \int_{x=0}^1 \frac{ab\theta T}{T + ab(\theta-1)(T-x)} u_{t-x} dx + \int_{x=0}^1 \frac{ab(\theta-1)T}{T + ab(\theta-1)(T-x)} v_{t-x} dx \quad (13)$$

2) Recall that firm j has a uniform distribution between $[0, 1]$ and the time variable s also uniformly distributed between $[0, T]$. To derive (9), we change the variables j to s by interchanging (7) from $s=0$ to T instead of integrating from $j=0$ to 1 and dividing whole terms by T . Only two terms in the middle of right hand side of equality depend on the time variable s . For example,

$$\int_{x=0}^1 y dx = \frac{1}{T} \int_{x=0}^T y ds, \text{ where } s \in [0, T].$$

In equation (13) the reduced form for output consists of two components. The first two terms on the right hand side give the full equilibrium level of output. These terms represent the output that would be achieved if wages were fully flexible. The full equilibrium does not respond to aggregate demand shocks. The second and third terms, which are represented as the sum of aggregate demand (D_t) and aggregate supply shocks (S_t), exhibit deviations of the actual value from its full equilibrium value. It is also noteworthy that, unlike Gray and Kandil (1991), a supply shock that occurred more than T periods ago still has cyclical impact today. By inspection, the deviation from demand shocks would be eliminated by full indexation ($\theta = 1$), while the deviation from full employment level of output would be stabilized by zero indexation ($\theta = 0$) in the case of supply disturbances.

2.2 Optimal Wage Indexation

In equation (13), given any contract length T , the output deviations are eliminated under full indexation ($\theta = 1$) in the absence of supply disturbances. Since indexing can completely insulate the real sector from nominal disturbances, full indexation is optimal. On the other hand, in an economy subject only to real shocks, θ is equal to zero. Now we want to find the optimal parameter θ^* which minimizes output variance in the model with both demand and supply shocks. The full equilibrium is determined at a situation in which price is equal to expected price. The full employment l_{jt}^* is obtained by substituting the market clearing level of wage rate into equation (3). Substituting l_{jt}^* for l_{jt} in equation (14) we have the full information level of output y_{jt}^* :

$$y_{jt}^* = a l_{jt}^* + S_t \quad (14)$$

Substituting (5) into (3) we have the expression for the full information level of output in equation (14),

$$y_{jt}^* = a d + \int_{x=0}^1 u_{t-x} dx \quad (15)$$

Use of (13) and (15) produces an expression for the deviation of output from full information output:

$$y_{jt} - y_{jt}^* = - \int_{x=0}^s \frac{ab\theta T}{T + ab(\theta - 1)(T - x)} u_{t-x} dx + \int_{x=0}^s \frac{ab(\theta - 1)T}{T + ab(\theta - 1)(T - x)} v_{t-x} dx \quad (16)$$

The firm j 's loss function is³⁾,

$$\begin{aligned} E[(y_{jt} - y_{jt}^*)^2] &= (abT)^2 [\theta^2 \sigma_u^2 + (\theta - 1)^2 \sigma_v^2] \int_{x=0}^s \left[\frac{1}{ab(\theta - 1)(T - x) + T} \right]^2 dx \\ &= \frac{s(abT)^2 [\theta^2 \sigma_u^2 + (\theta - 1)^2 \sigma_v^2]}{T[ab(\theta - 1)(T - s) + T] [ab(\theta - 1) + 1]} \end{aligned} \quad (17)$$

3) Minimizing this loss is consistent with maximizing the firm j 's profit. See Aizenman and Frenkel (1986).

Inspection of the firm's loss function shows that a firm's output loss depends on the indexation parameter, given contract length T .

Differentiating (17) with respect to θ yields

$$\frac{\partial E[(y_{1t} - y^*_{1t})^2]}{\partial \theta} = \frac{(ab)^2 Ts [\theta^2 \sigma_u^2 + (\theta - 1)^2 \sigma_v^2] ((\phi_1 + 1)[2\phi_2 - ab(T-s)] - ab[T(\phi_1 + 1) - \phi_s])}{(\phi_1 + 1)^2 \phi_2^2} \quad (18)$$

$$\begin{aligned} \text{where } \phi_1 &= ab(\theta - 1) \\ \phi_2 &= ab(\theta - 1)(T-s) + T \end{aligned}$$

Setting (18) equal to zero and solving for θ yields the equilibrium indexation decision :

$$\theta^* = \frac{(\delta + abT) \sigma_v^2 - (ab - 1) \delta \sigma_u^2 + \sqrt{[\sigma_v^2 + (ab - 1)^2 \sigma_u^2] [T^2 \sigma_v^2 + \delta^2 \sigma_u^2]}}{ab(2T - s) \sigma_v^2 - ab(2\delta + s) \sigma_u^2} \quad (19)$$

where $\delta = ab(T-s) - T$.

By inspection, this expression is bounded by 0 and 1, as in Gray(1976) and Ball(1988). Examination of the terms entering (19) reveals that the optimal indexing is a weighted average of the optimal corresponding to the extreme case of $\sigma_u^2 = 0$ and $\sigma_v^2 = 0$. Setting σ_u^2 equal to zero results in θ equal to one. On the other hadn, when $\sigma_v^2 = 0$, θ takes on the value zero. Full indexation ($\theta = 1$) arises if demand disturbances (σ_v^2) overwhelm supply disturbances (σ_u^2), which is the case of Gray and Kandil(1991). Therefore, in Gray and Kandil's analysis, which has no supply shocks, we can eliminate output deviations by full indexation regardless of the contract length. If supply disturbances overwhelm demand disturbances, non-indexation is optimal. The rationale behind this result rests on the distinction between real and nominal disturbances. While indexation insulates the real sector from nominal disturbances, it exacerbates the real effects of real disturbances. Therefore, in a system subject to both types of shocks, partial indexation is suggested. The optimal indexation parameter θ varies according to the shocks as is shown in equation (19). differentiating (19) with respect to σ_v^2 and σ_u^2 we have

$$\frac{\partial \theta^*}{\partial \sigma_v^2} > 0, \quad \frac{\partial \theta^*}{\partial \sigma_u^2} < 0.$$

As aggregate demand becomes more volatile a larger value of θ is needed to insulate the real sector from monetary shocks. On the other hand, as aggregate supply become more volatile θ goes to zero.

3. THE CASE OF ENDOGENOUS CONTRACT LENGTH

Firm j sets contract length T , taking as given T , the contract length chosen by all other firms. Following Gray and Kandil(1991), we assume that firm j selects T_j to minimize the expected average loss resulting from both of output deviations away from the full information level and of the actual cost of contracting, C . Firm j 's loss function is :

$$\text{Min } Z_j(T_j | T) = \frac{1}{T_j} \left\{ \lambda \int_{t=\tau}^{\tau+T_j} E[(y_{jt} - y^*)^2] dt + C \right\} \quad (20)$$

where γ represents the time at which the firm negotiates its contract and λ is a relative price that converts losses expressed as squared output deviations into losses expressed in the units of contracting cost. Using (17) we can rewrite (20) as follows :

$$\begin{aligned} Z_j(T_j | T) &= \frac{\lambda}{T_j} (abT)^2 [\theta^2 \sigma_u^2 + (\theta - 1)^2 \sigma_v^2] \int_{t=\tau}^{\tau+T_j} \int_{x=0}^s \left[\frac{1}{ab(\theta - 1)(T - x) + T} \right]^2 dx dt + \frac{C}{T_j} \end{aligned} \quad (21)$$

Changing the variable of integration from t to s in (21)⁴⁾, we have an expression for firm j 's expected costs per period.

$$\begin{aligned} Z_j(T_j | T) &= \frac{\lambda}{T_j} (abT)^2 [\theta^2 \sigma_u^2 + (\theta - 1)^2 \sigma_v^2] \int_{s=0}^{T_j} \int_{x=0}^s \left[\frac{1}{ab(\theta - 1)(T - x) + T} \right]^2 dx ds + \frac{C}{T_j} \end{aligned} \quad (22)$$

The contract length, T , chosen by other firms also enters equation (22) as a determinant of y_s , the firm j 's output, due to the fact that T affects the price level. Differentiating $Z_j(T_j | T)$ with respect to T , gives the first order condition :

$$\begin{aligned} \frac{\partial Z_j(T_j | T)}{\partial T_j} &= \frac{-\lambda}{T_j} (abT)^2 [\theta^2 \sigma_u^2 + (\theta - 1)^2 \sigma_v^2] \int_{s=0}^{T_j} \int_{x=0}^s \left[\frac{1}{ab(\theta - 1)(T - x) + T} \right]^2 dx ds \\ &\quad + \frac{\lambda}{T_j} (abT)^2 [\theta^2 \sigma_u^2 + (\theta - 1)^2 \sigma_v^2] \int_{x=0}^{T_j} \left[\frac{1}{ab(\theta - 1)(T - x) + T} \right]^2 dx - \frac{C}{T_j^2} \end{aligned} \quad (23)$$

The symmetric Nash equilibrium is found by setting $T_j = T$ in (23).

$$\frac{\partial Z_j}{\partial T_j} \Big|_{T_j=T}$$

4) Using a relationship between time variables s and t , $s = t - \tau$, we get $s = 0$ if $t = \tau$ and $s = T_j$ if $t = \tau + T_j$.

$$\begin{aligned}
&= -\lambda(ab)^2 [\theta^2 \sigma_u^2 + (\theta-1)^2 \sigma_v^2] \int_{x=0}^1 \int_{x=0}^1 \left[ab(\theta-1)(T-x) + T \right]^{-2} dx ds \\
&\quad + \lambda(ab)^2 [\theta^2 \sigma_u^2 + (\theta-1)^2 \sigma_v^2] \int_{x=0}^1 \left[ab(\theta-1)(T-x) + T \right]^{-2} dx - \frac{C}{T^2} \quad (24)
\end{aligned}$$

Setting the above expression equal to zero and solving for the optimal contract length T^* , we have⁵

$$\begin{aligned}
T^* &= -\frac{\beta}{2\gamma} - \frac{\sqrt{\left\{ \frac{\beta}{2\gamma} \right\}^2 + \frac{\beta(1+\beta)C}{(ab)^2 \gamma \lambda [\theta^2 \sigma_u^2 + (\theta-1)^2 \sigma_v^2]}}}{\beta(1+\beta)} \\
\text{where } \beta &= ab(\theta-1) > 0, \\
\gamma &= \frac{\beta - (1+\beta) \ln(1+\beta)}{\beta} < 0.
\end{aligned} \quad (25)$$

T^* has a real value only if $C \leq |(\beta/2\rho)[\rho \lambda(\theta^2 \sigma_u^2 + (\theta-1)^2 \sigma_v^2)]/\beta(1+\beta)|$. Contract length increases as the cost of negotiating contract, C , increases. Inspection of (25) gives an intuitive result that, if either σ_u^2 or σ_v^2 goes to infinity, then optimal contract length approaches to zero. This result is consistent with the arguments of Gray(1978) and Danziger(1988), which conclude that an increased variability shortens contract length. As is shown in (25), contract length is a function of the demand and supply variances. Differentiating (25) with respect to σ_u^2 , given θ , we have the following relationship (see Appendix 2):

$$\frac{\partial T^*}{\partial \sigma_u^2} < 0, \quad \frac{\partial T^*}{\partial \sigma_v^2} < 0. \quad (26)$$

The optimal contract length is shorter when the economy is more turbulent, regardless of the source of the turbulence. As demand or supply variances get smaller the optimal contract length increases. From the aggregate output schedule in equation (13), the stochastic integrals represent the cyclical component of aggregate output. The output variance is:

$$\begin{aligned}
\sigma_y^2 &= E[y_t - E(y)]^2 \\
&= E \left\{ ab \int_{x=0}^1 \frac{\theta(T-x)}{ab(\theta-1)(T-x) + T} u_t dx + \int_{x=0}^1 u_t dx \right\}^2 \\
&\quad + E \left\{ ab \int_{x=0}^1 \frac{(\theta-1)(T-x)}{ab(\theta-1)(T-x) + T} v_{t-x} dx \right\}^2 \quad (27)
\end{aligned}$$

5) The indexation parameter θ in this section represents the optimal value which is derived in the previous section. Recall that θ^* is a value between 0 and 1 in the presence of both demand and supply shocks. If we have only either one of these shocks the optimal indexation parameter has a value either 0 and 1. Thus, θ does not depend on T in two extreme cases.

Rewriting equation (27) gives

$$\sigma_v^2 = (ab)^2 [\theta^2 \sigma_u^2 + (\theta - 1)^2 \sigma_v^2] \int_{x=0}^T \left[\frac{T-x}{ab(\theta-1)(T-x)+T} \right]^2 dx \\ + ab\theta \sigma_u^2 \int_{x=0}^T \left[\frac{T-x}{ab(\theta-1)(T-x)+T} \right] dx + \int_{x=0}^T \sigma_v^2 dx \quad (28)$$

Differentiating equation (28) with respect to σ_v^2 and σ_u^2 , given θ and T , we have the following expressions :

$$\frac{\partial \sigma_v^2}{\partial \sigma_v^2} > 0, \quad \frac{\partial \sigma_v^2}{\partial \sigma_u^2} > 0. \quad (29)$$

The impacts of reductions in aggregate demand variability and supply disturbance on output stability are evaluated in the above equation (29). A decline in either the demand or supply disturbances directly causes a reduction in output variance. But σ_v^2 and σ_u^2 also enter equation (25) as determinants of contract length, T . The relationships between the optimal contract length, T , and σ_v^2 and σ_u^2 are shown in equation (26). That is, the optimal contract length T is a determinant of output variance, the value of T also affects output variability (see Appendix 4). Now, in a no-indexed economy ($\theta=0$), we vary T to find the relationship between output variance and contract length. In an indexed economy only with demand disturbance, output variance goes to zero regardless of contract length.

$$\frac{\partial \sigma_v^2}{\partial T} \Big|_{\sigma_u^2=0} > 0, \quad \frac{\partial \sigma_v^2}{\partial T} \Big|_{\sigma_v^2=0} > 0, \quad (30)$$

Increased contract length causes an increase in output variance in response to both demand and supply shocks in our model. This result contradicts with De Long and Summers' (1986). When contract length is considered as a wage flexibility parameter, increased wage flexibility is stabilizing in response to both demand and supply shocks. A decrease in variance of any disturbance in the system encourages longer contracts. Longer contracts, in turn, cause any particular aggregate demand shocks to have an impact on output variance. The indirect effect of a fall in σ_v^2 or σ_u^2 , acting through T , is to raise σ_v^2 . It is shown in the appendix 3 that the first effect outweighs the second, so that σ_v^2 has positive relationship with both of σ_v^2 and σ_u^2 .

4. CONCLUSION

We have investigated the relationship among the variabilities of aggregate demand and aggregate supply, contract length and the cyclical output fluctuation in a framework in which wage flexibility and output variability are endogenously determined. The results can be simply summarized :

i) Given any contract length T , the output deviations from the full equilibrium level of output are eliminated under full indexation ($\theta = 1$) in the absence of supply disturbances. The deviations are also eliminated under non-indexation ($\theta = 0$) in the absence of demand disturbances. Therefore, there need not be any concern with contract length in an indexed economy if there is either one of demand or supply shocks.

An optimal degree of indexing depends on the underlying stochastic structure of the economy set out in equations (1)~(11). It is shown that if demand becomes more volatile, θ approaches unity, while θ goes to zero if supply becomes more volatile. Indexing to unexpected change in the price level in our model completely neutralizes monetary variability.

ii) Our result supports the Gray and Kandil's argument that a smaller variance of aggregate demand shocks in the postwar period can explain the observations of reduced output variability with reduced wage and price flexibility in the postwar period. Increased contract length causes an increase in output variance in response to both demand and supply shocks. This result contradicts with De Long and Summers' (1986). When contract length is considered as a wage flexibility parameter, increased wage flexibility (shorter T) is stabilizing in response to both demand and supply shocks. A decrease in variance of any disturbance in the system encourages longer contracts. Longer contracts, in turn, cause aggregate demand shocks to have an impact on output variance. The indirect effect of a fall in σ_v^2 or σ_μ^2 , acting through T , is to raise σ_y^2 . It is shown in the appendix 4 that the first effect outweighs the second, so that σ_y^2 has positive relationship with both or σ_v^2 and σ_μ^2 .

Therefore, we conclude that an increased stability of aggregate demand or supply reduces output variability in the postwar period. Unlike the previous literature, increased wage flexibility (shorter T) is stabilizing in response to both demand and supply shocks.

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APPENDIX 1

We solved the model by using the method of undetermined coefficients. From equations (9), (10), and (11) we can produce the following expression for the price level.

$$p_t = \alpha - ad - \int_{x=0}^t u_{t-x} dx + \int_{x=0}^t v_{t-x} dx \quad (2a)$$

$$- \frac{1}{T} \left\{ ab(\theta - 1) \int_{s=0}^T (p_t - p_{t-t-s}) ds - ab \int_{s=0}^T \int_{x=0}^s u_{t-x} dx ds \right\}$$

Our postulated solution for price is :

$$p_t = A + \int_{x=0}^t B(x) u_{t-x} dx + \int_{x=0}^t C(x) v_{t-x} dx. \quad (2b)$$

Then, the error of forecasting p_t "s" periods ahead is :

$$p_t - p_{t-s} = \int_{x=0}^s B(x) u_{t-x} dx + \int_{x=0}^s C(x) v_{t-x} dx. \quad (2c)$$

Substituting equation (2c) into (2a) gives

$$p_t = \alpha - ad - \frac{ab(\theta - 1)}{T} \left\{ \int_{s=0}^t \int_{x=0}^s B(x) u_{t-x} dx + \int_{s=0}^T \int_{x=0}^s C(x) v_{t-x} dx \right\} \quad (2d)$$

$$+ \frac{ab}{T} \int_{s=0}^T \int_{x=0}^s u_{t-x} dx ds - \int_{x=0}^t u_{t-x} dx + \int_{x=0}^t v_{t-x} dx.$$

Now, we match coefficients across equations (2a) and (2d) and we determine the coefficients as follows :

$$A = \alpha - ad$$

$$B(x) = \frac{ab(T-x)-T}{ab(\theta-1)(T-x)+T}, \quad \text{for } x \in [0, T],$$

$$B(x) = -1, \quad \text{for } x \in [T, t].$$

$$C(x) = \frac{T}{T+ab(\theta-1)(T-x)}, \quad \text{for } x \in [0, T],$$

$$C(x) = 1, \quad \text{for } x \in [T, t].$$

APPENDIX 2

i) The Relationship between T^* and σ_v^2

Differentiating (25) with respect to σ_v^2 we have

$$\frac{\partial T^*}{\partial \sigma_v^2} = \left\{ \left\{ \frac{\beta}{4\gamma^2} \right\} + \frac{(1+\beta)C}{(ab)^2 \gamma [\theta^2 \sigma_u^2 + (\theta-1)^2 \sigma_v^2] \lambda} \right\} \left\{ \frac{(1+\beta)(\theta-1)^2 C}{(ab)^2 \gamma \lambda [\theta^2 \sigma_u^2 + (\theta-1)^2 \sigma_v^2]^2} \right\} \geq 0$$

In the above equation, both terms on the right hand side are negative. Therefore,

$$\frac{\partial T^*}{\partial \sigma_v^2} < 0.$$

ii) The Relationship between T^* and σ_u^2

Differentiating (25) with respect to σ_u^2 we have

$$\frac{\partial T^*}{\partial \sigma_u^2} = \frac{1}{2} \left\{ \left\{ \frac{\beta}{2\gamma} \right\}^2 + \frac{\beta(1+\beta)C}{(ab)^2 \gamma \lambda [\theta^2 \sigma_u^2 + (\theta-1)^2 \sigma_v^2]} \right\} \left\{ \frac{\beta(1+\beta)\theta^2 C}{(ab)^2 \gamma \lambda [\theta^2 \sigma_u^2 + (\theta-1)^2 \sigma_v^2]} \right\} \geq 0$$

In the above expression, the first term on the right hand side is positive, while the second term is negative because $\gamma < 0$. Therefore,

$$\frac{\partial T^*}{\partial \sigma_u^2} < 0.$$

APPENDIX 3

The effect of σ_v^2 and σ_u^2 on σ_v^2

By inspection of equation (26), we easily can find that output variance is a linear combination of σ_u^2 and σ_v^2 under the assumption that σ_u^2 and σ_v^2 are uncorrelated. Therefore, we can separate the effect of T on σ_v^2 in response to σ_u^2 and the effect of T on σ_v^2 in response to σ_v^2 .

i) The effect of σ_v^2 on σ_v^2

Equation (25) of the text can be solved for σ_v^2 to produce

$\sigma_v^2 = 4\gamma\beta(1+\beta)C(ab)^2\lambda(\theta-1)[4\gamma^2T^2+4\gamma T\beta+\beta+\beta^2]$ Substituting the above equation into equation (28) of the text produces

$$\sigma_v^2 | \sigma_u^2=0 = \frac{4\gamma\beta(1+\beta)C}{\lambda[4\gamma^2T^2+4\gamma T\beta+\beta+\beta^2]} \int_{x=0}^T \left[\frac{(T-x)}{ab(\theta-1)(T-x)+T} \right]^2 dx$$

Differentiating the above expression with respect to T gives

$$\begin{aligned} \frac{\partial \sigma_v^2}{\partial T} | \sigma_u^2=0 &= \frac{-4\gamma^2\beta(1+\beta)CT(8\gamma+4)}{A^2} \int_{x=0}^T \left[\frac{T-x}{ab(\theta-1)(T-x)+T} \right]^2 dx \\ &+ \frac{4\gamma\beta(1+\beta)C}{A} \left\{ ab(\theta-1) \left[\frac{-2\ln[1-ab(\theta-1)]}{[ab(\theta-1)]^2} - \frac{1}{1+ab(\theta-1)} + \frac{2}{ab(\theta-1)} \right] \right\} \end{aligned}$$

$$\text{where } A = \lambda[4\gamma^2T^2+4\gamma T\beta+\beta+\beta^2] > 0$$

By inspection, the coefficient on an integral term is negative and the second term on the righthand side is negative because of $\gamma < 0$. Thus, we get $\partial \sigma_v^2 / \partial T < 0$. Therefore, from equation (25) we know that T is a decreasing function of σ_v^2 which, together with the above relationship, implies that $\partial \sigma_v^2 / \partial \sigma_v^2 > 0$.

ii) The effect of σ_u^2 on σ_v^2

Equation (25) also can be solved for σ_u^2 to produce

$$\sigma_v^2 | \sigma_u^2=0 = (ab\theta)B \int_{x=0}^T \left[\frac{T-x}{ab(\theta-1)(T-x)+T} \right]^2 dx + ab\theta B \int_{x=0}^T \left[\frac{T-x}{ab(\theta-1)(T-x)+T} \right] dx$$

$$\text{where } B = \frac{4\gamma\beta(1+\beta)C}{(ab)^2\lambda\theta^2[4\gamma^2T^2+4\gamma T\beta+\beta+\beta^2]} > 0$$

Differentiating the above expression with respect to T gives

$$\frac{\partial \sigma_v^2}{\partial T} | \sigma_u^2=0 < 0.$$

Finally, from the relationship (26) we know that T is a decreasing function of σ_u^2 which together with the above relationship, implies that $\partial \sigma_v^2 / \partial \sigma_u^2 > 0$.

APPENDIX 4

The relationship between σ_v^2 and T

By inspection of equation (27), we easily can find that output variance is a linear combination of σ_u^2 and σ_v^2 because of the implicit assumption that σ_u^2 and σ_v^2 are uncorrelated. Therefore, we can separate the effect of T on σ_v^2 in response to σ_u^2 and the effect of T on σ_v^2 in response to σ_v^2 . Differentiating σ_v^2 with respect to T , given $\sigma_u^2=0$, we have

$$\frac{\partial \sigma_v^2}{\partial T} \Big|_{\sigma_u^2=0} = -\frac{s[ab(\theta-1)]^2[ab(\theta-1)+1]}{ab(\theta-1)(T-s)+T} < 0 \quad (4a)$$

Differentiating σ_v^2 with respect to T , given $\sigma_v^2=0$, we have

$$\frac{\partial \sigma_v^2}{\partial T} \Big|_{\sigma_v^2=0} \geq 0$$

$$\begin{aligned} \leftrightarrow & \text{abs}(ab-2)^2(\theta-1)[\text{abs}(\theta-1)]^2 \\ & - 2(\text{abs})^2(\theta-1)(ab-2)[ab(\theta-1)+1] \\ & - Tab(\theta-1)^2s[1+ab(\theta-1)] - \text{abs}[ab(\theta-1)+1]^2 \\ & - \text{abs}^2(\theta-1)(ab+2)[ab(\theta-1)+1] \geq 0 \end{aligned}$$

All of the terms except the second one on the right hand side are positive. However, the negative second term is less than the first one. Thus, we have the relationship (30) in text.