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MARKET STRUCTURE, AGGREGATE UNCERTAINTY AND SOCIAL VALUE OF INFORMATION IN A GENERAL EQUILIBRIUM MODEL*

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There are many economic parameters which would affect social value of public informaton in a general equilibrium model. This paper examines social value of public information in conjunction with preferences, market structures and aggregate uncertainty. An exact condition for information to have no social value is derived in terms of equilibrium prices when markets are complete. Also, when information is disseminated into the economy under individual uncertainty but no aggregate uncertainty, it is pointed out that information will not only have no social value, but also will induce a possibility to make everyone worse off in a non-trivial way.

I. INTRODUCTION

When public information is disseminated into the economy under uncertainty before any binding contracts are made, it might well be expected that the welfare levels of economic agents will be affected in many different ways. Although any type of information is valuable from individual point of view, it is not so obvious from a general equilibrium point of view whether information is socially valuable or not in *ex ante* sense.[1] Since the seminal paper by Hirshleifer(1971), many works have been done on this issue. Hakansson, Kunkel and Ohlson(1982) have examined sufficient and necessary conditions for information to have social value in a simple pure exchange economy. They have characterized such conditions in terms of market structure, prior beliefs, homogeneity of information structure and preferences. But, their argument was based largely upon the assumption that endowment is an equilibrium allocation. Kunkel(1982) has also examined sufficient conditions for informa-

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[1] *Ex ante* information is received before binding contracts are made while *ex post* information is received after binding contracts are made.

tion to have social value in a production economy. But, his argument was also much dependent on the same assumption. Jaffe(1975) has examined *ex post* value of public information depending on markets being complete or not. His argument has some limit in the sense that it is more natural to evaluate social value of information in *ex ante* sense. Ng(1977) has dealt the issue of authentic information[2] and characterized the conditions under which authentic information has social value. Arrow(1984) has given simple examples that public information can be socially harmful rather than valuable. His examples are full of insights, but can not be easily generalized.

Since there are many economic parameters affecting social value of information, it is not easy to come up with a general theory of social value of information in a general equilibrium framework. What we may achieve is to classify economic situations which respond differently but systematically to the release of information. In this sense, Ohlson and Buckman(1980)(1981) have given us a relatively general theory of social value of information with a nice summary of previous works. But, they have addressed the issue largely from a planner's point of view rather than a general equilibrium point of view. Since a planner's point of view is identical to an individual point of view in the sense that more information is always valuable, quite different results can be expected when the issue is examined from a general equilibrium point of view. Also, their argument about the welfare aspect of information is based upon incomplete market structure with a single commodity. But, we have to be very careful about the welfare aspect of economies with incomplete markets because there is a fundamental difficulty in extending the argument to economies with more than one commodity. Their argument is very much restrictive in this sense. Ohlson (1988) has also dealt the problem of social value of information in a production economy. But, his argument was not rigorous enough for the analysis of social value of information in a general equilibrium framework. Moreover his focus was on the welfare comparison of two different information structures.

Among many economic parameters which affect social value of information, I will assume that prior beliefs and hence signal beliefs are homogeneous[3] and focus on the role of market structure, preferences and aggregate uncertainty on social value of information. The outline of the paper is as follows. In section II, I discuss the relationship between equilibrium price vectors and social value of information so that we can draw a general conclusion about social value of

[2] Authentic information is defined as the observation(signal) that leads to a convergence in individuals' beliefs when it arrives. Thus, his definition of authentic information is different from the usual definition of public information.

[3] Since heterogeneous prior beliefs and hence heterogeneous signal beliefs are obviously necessary conditions for information to have social value, they are ruled out here for us to concentrate on other economic parameters. See Hakansson and et al.(1982) for the detail.

information merely by taking a look at and comparing equilibrium price vectors. Section III deals with the role of aggregate uncertainty on the welfare implication of information in a pure exchange and also in a production economy. Section IV makes some concluding comments and suggestions for future research.

II. MARKET STRUCTURE AND SOCIAL VALUE OF INFORMATION

2.1. Complete Markets and Social Value of Information

Consider a pure exchange economy which lasts two periods. There are uncertain states in period $t=1$, denoted by $s \in S=\{s{:}s=1,2,.....,n\}$. There is a single commodity which can be traded in complete contingent claims markets in period $t=0$. There are finitely many consumers, $h \in H=\{h{:}h=1,2,.....,m\}$. Each consumer has a state dependent endowment $e_h=(e_h(0),e_h(1),....,e_h(n))$. His consumption plan is denoted by $x_h=(x_h(0),\ x_h(1),....,x_h(n))$. Contingent claims prices are denoted by $p=(p(0),p(1),.....,p(n))$. Each consumer's preference is expressed by a twice continuously differentiable, strictly concave Von-Neumann and Morgenstern utility function $U_h(x_h(0),x_h(s))$ when state "s" is supposed to occur. Also, consumers are assumed to hold homogeneous prior belief about states $\pi(1),....,\pi(n)$ with $\sum_s \pi(s)=1$. His preference under uncertainty is represented by expected utility theorem so that

$$V_h(x_h)=\sum_s \pi(s)U_h(x_h(0),x_h(s)).$$

Ifutility fuction is additively separable, $V_h(x_h)=U_h(x_h(0))+\sum_s \pi(s)U_h(x_h(s))$.

These are the basic features of the model considered here. Some of them can be modified or relaxed if necessary.

Now, if utility function is additively separable and there is no public information available under uncertainty, each consumer will maximize his expected utility in the standard way:

$$\begin{aligned}&\text{Maximize } [U_h(x_h(0))+\sum_s \pi(s)U_h(x_h(s))] && \\ &\text{subject to} && (\mathrm{I})\\ &p(0)x_h(0)+\sum_s p(s)x_h(s)=p(0)e_h(0)+\sum_s p(s)e_h(s) \qquad (\lambda_h) &&\end{aligned}$$

The first order conditions (f.o.c.) are obviously as follows :

$$\partial U_h(x_h(0))/\partial x_h(0)-\lambda_h p(0)=0 \qquad ....1)$$

$$\pi(s)\partial U_h(x_h(s))/\partial x_h(s) - \lambda_h p(s) = 0 \text{ for } s=1,\dots,n \text{ and } h=1,\dots,m \qquad \dots.2)$$

and budget constraints are satisfied with equalities. This problem is well defined and there exists a competitive equilibrium with complete contingent claims markets (C.C,C.M.). Let $p^* = (p^*(0), p^*(1), \dots\dots\dots, p^*(n))$ and $x^* = (x_1^*, \dots\dots, x_m^*)$ be a competitive equilibrium of this economy. Also, let $V_h(Y_0) = U_h(x_h^*(0)) + \sum_s \pi(s) U_h(x_h^*(s))$ denote the value of no information for h. Now, suppose that public information becomes available in period $t=0$ before any binding contracts are made. The question that we can raise is in which direction public information will affect consumers' welfare. We can think of four possible cases :

i) No consumer is worse off and at least one consumer is strictly better off.
ii) some consumers are better off, but there are some others who are made strictly worse off.
iii) No consumer's welfare is changed.
iv) Public information makes no one better off and makes at least one worse off.

Public information is said to have social value if i) holds. Public information is said to have no social value if ii) or iii) holds. But, we have to be more careful about case ii). It is more appropriate to say that public information has social value potentially in case ii) if every one is made better off by some transfers.[4] The converse may hold in case ii). Public information is socially harmful in case iv).

There are many parameters affecting the welfare property of public information. Some of them are market structure, signal-contingent trading opportunities (which are called "market regime" in the literature), separability of utility function, homogeneity of prior beliefs and property of information structure. It is almost impossible to construct a general theory of social value of information relative to every possible combination of the above parameters. Thus, it is necessary to specify those parameters in detail in order to derive a somewhat general conclusion.

First, we consider the welfare implication of perfect noisyless information as a benchmark. Suppose that consumers are supposed to have chances to observe a random signal y_s from the set of possible signals Y_P before any binding con-

[4] This implies that we can apply the well known Kaldor-Hicks "compensation principle" for the evaluation of the social value of public information.

tracts are made. Perfect noisyless information implies that there exists a function η :$S \rightarrow Y_p$, which is one to one and onto. In other word, the set S is partitioned into "n" singleton sets. If any additional trading opportunities contingent on signals are not available, then consumers will maximize their conditional expected utilities as follows :

For some $y_s \in Y_p$,

$$\text{Maximize } [U_h(x_h(0, y_s)) + U_h(x_h(s, y_s))]$$
$$\text{subject to} \qquad \text{(II)}$$
$$p(0, y_s)x_h(0, y_s) + p(s, y_s)x_h(s, y_s) = p(0, y_s)e_h(0) + p(s, y_s)e_h(s) \qquad (\delta_{hs})$$

The first order conditions are again as follows :

$$\partial U_h(x_h(0, y_s))/\partial x_h(0, y_s) - \delta_{hs}p(0, y_s) = 0 \qquad ....3)$$

$$\partial U_h(x_h(s, y_s))/\partial x_h(s, y_s) - \delta_{hs}p(s, y_s) = 0 \text{ for all } h \qquad ....4)$$

and budget constraint is satisfied with equality.

Let $p'(y_s) = (p'(0, y_s), p'(s, y_s))$ and $x'(y_s) = (x_1'(y_s)$, ... $x_m'(y_s))$ such that $x_h'(y_s) = (x_h'(0, y_s), x_h'(s, y_s))$ be a competitive equilibrium of the conditional markets relative to $y_s \in Y_p$. Also, let $V_h(y_s) = U_h(x_h'(0, y_s)) + U(x_h'(s, y_s))$. Then, the value of information in the conditional markets is defined as follows:

$$V_h(Y_p) = \sum_s \pi(y_s)V_h(y_s)$$

Observe that $\pi(y_s) = \pi(s)$ due to the property of perfect information. What is instresting to us here is whether consumers are made better off with the introduction of public information. This can be checked by comparing $V_h(Y_0)$ and $V_h(Y_p)$ for each $h \in H$. Intuitively, public information would reduce the degree of uncertainty on the one hand, but introduce what Hirshleifer called "distributive risk" into the economy on the other hand. So, overall effect of public information may depend upon which effect is dominant. To make this point clear, consider the following market regime which is called the "standard regime" in the literature.[5] Suppose that contracts are available contingent not only on states, but also on signals. Then, consumers will face the following maximization problem :

[5] According to the availablility of signal-contingent tradings, four types of market regimes are considered in the literature : Standard regime, Arrow regime, Iterated market regime and Non-iterated market regime. See Ohlson and Buckman (1981) for the detail.

Maximize $\sum_s \pi(y_s)[U_h(x_h(0, y_s))+U_h(x_h(s, y_s))]$

subject to (III)

$$\sum_s p(0, y_s)x_h(0, y_s)+\sum_s p(s, y_s)x_h(s, y_s)=\sum_s p(0, y_s)\,e_h(0) + \sum_s p(s, y_s)e_h(s) \qquad (\mu_h)$$

Let's take a look at the first order conditions of this problem.

$$\pi(y_s)\partial U_h(x_h(0, y_s))/\partial x_h(0, y_s)-\mu_h p(0, y_s)=0 \qquad ....5)$$

$$\pi(y_s)\partial U_h(x_h(s, y_s))/\partial x_h(s, y_s)-\mu_h p(s, y_s)=0 \quad \text{for } s=1,..,n \qquad ....6)$$

and budget constraints are satisfied with equalities.

Let $(p^*(SY_p),x^*(SY_p))$ denote a competitive equilibrium of a standard regime such that $p^*(SY_p)=(p^*(0, y_1),p^*(1, y_1),....,p^*(0, y_n),p^*(n, y_n))$ and $x_h^*(SY_p)=(x_h^*(0, y_1),x_h^*(1, y_1),....,x_h^*(0, y_n),x_h^*(n, y_n))$. It is easy to see that this equilibrium is full Pareto optimal with respect to information Y_p. Again, define $V_h(SY_p)$ such that $V_h(SY_p)=\sum_s \pi(y_s)[U_h(x_h^*(0,y_s))+U_h(x_h^*(s,y_s))]$

DEFINITION 1

$x(SY_p)\equiv x$ if $x_h(0, y_s)=x_h(0)$ and $x_h(s, y_S)=x_h(s)$ for each $y_s\in Y_p$ and $h\in H$.

PROPOSITION 1

If x^* is an equilibrium allocation of C.C.C.M. without information, then $x(SY_p)\equiv x^*$ is an equilibrium allocation of the standard regime with perfect information.

PROOF

Let's define a price vector and Lagrangian multiplier in the following way : $p^*(0, y_s)=\pi(y_s)p^*(0)$, $p^*(s, y_s)=p^*(s)\mu_h=\lambda_h^*$ for all h where λ_h^* is the Lagrangian multiplier of consumer h, supporting the equilibrium allocation in (I). Then, it is easy to check that $x_h(Y_p)$ satisfies 5) and 6) at this price vector. Budget constraint is also satisfied at this allocation and price vector.

$$\sum_s p^*(0, y_s)x_h^*(0, y_s)+\sum_s p^*(s, y_s)x_h^*(s, y_s)$$
$$=\sum_s \pi(y_s)p^*(0)x_h^*(s)+\sum_s p^*(s)x_h^*(s)$$

$$=\sum_{s} \pi(y_s)p^*(0)e_h(0) + \sum_{s} p^*(s)e_h(s)$$
$$=\sum_{s} p^*(0, y_s)e_h(0) + \sum_{s} p^*(s, y_s)e_h(s).$$

Market clearing conditions are automatically satisfied from the definition of $x(SY_p)$. Therefore, $x(SY_p)$ is an equilibrium allocation of the standard regime with perfect information. Q.E.D

LEMMA 1

If $x^*(SY_p)$ is an equilibrium allocation of the standard regime with perfect information, then $x_h^*(0,y_1) = ... = x_h^*(0,y_n)$ for all $h \in H$.

PROOF

Supose not. Then, there exist some $i \in H$ and some $y_j, y_k \in Y_p$ such that $x_i^*(0, y_j) \neq x_i^*(0, y_k)$. Define the average allocation over signals.

$$x_h'(0) = \sum_{s} \pi(y_s)x_h^*(0, y_s) \qquad \text{for all } h.$$

Then, $\sum_{s} \pi(y_s)U_h(x_h'(0)) = U_h(x_h'(0)) \geq \sum_{s} \pi(y_s)U_h(x_h^*(0, y_s))$ from the strict concavity of the utility function for all h with at least one strict inequality. Also, this is feasible since ;

$$\sum_{h} x_h'(0) = \sum_{h}\sum_{s} \pi(y_s)x_h^*(0,y_s) = \sum_{s} \pi(y_s)\sum_{h} x_h^*(0,y_s) = \sum_{s} \pi(y_s)\sum_{h} e_h(0) = \sum_{h} e_h(0).$$

This implies that $x(SY_p)$ is not a Pareto optimal allocation, which is a contradiction. Q.E.D

Thus, we can establish the equivalence between C.C.C.M. without information and the standard regime with perfect information in the sense that both yields exactly the same set of equilibrium allocations. That is, if $E(x)$ and $E(x(SY_p))$ denote the sets of equilibrium allocations respectively, then we can extend proposition 1 by dint of lemma 1 so as to conclude that $E(x) \equiv E(x(SY_p))$. Next, consider the following problem.

$$\text{Maximize } \sum_{s} \pi(y_s)[U_h(x_h(0, y_s)) + U_h(x_h(s, y_s))]$$

subject to (IV)

$$p(0, y_1)x_h(0, y_1) + p(1, y_1)x_h(1, y_1) = p(0, y_1)e_h(0) + p(1, y_1)e_h(1) \qquad (\delta_{h^1})$$
$$\vdots$$
$$p(0, y_n)x_h(0, y_n) + p(n\ y_n)x_h(n, y_n) = p(0, y_n)e_h(0) + p(n, y_n)e_h(n) \qquad (\delta_{h^n})$$

This problem is nothing but the summation of the maximization problems in (II). Hence the equilibrium allocation as a solution to (IV) is exactly identical to that of (II) relative to each $y_s \in Y_p$. Therefore, $V_h(Y_p)$ is also the value of information of an equilibrium allocation in (IV).

Now, let z_h denote an excess demand vector which is suitably rearranged as follows : $z_h=(z_h(0,y_1), z_h(1,y_1), z_h(0,y_2), z_h(2,y_2),\ldots\ldots, z_h(0,y_n), z_h(n,y_n))$. Here, define the price systems of (III) and (IV) in the following way :

$[p(SY_p)]=(p(0, y_1), p(1, y_1),\ldots\ldots, p(0, y_n), p(n, y_n))$ for (III) and for (IV)

$$[p(Y_p)]=\begin{bmatrix} p(0,y_1) & p(1,y_1) & 0 & \cdots & & & & 0 \\ 0 & 0 & p(0,y_2) & p(2,y_2) & 0 & \cdots & & 0 \\ \cdot & & & \cdot & & & & \cdot \\ \cdot & & & & & \cdot & & \cdot \\ \cdot & & & & & & \cdot & 0 \\ 0 & \cdots & & & & 0 & p(0,y_n) & p(n,y_n) \end{bmatrix}$$

where price vectors are suitably normalized, that is, $p(0, y_1)=1$ in (III) and $p(0,y_s)=1$ for each $y_s \in Y_p$ respectively.

Next, define budget sets $B_h([p(SY_p)])$ and $B_h([p(Y_p)])$ as follows :

$$B_h([p(SY_p)])=\{z_h \in R^{2n} : [p(SY_p)]z_h=0\} \text{ for (III) and}$$
$$B_h([p(Y_p)])=\{z_h \in R^{2n} : [p(Y_p)]z_h=0\} \text{ for (IV)}$$

Thus, whether $B_h([p(Y_p)])$ is a subspace of $B_h([p(SY_p)])$ or not depends on the relationship between price systems prevailing in both cases.

Suppose that $[p^*(SY_p)]$ and $[p'(Y_p)]$ are equilibrium price systems of (III) and (IV) respectively. There are two possible cases. That is, there is a unique vector $\omega \in R^n$ such that $\omega^T[p'(YP)]=[p^*(SY_p)]$[6] or there is no such vector. Now, let $x^*(SY_p)$ and $x'(Y_p)$ denote an equilibrium allocation in (III) and (IV) respectively.

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[6] This can be interpreted as a complement to the relationship between equilibrium prices with and without information, which was pointed out by Ohlson(1984) and Ohlson and Buckman(1981). They argued that prices in the economy with no information are unbiased estimators of prices in the economy with information, provided that information does not affect the final allocation. I claim that their statement is reversed. The correct statement should be the other way round. Note that this relationship is guaranteed whenever endowment is an equilibrium allocation. We can extend this relationship to the comparison between equilibrium without information and equilibrium with information, but with no signal-contingent tradings via the proposed price relationship.

LEMMA 2

$x^*(SY_p) \equiv x'(Y_p)$ only if there exists a unique vector $\omega \in R^n$ such that $\omega^T[p'(Y_p)] = [p^*(SY_p)]$.

PROOF

Define normalized gradient vectors evaluated at $x_h^*(SY_p)$ and $x_h'(Y_p)$ by $D^N V_h(x_h^*(SY_p))$ and $D^N V_h(x_h'(Y_p))$ respectively. This normalization is done by dividing the gradient vector $DV_h(x_h^*(SY_p))$ by $\pi(y_1)\partial U_h(x_h^*(0,y_1))/\partial x_h(0,y_1)$ and dividing $DV_h(x_h'(Y_p))$ by $\pi(y_1)\, U_h(x_h'(0, y_1))/\, x_h(0, y_1)$.

Then, the following relationships must hold from the f.o.c. :

$$D^N V_h(x_h^*(SY_p)) = [p^*(Y_p)] \text{ and } D^N V_h(x_h'(Y_p)) = \gamma_h^T[p'(Y_p)] \qquad \text{.... 7)}$$

where $\gamma_h = (1, (\delta_{h^2}/\delta_{h^1}), \ldots\ldots, (\delta_{h^r}/\delta_{h^1}))$ evaluated at the equilibrium allocation. Now, suppose that $x^*(SY_p) \equiv x'(Y_p)$, but there exists no vector ω such that $\omega^T[p'(Y_p)] = [p^*(SY_p)]$. Then, since $\gamma_{h^T}[p'(Y_p)] = [p^*(SY_p)]$ for all h from 7) and $[p'(Y_p)]$ has a full row rank, γ_h must be independent of $h \in H$. So, we can set $\gamma_h = \omega$ without loss of generality. Q.E.D

The converse of lemma 2 does not hold in general. That is, the existence of such a vector ω does not guarantee the equivalence of equilibrium allocations in both regimes. But, if such a vector ω exists, it gives us a nice interpretation about the welfare effect of information on each consumer.

Suppose that $\omega^T[p'(Y_p)] = [p^*(SY_p)]$ holds at equilibrium. Then, the set "H" is partitioned into two subsets :

$$H_1 = \{h \in H : \gamma_h = \omega\} \text{ and } H_2 = \{h \in H : \gamma_h \neq \omega\}$$

Those utility levels in H_1 are unchanged when information is disseminated without any creation of signal-contingent markets while those in H_2 are strictly worse off. This is because $B_h([p'(Y_p)]) \subset B_h([p^*(SY_p)])$ for $h \in H_2$ and $x_h^*(SY_p)$ is the greatest element in $B_h([p^*(SY_p)])$.

LEMMA 3

If there exists no $\omega \in R^n$ such that $\omega^T[p'(Y_p)] = [p^*(SY_p)]$, then either i) $V_h(SY_p) > V_h(Y_p)$ for some h and $V_j(SY_p) < V_j(Y_p)$ for some other j or ii) $V_h(SY_p) \geq V_h(Y_p)$ for all h with at least one strict inequality.

PROOF

This is obvious from lemma 2 and Pareto optimality of $x^*(SY_p)$. Q.E.D

PROPOSITION 2

When information is disseminated into the economy with complete contingent claims markets without signal-contingent trading opportunities, then either information will have no social value or it will make no one better off with someone being made strictly worse off.

PROOF

This is straightforward from proposition 1 and lemma 1, 2 and 3. Q.E.D

REMARK 1

1) It follows immediately that all the above arguments hold exactly to any case of noisyless, but imperfect information structure. The choice of perfect information is just for the expository convenience.

2) It may be expected that all the previous arguments will not be applied to the case of noisy information structure. Noisy information is different from noisyless information in the sense that the former will introduce distributive risk without reducing the degree of uncertainty in general. But, it can be shown that nothing would be altered even if noisy information were considered.

Suppose that noisy information is disseminated into the economy. We will examine whether all the previous arguments with noisyless information can be maintained or not. First, note that there is no fundamental change in consumer's maximization problem due to noisy information. Second, what we have to be careful about is the difference in the treatment of conditional probabilities. That is, $\pi(y_i/s)=1$ if $s \in S(y_i)$ or $\pi(y_i/s)=0$ otherwise in case of noisyless information. On the other hand, $\pi(y_i/s) \neq 0,1$ in general in case of noisy information. It is enough for us to take into account this difference in the characterization of equilibrium price and allocation.

Suppose that $Y_N = \{y_i : i = 1,2,....,k\}$ contains "k" random signals and there is a correspondence, not a function between S and Y_N. In the standard regime, consumers will face the following problem :

$$\text{Maximize } \sum_i \pi(y_i) \sum_s \pi(s/y_i)[U_h(x_h(0, y_i)) + U_h(x_h(s, y_i))]$$

subject to

$$\sum_i p(0, y_i)x_h(0, y_i) + \sum_i \sum_s p(s, y_i)x_h(s, y_i) = \sum_i p(0, y_i)e_h(0) + \sum_i \sum_s p(s, y_i)e_h(s) \qquad (\mu_h)$$

The first order conditions are as follows :

$$\pi(y_i)\partial U_h(x_h(0, y_i))/\partial x_h(0, y_i) - \mu_h p(0, y_i) = 0 \qquad .....8)$$

$\pi(s, y_i)\partial U_h(x_h(s, y_i))/\partial x_h(s, y_i) - \mu_h p(s, y_i) = 0$ .....9)
for $s \in S(y_i)$ and $y_i \in Y_N$

and budget constraints are satisfied with equalities.

PROPOSITION 3

If (p^*, x^*) is a competitive equilibrium of C.C.C.M. without information, then $(p^*(Y_N), x^*(Y_N))$, satisfying the following conditions, is a competitive equilibrium of the standard regime with noisy information :

$p^*(0, y_i) = \pi(y_i)p^*(0)$, $p^*(s, y_i) = \pi(y_i/s)p^*(s)$ and
$x_h^*(0, y_i) = x_h^*(0)$, $x_h^*(s, y_i) = x_h^*(s)$ for all $y_i \in Y_N$ and $s \in S$

PROOF

The same arguments as in the proof of proposition 1 hold for the proof here. Q.E.D

Note that $x^*(Y_N)$ is independent of signals. The question is whether there is another equilibrium of the standard regime with noisy information which is dependent on signals. The answer is no. To make the argument complete, suppose that $x(Y_N)$ is an equilibrium allocation of a standard regime with noisy information, which is dependent on signals. Then, take an average allocation over signals $x'(Y_N)$ again as follows.

$x_h'(0) = \sum_i \pi(y_i)x_h(0, y_i)$ and $x_h'(s) = \sum_i \pi(y_i/s)x_h(s, y_i)$ for $s \in S$ and all h.

Then, $x'(Y_N)$ is feasible since

$\sum_h x_h'(0) = \sum_h \sum_i \pi(y_i)x_h(0, y_i) = \sum_i \pi(y_i) \sum_h (0, y_i) = \sum_h e_h(0)$ and
$\sum_h x_h'(s) = \sum_h \sum_i \pi(y_i/s)x_h(s, y_i) = \sum_i \pi(y_i/s) \sum_h x_h(s, y_i) = \sum e_h(s)$.

$$\begin{aligned}
V_h(x_h'(Y_N)) &= \sum_i \pi(y_i) \sum_s \pi(s/y_i)[U_h(x_h'(0)) + U_h(x_h'(s))] \\
&= \sum_i \sum_s \pi(s, y_i)[U_h(x_h'(0)) + U_h(x_h'(s))] \\
&= U_h(x_h'(0)) + \sum_s \pi(s)U_h(x_h'(s)) \\
&= U_h(\sum_i \pi(y_i)x_h(0, y_i)) + \sum_s \pi(s)U_h(\sum_i \pi(y_i/s)x_h(s, y_i)) \\
&\geq \sum_i \pi(y_i)U_h(x_h(0, y_i)) + \sum_s \pi(s) \sum_i \pi(y_i/s)U_h(x_h(s, y_i)) \\
&= \sum_i \pi(y_i) \sum_s \pi(s/y_i)[U_h(x_h(0, y_i)) + U_h(x_h(s, y_i))] = V_h(x_h(Y_N))
\end{aligned}$$

with at least one strict inequality.

This is a contraction to Pareto optimality of an equilibrium allocation of a standard regime. Therefore, there exists no equilibrium allocation of standard regime with noisy information, which is dependent on signals. This implies that the converse of proposition 3 holds also as in the case of noisyless information. Thus, there is an isomorphism between C.C.C.M. without information and the standard regime with information regardless of types of information structures. This implies that all the previous arguments about the welfare comparison hold regardless of types of information structures.

REMARK 2

1) Note also that all the previous arguments can be extended to the case of many commodities without any difficulty. As long as the original markets are complete, the number of commodities does not matter. But, this matters if the original markets are incomplete.

2) We have maintained the assumption that utility function is additively separable. Since this may be a restrictive assumption in some sense, it will be shown that this assumption can be relaxed if no aggregate uncertainty is introduced. Then, we will check whether the previous isomorphism and welfare implication are maintained without the separability assumption.

2.2. Incomplete Markets and Social Value of Information : On some Fundamental Issues

It has become clear now that one of most important necessary conditions for information to have social value is that the original market structure must be incomplete. This is true whether additional trading opportunities contingent on signals are available or not. But, due to the complexity associated with the welfare property of equilibrium allocation with or without information when the markets are incomplete, almost nothing has been done about social value of information with incomplete markets except some work by Ohlson and Buckman(1980) (1981) and Ohlson(1987). They have examined the issue in a model of incomplete markets with a single commodity and with real securities largely from a planner's point of view and compared the welfare property of different market regimes with respect to given public information. Their arguments were heavily dependent on the fact that a competitive equilibrium allocation with or without information in suitably chosen market regimes is always constrained Pareto optimal, which can not be generalized any more.

As was pointed out in the recent literature about economies with incomplete markets, equilibrium with incomplete markets exhibits many different properties from that with complete markets. For instance, there are infinite number of equilibria in economies with incomplete financial markets. On the other hand there may not exist an equilibrium or exists a finite number of equilibria in economies with real securities such as commodities futures or stocks. So, we have to be careful about introducing information and evaluating its welfare implication in a general equilibrium model with incomplete markets. If there are infinite number of equilibria in the original economy, it is difficult to evaluate the effect of information on the welfare property of equilibria when the original markets are maintained. Also, it has been pointed out by Dreze, et al.(1990) and Geanakoplos and Polemarchakis(1986) that equilibrium with incomplete markets is generically constrained Pareto sub-optimal. Although their arguments are based on some restrictive assumptions, this can severely restrict the discussion about social value of information in economies with incomplete markets. Since most of the arguments made by Ohlson and Buckman(1981) are based on constrained Pareto optimality of equilibrium in an economy with a single commodity, their arguments can't hold any more if equilibrium allocation is not constrained Pareto optimal and also can't be extended to the case with many commodities.

When the original economy is incomplete, the benchmark that information may have positive social value is that markets become conditionally complete with respect to information structure.[7] Now, let's take a look at the following examples to understand the difficulty inherent in evaluating social value of information with incomplete markets and the role of conditionally complete markets.

[Example 1]

Consider a simple pure exchange economy which lasts two periods with 4 uncertain states in period 1, $s=s_1, s_2, s_3, s_4$. There are two consumers $h=1,2$. Also, there are two commodities and one real security with the following state dependent returns in terms of numeraire commodity :

$$[R]=\begin{bmatrix} a(s_1) \\ a(s_2) \\ a(s_3) \\ a(s_4) \end{bmatrix}$$

Then, each consumer h will solve the following problem :

[7] It is defined that the market structure is conditionally complete with respect to information if there are sufficient number of instruments(assets) against states conditional on each information signal. Then, conditionally complete markets, together with signal-contingent trading opportunities, will achieve a Pareto optimal equilibrium allocation with respect to information structure. See Ohlson and Buckman(1981) for the detail.

Maximize $\sum_s \pi(s)U_h(x_h(0), x_h(s))$

subject to

$p(0)x_h(0)+qb_h=p(0)e_h(0)$

$p(s)x_h(s)=p(s)e_h(s)+p_1(s)a(s)b_h$ for $s=s_1,..,s_4$ and $h=1,2$

where q and b_h denote security price and consumer h's security holding respectively.

As was mentioned, a competitive equilibrium allocation is not constrained Pareto optimal generically. Let $V(Y_0)=(V_1(Y_0), V_2(Y_0))$ denote a vector of utilities evaluated at the equilibrium allocation. If $UPF(Y_0)$[8] denotes a set of all "V" corresponding to constrained Pareto optimal allocations, then $V(Y_0)$ will not belong to $UPF(Y_0)$ generically.

Now, suppose that public information is introduced into the economy before binding contracts are made. Let $Y=\{y_1, y_2\}$ be the set of information signals, which induces a partition on "S" such that $S_1=\{s_1,s_2\}$ and $S_2=\{s_3,s_4\}$. If signal-contingent tradings are available, then each consumer h will solve the following problem :

Maximize $\sum_i \pi(y_i)\sum_s \pi(s/y_i)U_h(x_h(0, y_i), x_h(s, y_i))$

subject to

$\sum_i p(0, y_i)x_h(0, y_i)+\sum_i q(y_i)b_h(y_i)=\sum_i p(0, y_i)e_h(0)$

$p(s_1, y_1)x_h(s_1, y_1)=p(s_1, y_1)e_h(s_1)+p_1(s_1, y_1)a(s_1, y_1)b_h(y_1)$

$p(s_2, y_1)x_h(s_2, y_1)=p(s_2, y_1)e_h(s_2)+p_1(s_2, y_1)a(s_2, y_1)b_h(y_1)$

$p(s_3, y_2)x_h(s_3, y_2)=p(s_3, y_2)e_h(s_3)+p_1(s_3, y_2)a(s_3, y_2)b_h(y_2)$

$p(s_4, y_2)x_h(s_4, y_2)=p(s_4, y_2)e_h(s_4)+p_1(s_4, y_2)a(s_4, y_2)b_h(y_2)$

Here, consumers are faced with the following conditional security returns :

$$[R(Y)]=\begin{bmatrix} a(s_1, y_1) & 0 \\ a(s_2, y_1) & 0 \\ 0 & a(s_3, y_2) \\ 0 & a(s_4, y_2) \end{bmatrix}$$

Thus, we can easily verify that markets are not conditionally complete with respect to information. This implies that competitive equilibrium of this stan-

[8] $UPF(Y_0)$ is nothing but the usual utility possibilities frontier corresponding to constrained Pareto optimal allocation when the planner uses the same instruments as the markets do for resource allocation.

dard regime is not constrained Pareto optimal generically. Again, let $V(SY) = (V_1(SY), V_2(SY))$ denote a vector of utilities evaluated at the equilibrium allocation and $UPF(SY)$ be the set of all "V" corresponding to constrained Pareto optimal allocations with respect to information structure, Y. Then, $V(SY)$ will not belong to $UPF(SY)$ generically. Therefore, it is very difficult to evaluate whether information have social value or not even though $UPF(SY)$ will lie outside of $UPF(Y_0)$. This problem is more conspicuous if we compare the original economy with the one without signal-contingent tradings, i.e., conditional markets. We can not use the well defined relationship developed in the previous section with complete markets. This is the fundamental problem associated with social value of information with incomplete markets.

[Example 2]

All the features of the economy are identical to those in Example 1 except that one more real security is introduced. Hence, return matrices $[R]$ and $[R(Y)]$ are modified as follows :

$$[R] = \begin{bmatrix} a_1(s_1) & a_2(s_1) \\ a_1(s_2) & a_2(s_2) \\ a_1(s_3) & a_2(s_3) \\ a_1(s_4) & a_2(s_4) \end{bmatrix}$$

$$[R(Y)] = \begin{bmatrix} a_1(s_1, y_1) & a_2(s_1, y_1) & 0 & 0 \\ a_1(s_2, y_1) & a_2(s_2, y_1) & 0 & 0 \\ 0 & 0 & a_1(s_3, y_2) & a_2(s_3, y_2) \\ 0 & 0 & a_1(s_4, y_2) & a_2(s_4, y_2) \end{bmatrix}$$

Suppose that $[R]$ has a full rank and all its submatrices have full rank too. It is easy to see that markets are still incomplete with respect to no information, Y_0, but markets become conditionally complete with respect to information, Y. As before a competitive equilibrium without information is not constrained Pareto optimal generically. But, a competitive equilibrium with information will achieve a full Pareto optimality when signal-contingent tradings are available. Since $UPF(Y_0)$ lies strictly inside of $UPF(SY)$, it is the case that public information will have social value at least potentially if signal-contingent tradings are available. What happens to social value of information if signal-contingent tradings are not available ? Then, it is not clear whether public information has social value or not since a competitive equilibrium of this economy is not necessary constrained Pareto optimal with respect to information, Y and hence we can't draw any definite conclusion. If it turns out to be constrained Pareto optimal while the original equilibrium is not, then we can safely say that public information will have social value at least potentially.

We are basically interested in the welfare effect of public information with or without any creation of new markets. Since it is not easy to set up new markets, we have to pay much attention to the conditions under which public information will have social value without signal-contingent tradings being introducing in particular. Almost nothing has been done in this field and this is a serious topic left for future research.

III. AGGREGATE UNCERTAINTY AND SOCIAL VALUE OF INFORMATION

3.1. Aggregate Uncertainty and Social Value of Information in a Pure Exchange Economy

One interesting feature of an economy under uncertainty is that individuals are subject to individual uncertainty, but there is no aggregate uncertainty in the economy as a whole. That is, there is individual risk, but no aggregate risk. This is a probable economic situation in the economy with large markets as was suggested by Malinbaud(1972). If this is the case, the effect of public information on the welfare levels of economic agents will turn out to be different from the case with aggregate uncertainty. Social value of information will be affected by the nature of uncertainty in a non-trivial way.

Furthermore, we can relax the assumption of additively separable utility function when there is no aggregate uncertainty. So, we will examine the problem with a broad class of utility functions from now on.

DEFINITION 2

If endowment profile satisfies the following condition, then it is said that there is individual uncertainty, but no aggregate uncertainty[9]:

$$\sum_h e_h(1) = \ldots\ldots = \sum_h e_h(n)$$

LEMMA 4

If there is individual uncertainty, but no aggregate uncertainty when there are complete contingent claims markets, then an equilibrium allocation x^* must satisfy the following condition :

$$x_h^*(1) = \ldots\ldots = x_h^*(n) \text{ for all } h \in H.$$

[9] This is a simple interpretation of "no aggregate uncertainty". See Malinbaud (1972) for the detail.

PROOF

Suppose not. Then, there must be at least a some consumer $i \in H$ and some states s, s' such that $x_i^*(s) \neq x_i^*(s')$. Now, take an average allocation over states and define a new allocation $x_h'(1)$:

$$x_h'(1) = \sum_s \pi(s) x_h^*(s)$$

Then, $\sum_h x_h'(1) = \sum_h \sum_s \pi(s) x_h^*(s) = \sum_s \pi(s) \sum_h x_h^*(s) = \sum_h e_h(s)$, which implies that x' is feasible. Furthermore, it is true due to the concavity of utility function :

$$\sum_s \pi(s) U_h(x_h^*(0), x_h'(1)) = U_h(x_h^*(0), x_h'(1)) = U_h(x_h^*(0), \sum_s \pi(s) x_h^*(s))$$
$$= U_h(\sum_s \pi(s) x_h^*(0), \sum_s \pi(s) x_h^*(s)) \geq \sum_s \pi(s) U_h(x_h^*(0), x_h^*(s))$$

with at least one strict inequality, which is a contradiction to the fact that x^* is a Pareto optimal allocation. Q.E.D

LEMMA 5

Suppose that there is individual uncertainty, but no aggregate uncertainty. Let x^* be an equilibrium allocation of C.C.C.M.. Then, $x^*(SY) \equiv x^*$ for any public information structure, Y where $x^*(SY)$ is an equilibrium allocation of the standard regime with information.

PROOF

The following conditions must hold from the first order conditions and by lemma 4 at an equilibrium allocation of C.C.C.M. :

$$\sum_s (\pi(s) \partial U_h(x_h^*(0), x_h^*(s)) / \partial x_h(0)) = (\partial U_h(x_h^*(0), x_h^*(1)) / \partial x_h(0)) \sum_s \pi(s)$$
$$= \partial U_h(x_h^*(0), x_h^*(1)) / \partial x_h(0) = \lambda_h^* p^*(0)$$

$$\pi(s) \partial U_h(x_h^*(0), x_h^*(s)) / \partial x_h(s) = \lambda_h^* p^*(s) \text{ for each } s \in S$$

Now, let $\mu_h^* = \lambda_h^*$ and $p^*(0, y_i) = \pi(y_i) p^*(0)$, $p^*(s, y_i) = \pi(y_i / s) p^*(s)$. Then, it is easy to check that $(p^*(SY), x^*(SY))$ is a competitive equilibrium of the standard regime due to the same argument in the previous section. Q.E.D

Note that the converse of lemma 5 holds too as before, which implies that there is an isomorphism again between these two market regimes. This means that all the arguments in section 2.1. holds here without the separability assumption on the utility function. Moreover, it may be the case that public information would make everyone worse off unless signal-contingent tradings are available under no aggregate uncertainty. The following simple example is illustrated here for the expository purpose.[10)]

[Example 3]

There are two agents in this simple pure exchange economy, denoted by $h=1,2$. There is one commodity and there are two uncertain states, denoted by $s=a, b$ in period $t=1$. There is no consumption in $t=0$. Prior probabilities are summarized by the objective probabilities π^a, π^b respectively. Each consumer's endowment of good in period $t=1$ is assumed to be given as follows :

$$e_1=(e_1^a, e_1^b)=(\varepsilon^a, \varepsilon^b), \quad e_2=(e_2^a, e_2^b)=(1-\varepsilon^a, 1-\varepsilon^b)$$

Each consumer's preference is represented by Von-Neumann and Morgenstern utility function : $V_h(x_h)=\pi^a U_h(x_h^a)+\pi^b U_h(x_h^b)$ where u_h is strictly concave. Also, there are complete contingent claims markets in period $t=0$. Now, consider several cases and compare the welfare properties of equilibrium allocati-ons.

1) C.C.C.M. without information

In this case without aggregate uncertainty, equilibriun allocation must be on the diagonal of the Edgeworth box with the usual tangency condition between marginal rate of substitutions. Thus, the supporting equilibrium prices must satisfy the condition $p^{a^*}/p^{b^*}=\pi^a/\pi^b$.

Then, equilibrium allocation is obtained as follows :

$$x_1^{a^*}=x_1^{b^*}=\varepsilon^a\pi^a+\varepsilon^b\pi^b \text{ and } x_2^{a^*}=x_2^{b^*}=(1-\varepsilon^a)\pi^a+(1-\varepsilon^b)\pi^b$$

The utility level of each consumer can be obtained as follows :

$$V_1(x_1^*)=V_1(Y_0)=\pi^a U_1(x_1^{a^*})+\pi^b U_1(x_1^{b^*})=U_1(\varepsilon^a\pi^a+\varepsilon^b\pi^b)$$

$$V_2(x_2^*)=V_2(Y_0)=\pi^a U_2(x_2^{a^*})+\pi^b U_2(x_2^{b^*})=U_2((1-\varepsilon^a)\,\pi^a+(1-\varepsilon^b)\,\pi^b)$$

2) C.C.C.M with perfect information

Suppose that there are two possible signals available $y=\alpha, \beta$ before binding contracts are made and information is perfect, that is there is a function "η"

[10] This example is the generalization of that used by Arrow(1984).

such that $\eta(a)=\alpha$, $\eta(b)=\beta$.

Then, whether α or β is observed, no contingent claims contracts will be made and hence endowment point is equilibrium allocation. Hence, utility levels are as follows :

When α is observed, $V_1(\alpha)=U_1(\varepsilon^a)$, $V_2(\alpha)=U_2(1\text{-}\varepsilon^a)$
When β is observed, $V_1(\beta)=U_1(1\text{-}\varepsilon^a)$, $V_2(\beta)=U_2(1\text{-}\varepsilon^b)$

Then, the value of information for each consumer is as follows :

$$V_1(Y_p)=\pi(\alpha)U_1(\varepsilon^a)+\pi(\beta)U_1(\varepsilon^b)=\pi^aU_1(\varepsilon^a)+\pi^bU_1(\varepsilon^b)$$
$$V_2(Y_p)=\pi(\alpha)U_2(1-\varepsilon^a)+\pi(\beta)U_2(1-\varepsilon^b)=\pi^aU_2(1-\varepsilon^a)+\pi^bU_2(1-\varepsilon^b)$$

Now, comparing their utility levels in both cases, we'll get the following relationship by the concavity of utility function :

$$V_1(Y_0)>V_1(Y_p) \text{ and } V_2(Y_0)>V_2(Y_p)$$

This implies that perfect public information will make both consumers worse off by eliminating risk sharing opportunities and instead introducing distributive risk. So, perfect public information is socially harmful here unless signal-contingent trading opportunities are available.

3) C.C.C.M. with noisy information

Now, suppose that noisy information is available before binding contracts are made. Noisy information is represented by the following Markov matrix :

$$\Pi=\begin{bmatrix} \pi(\alpha/a) & \pi(\beta/a) \\ \pi(\alpha/b) & \pi(\beta/b) \end{bmatrix}$$

where each element is non-zero.

When α is observed, contingent claims contracts are made as in case 1 and the resulting equilibrium allocation is obtained as follows :

$$p^*(a/\alpha)/p^*(b/\alpha)=\pi(a/\alpha)/\pi(b/\alpha)$$
$$x_1^{a^*}=x_1^{b^*}=\varepsilon^a\pi(a/\alpha)+\varepsilon^b\pi(b/\alpha)$$
$$x_2^{a^*}=x_2^{b^*}=(1-\varepsilon^a)\pi(a/\alpha)+(1-\varepsilon^b)\pi(b/\alpha)$$

$$\text{Hence, } V_1(\alpha)=\pi(a/\alpha)U_1(x_1^{a^*})+\pi(b/\alpha)U_1(x_1^{b^*})$$
$$=U_1(\varepsilon^a\pi(a/\alpha)+\varepsilon^b\pi(b/\alpha))$$
$$V_2(\alpha)=\pi(a/\alpha)U_2(x_2^{a^*})+\pi(b/\alpha)U_2(x_2^{b^*})$$
$$=U_2((1-\varepsilon^a)\pi(a/\alpha)+(1-\varepsilon^b)\pi(b/\alpha))$$

When β is observed, the above process will be repeated and hence the resulting utilty levels are as follows :

$$V_1(\beta)=\pi(a/\beta)U_1(x_1^{a^*})+\pi(b/\beta)U_1(x_1^{b^*})$$
$$=U_1(\varepsilon^a\pi(\mathrm{a}/\beta)+\varepsilon^b\pi(\mathrm{b}/\beta))$$
$$V_2(\beta)=\pi(a/\beta)U_2(x_2^{a^*})+\pi(b/\beta)U_2(x_2^{b^*})$$
$$=U_2((1-\varepsilon^a)\pi(a/\beta)+(1-\varepsilon^b)\pi(b/\beta))$$

Now, the value of information for each consumer is as follows :

$$V_h(Y_N)=\pi(\alpha)V_h(\alpha)+\pi(\beta)V_h(\beta) \qquad \text{for } h=1,2$$

From the strict concavity of utility function,

$$\pi(\alpha)V_1(\alpha)+\pi(\beta)V_1(\beta)$$
$$< U_1(\varepsilon^a\pi(a,\alpha)+\varepsilon^b\pi(b,\alpha)+\varepsilon^a\pi(a,\beta)+\varepsilon^b\pi(b,\beta))$$
$$= U_1(\varepsilon^a\pi^a+\varepsilon^b\pi^b)=V_1(Y_0) \text{ and this is same for } h=2.$$

Thus, $V_1(Y_0)>V_1(Y_N)$ and $V_2(Y_0)>V_2(Y_N)$.

Moreover, $V_h(Y_0)>V_h(Y_N)>V_h(Y_P)$ for $h=1,2$ since there were exchanges of claims under noisy information while no tradings occured under perfect information. Again, public information is socially harmful and more information is more harmful than less information.

REMARK 3

1) The main implication of this example is that public information is socially harmful in the sense of making everyone worse off regardless of the property of information structures if there are complete markets and there exists only individual uncertainty. The reason is simply because information introduces distributive risk and market process is just another version of fair game in this context. Thus risk averse consumers will be worse off by participating in the markets with information. This argument can be extended to a general model with many consumers, many states and many commodities under no aggregate

uncertainty.

2) The argument that more information is always valuable from an individual or a planner's point of view is reversed here. That is, more information is more harmful than less information in a general equilibrium model under no aggregate uncertainty. This argument can also be extended to a general model as long as a sequence of information structures induce a sequence of fair market games.

3.2. Aggregate Uncertainty and Social Value of Information in a Production Economy

When we discuss the issue of social value of information in a general equilibrium model, it might well be expected that public information has a more positive welfare implication in a production economy than in a pure exchange economy as was mentioned by Hirshleifer(1971) and Arrow(1984). But, it is not always obvious whether there will be welfare gain in any production economy regardless of types of information structures. So, it may be safe to say that whether public information can have social value or not in a production economy will also depend on fundamentals of market economies such as preferences, market structure, market regime and technological properties.

Here we can extend the argument in the previous section to an economy with production so as to make a general observation on the welfare implication of public information. Suppose that there are C.C.C.M. in a two-period economy as before. Other features of our economy are identical except that each individual is endowed with commodities and initial share of firms and there are "F" number of firms, $f=1, 2, ..., F$ with the following production technology :

Each production plan by firm "f" is represented by $a_f=(a_f(0), a_f(1),..., a_f(n))\in A_f$ where A_f is a compact and convex production possibility set.

Alternatively, $a_f=(a_f(0), g_{f1}(a_f(0)), ..., g_{fn}(a_f(0)))\in A_f$ where $g_{fs}(.)$ is a concave and state dependent production function for $s=1,...,n$ and $f=1,...,F$.

Now, suppose that each firm is subject to technological uncertainty, but it is cancelled out from overall point of view so that there is no aggregate technological uncertainty in the economy as a whole.

DEFINITION 3

There is no aggregate technological uncertainty if aggregate level of production is independent of states. That is, $\sum_f a_f(s)=\sum_f a_f(1)$, $s=2,....,n$, for every choice of $a_f(0)$, $f=1,..,F$. When there are C.C.C.M. without information, each consumer will face the following problem :

Maximize $\sum_{s} \pi(s) U_h(x_h(0), x_h(s))$

subject to $p(0)x_h(0) + \sum_{s} p(s)x_h(s) = p(0)e_h(0) + \sum_{f} v_f(a_f)\, \underline{\theta}_{hf}$ ....10)

where $V_f(a_f)$ denotes firm f's profit when production plan a_f is chosen and $\underline{\theta}_{hf}$ is the initial shareholding of firm f by consumer h. Each firm will face the following problem :

Maximize $[\sum_{s} p(s)a_f(s) - p(0)a_f(0)]$ subject to $a_f \in A_f$, for $f = 1, \ldots, F$ ....11)

Now, let a triple (p^*, x^*, a^*) be a competitive equilibrium of this economy. Then, x^* will satisfy the following property due to the nature of no aggregate technological uncertainty.

LEMMA 6

If x^* is an equilibrium allocation of this economy, it must be the case that $x_h^*(s) = x_h^*(1)$ for $s = 2, \ldots, n$, for every $h \in H$.

PROOF

The same argument in a pure exchange economy is straightforwardly applied here due to no aggregate technological uncertainty. Q.E.D

Next, suppose that perfect public information is introduced and tradings opportunities contingent not only on states, but also on signals are available. In this standard regime with perfect information and production, each consumer and firm will face the following problems :

Maximize $\sum_{s} \pi(y_s) U_h(x_h(0, y_s), x_h(s, y_s))$

subject to .... 12)

$\sum_{s} p(0, y_s)x_h(0, y_s) + \sum_{s} p(s, y_s)x_h(s, y_s) = \sum_{s} p(0, y_s)e_h(0) + \sum_{f}\sum_{s} v_f(a_f(y_s))\underline{\theta}_{hf}$

and

Maximize $\sum_{s} [p(s, y_s)a_f(s, y_s) - p(0, y_s)a_f(0, y_s)]$

subject to $a_f(y_s) \in A_f(y_s)$ for all s .... 13)

where $a_f(y_s) = (a_f(0, y_s), a_f(s, y_s))$ and $A_f(y_s)$ is the projection of A_f on two relevant coordinates, which is also convex for each $s \in S$.

Let $p(SY_p)$, $x(SY_p)$ and $a(SY_p)$ be defined as follows:

$$p(SY_p)=(p(y_1),....,p(y_n)),$$
$$x(SY_p)=(x_1(SY_p),....,x_m(SY_p)),$$
$$a(Y_p)=(a_1(SY_p),....,a_f(SY_p))$$

where $x_h(Y_p)=(x_h(y_1),...,x_h(y_n))$ and $a_f(Y_p)=(a_f(y_1),...,a_f(y_n))$

DEFINITION 4

$(p^*(SY_p), x^*(SY_p), a^*(SY_p))$ is a competitive equilibrium of the standard regime with perfect information if i) $x_h^*(SY_p)$ is the optimal solution to (12) relative to $p^*(SY_p)$ ii) $a_f^*(SY_p)$ is the optimal solution to (13) relative to $p^*(SY_p)$ and iii) $\sum_h x_h^*(0,y_s)+\sum_f a_f^*(0,y_s)=\sum_h e_h(0)$, $\sum_h x_h^*(s, y_s)=\sum_f a_f^*(s,y_s)$ for $s=1,....,n$

LEMMA 7

If $x^*(SY_p)$ is an equilibrium allocation of the standard regime with perfect information, it must be the case that $x_h^*(0,y_1)=x_h^*(0,y_s)$ and $x_h^*(1,y_1) = x_h^*(s,y_s)$ for $s=2,...,n$ and all $h\in H$.

PROOF

Suppose not. Then, there is at least one consumer $j\in H$ with an equilibrium allocation violating one of the above equalities. Now, define another allocation x' such that $x_h'(0)=\sum_s \pi(y_s)x_h^*(0,y_s)$, $x_h'(i)=\sum_s \pi(y_s)x_h^*(s,y_s)$ for $i=1,..,n$ and $h\in H$. It must be checked whether this allocation x' is feasible under the given technology.

1) feasibility

$$\sum_h x_h'(0)=\sum_h\sum_s \pi(y_s)x_h^*(0,y_s)=\sum_s \pi(y_s)(\sum_h e_h(0)-\sum_f a_f^*(0,y_s))$$
$$=\sum_h e_h(0)-\sum_f(\sum_s \pi(y_s)a_f^*(0,y_s))$$
$$\sum_h x_h'(i)=\sum_h\sum_s \pi(y_s)x_h^*(s,y_s)=\sum_s \pi(y_s)\sum_f a_f^*(s,y_s)$$
$$=\sum_f(\sum_s \pi(y_s)a_f^*(s,y_s))$$

Now, define $a_f'=(a_f'(0), a_f'(1),....,a_f'(n))$ such that $a_f'(0)=\sum_s \pi(y_s)a_f^*(0,y_s)$ and $a_f'(i)=g_{fi}(a_f'(0))=g_{fi}(\sum_s \pi(y_s)a_f^*(0,y_s))$ for $i=1,...,n$, Then

$$\sum_f a_f'(i)=\sum_f g_{fi}(\sum_s \pi(y_s)a_f^*(0,y_s))$$
$$\geq \sum_f\sum_s \pi(y_s)g_{fi}(a_f^*(0,y_s)) \text{ by the concavity of } g_{fi}(\cdot)$$

$$=\sum_s \pi(g_{fi})\sum_f g_f 6^i(a_f^*(0,y_s)) \qquad \text{.... 14)}$$

Note that $\sum_h x_h'(i)=\sum_f\sum_s \pi(y_s)g_{fs}(a_f^*(0,y_s))$

$$=\sum_s \pi(y_s)\sum_f g_{fs}(a_f^*(0,y_s)) \qquad \text{.... 15)}$$

In 14) and 15), $\sum_f g_{fi}(a_f^*(0,y_s)) = \sum_f g_{fs}(a_f^*(0,y_s))$ by no aggregate technological uncertainty. So, it holds that $\sum_h x_h'(i)\le\sum_f a_f'(i)$ for $i=1,...,n$. Therefore, x' is a feasible allocation under the given technology.

2) Pareto domination

$$\sum_s \pi(y_s)U_h(x_h^*(0,y_s),x_h^*(s,y_s))\le U_h(\sum_s \pi(y_s)x_h^*(0,y_s),\ \sum_s \pi(y_s)x_h^*(s,y_s))$$
$$=U_h(x_h'(0),x_h'(i))$$

for all $h\in H$ with at least one strict inequality. This is a contradiction to the fact that $x^*(SY_p)$ is a Pareto optimal allocation as an equilibrium of the standard regime. Q.E.D

LEMMA 8

Suppose that (p^*,x^*,a^*) is a competitive equilibrium of C.C.C.M. without information. Then, $x^*(SY_p)\equiv x^*$ is a competitive equilibrium allocation of the standard regime with perfect information.

PROOF

First, set the price system of the standard regime as follows :

$p^*(0,y_s)=\pi(y_s)p^*(0)$, $p^*(s,y_s)=p^*(s)$.

It is easy to check that all the f.o.c. are satisfied at this price system. For the choice of optimal production plan, consider the following manipulation:

Maximize $\sum_s[p^*(s,y_s)a_f(s,y_s)-p^*(0,y_s)a_f(0,y_s)]$

$\Leftrightarrow$ Maximize $\sum_s[p^*(s)a_f(s,y_s)-\pi(y_s)p^*(0)a_f(0,y_s)]$

$\Leftrightarrow$ Maximize$[\sum_s p^*(s)a_f(s,y_s)-p^*(0)(\sum_s \pi(y_s)a_f(0,y_s))]$

subject to $a_f(y_s)\in A_f(y_s)$ for all s

If firm f chooses $a_f^*(SY_p)$ such that $a_f^*(0,\ y_s)=a_f^*(0)$, $a_f^*(s,y_s)=a_f^*(s)$, it is a profit maximizing plan relative to the price system. Thus, there exist a production plan $a^*(SY_p)$ supporting $x^*(SY_p)$ as a feasible allocation and consumers' budget constraints are satisfied. Obviously all markets clear at these consump- tion and production plans. Q.E.D

Now, let $E(x)$ and $E(x(SY_p))$ denote the sets of equilibrium allocations of a production economy with each market regime respectively under no aggregate technological uncertainty. Again, it holds that $E(x)\equiv E(x(SY_p))$ from lemma 7 and lemma 8 as in a pure exchange economy.

Next, consider a competitive equilibrium of the conditional markets and evaluate social value of perfect information by comparing utility levels in this regime with those in C.C.C.M.. Then, the value of information of each consumer is equivalent to the utility level evaluated at a competitive equilibrium allocation as a solution to the following problem as in a pure exchange economy :

Maximize $\sum_s \pi(y_s)U_h(x_h(0,y_s),x_h(s,y_s))$

subject to

$$p(0,y_1)x_h(0,y_1)+p(1,y_1)x_h(1,y_1)=p(0,y_1)e_h(0)+\sum_f v_f(a_f(y_1))\underline{\theta}_{hf}$$

$$\vdots \qquad\qquad \vdots$$

$$p(0,y_n)x_h(0,y_n)+p(n,y_n)x_h(n,y_n)=p(0,y_n)e_h(0)+\sum_f v_f(a_f(y_n))\underline{\theta}_{hf}$$

and for each firm "f" and each signal $y_s\in Y_p$

Maximize $[p(s,y_s)a_f(s,y_s)-p(0,y_s)a_f(0,y_s)]$

subject to $a_f(y_s)\in A_f(y_s)$

Let $(p'(Y_p),x'(Y_p),a'(Y_p))$ be a competitive equilibrium of the conditional markets. It is obvious that all the arguments about the welfare comparison in a pure exchange economy are straightforwardly applied to a production economy here. So, let $[p'(Y_p)]$ and $[p^*(SY_p)]$ denote matrix representation of equilibrium prices as before.

REMARK 4

1) $x^*(SY_p)\equiv x'(Y_p)$ only if there exists a vector $\omega\in R^n$ such that $\omega^T[p'(Y_p)]=[p^*(SY_p)]$. If there exists such ω, then $B_h([p'(Y_p)])$ is a subspace of $B_h([p^*(SY_p)])$ for all h. But, there is no guarantee that $\gamma_h=\omega$ for all h. Thus, $x_h'(Y_p)=x_h^*(SY_p)$, implying $V_h(Y_p)=V_h(SY_p)$ if $\gamma_h=\omega$ and $x_j'(Y_p)\neq x_j^*(SY_p)$, implying $V_j(Y_p)<V_j(SY_p)$ otherwise.

2) If there exists no vector $\omega\in R^n$ such that $\omega^T[p\,(Y_p)]=[p^*(SY_p)]$, then

either i) $V_h(SY_p)>V_h(Y_p)$ for some h and $V_j(SY_p)<V_j(Y_p)$ for some j or ii) $V_h(SY_p)\geq V_h(Y_p)$ for all h with at least one strict inequality. This is obvious from the Pareto optimality of $x^*(SY_p)$ and the non-existence of such ω.

PROPOSITION 4

If there are C.C.C.M. without information in a production economy under no aggregate technological uncertainty and if no signal-contingent tradings are available, then perfect information will have no social value or make no one better off with someone being made strictly worse off.

PROOF

This is straightforward from the above remark and lemma 7, 8. Q.E.D

Next, we have to examine whether all the arguments made so far can be maintained when noisyless information is not perfect or even when information is noisy. It is easy to confirm that all the previous arguments hold in the case with imperfect information. Although it is expected that this is also true in the case with noisy information, we will examine social value of noisy information for the heuristic purpose. In the standard regime with noisy information, consumers and firms will face the following problems as before :

$$\text{Maximize } \sum_i \pi(y_i)\sum_s \pi(s/y_i)U_h(x_h(0,y_i),x_h(s,y_i))$$

subject to

$$\sum_i p(0,y_i)x_h(0,y_i)+\sum_i\sum_s p(s,y_i)x_h(s,y_i)=\sum_i p(0,y_i)e_h(0)+\sum_f\sum_i v_f(a_f(y_i))\underline{\theta}_{hf}$$

and

$$\text{Maximize } [\sum_i\sum_s p(s,y_i)a_f(s,y_i)-p(0,y_i)a_f(0,y_i)]$$

subject to $a_f(y_i)\in A_f(y_i)$ for $i=1,\ldots,k$

Suppose that $(p^*(SY_N), x^*(SY_N), a^*(SY_N))$ is a competitive equilibrium of the standard regime. Let's define the following allocation $x'(SY_N)$ as before. $x_h(0)=\sum_i\sum_s \pi(s,y_i)x_h^*(0,y_i)$ and $x_h'(s')=\sum_i\sum_s \pi(s,y_i)x_h^*(s,y_i)$ for $s'=1,\ldots,n$ and all h. We have to check whether this allocation is feasible under the given technology and information.

$$\sum_h xh'(0)=\sum_h\sum_i\sum_s \pi(s,y_i)x_h^*(0,y_i)$$
$$=\sum_h\sum_i \pi(y_i)x_h^*(0,y_i)$$

$$=\sum_h e_h(0)-\sum_f(\sum_i \pi(y_i)a_f{}^*(0,y_i))$$

$$\sum_h xh'(s')=\sum_h\sum_i\sum_s \pi(s,\, y_i)x_h{}^*(s,\, y_i)$$

$$=\sum_i\sum_s \pi(s,\, y_i)\sum_f a_f{}^*(s,\, y_i)$$

$$=\sum_f\sum_i a_f{}^*(s,\, y_i)\sum_s \pi(s,\, y_i),\text{ since }\sum_f a_f{}^*(s,\, y_i)\text{ is independent of } s\in S(y_i)\text{ by no aggregate technological uncertainty,}$$

$$=\sum_f(\sum_i \pi(y_i)a_f{}^*(s,\, y_i))$$

So, x' is also a feasible allocation by the same argument as before. Moreover,

$$\sum_i \pi(y_i)\sum_s \pi(s/y_i)U_h(x_h{}^*(0,y_i),x_h{}^*(s,y_i))$$
$$=\sum_i\sum_s \pi(s,y_i)U_h(x_h{}^*(0,y_i),x_h{}^*(s,y_i))$$
$$\leq U_h(\sum_i\sum_s \pi(s,y_i)x_h{}^*(0,y_i),\ \sum_i\sum_s \pi(s,y_i)x_h{}^*(s,y_i))$$
$$=U_h(x_h{}'(0),\ x_h{}'(s'))$$

for all h with at least one strict inequality from the strict concavity of utility function.

COROLLARY 1

Suppose that $(p^*(SY_N),\, x^*(SY_N),\, a^*(SY_N\,))$ is a competitive equilibrium of the standard regime. Then, it must be the case that $x^*(Y_N)$ is independent of states and signals and investment decisions by firms are also independent of signals.

PROOF

This is obvious from the feasibility of x' and Pareto optimality of an equilibrium allocation of the standard regime. Q.E.D

COROLLARY 2

If $(p^*,\ x^*,\ a^*)$ is a competitive equilibrium of C.C.C.M. without information, then $x^*(SY_N\,)\equiv x^*$ is also an equilibrium allocation of the standard regime, supported by the following price system :

$$p^*(0,y_i)=\pi(y_i)p^*(0),\ p^*(s,y_i)=\pi(y_i/s)p^*(s)\text{ for all } s\in S\text{ and } y_i\in Y_N.$$

PROOF

$x_h{}^*(SY_N)$ is the optimal solution relative to the above price system and production plans such that $a_f{}^*(SY_N)\equiv a_f{}^*$ is also the optimal production plan for each $f\in F$. Furthermore budget constraints and market clearing conditions are all met at such an allocation. Q.E.D

REMARK 5

1) We can establish again the isomorphism between C.C.C.M. without information and the standard regime with noisy information in a production economy under no aggregate technological uncertainty by corollary 1 and 2. Thus, no matter what type of information is disseminated into the economy, it will make no difference between a standard regime with information and C.C. C.M. without information from a general equilibrium point of view.

2) When information is disseminated and there are no signal-contingent markets, any type of information will have no social value or make no one better off with someone being made strictly worse off in a production economy under no aggregate technological uncertainty.[11] This is somewhat different from the welfare implication of information in a pure exchange economy. Information plays the role of giving consumers chances to participate in fair market games and hence makes everyone worse off. But, since production efficiency can be improved by information in a production economy, there will be a little chance that all consumers are made strictly worse off.

3) No aggregate uncertainty may be a strong assumption. But, it gives us a benchmark to deal with the issue of social value of information in a pure exchange and a production economy. If there is individual uncertainty and aggregate uncertainty is not "large", then it can be conjectured that all the previous arguments about social value of information with or without creation of new markets would hold because of the continuity property of utility function and production function. Then, we can extend our discussion to a more broad set of endowment profiles and production technologies.

Ⅳ. CONCLUDING REMARKS

We have examined some basic problems related to social value of public information. When the original markets are complete, information dissemination will have no social value or even make everyone worse off unless signal-contingent trading opportunities become available. The exact conditions for social value of information are derived in terms of the relationship between equilibrium prices with and without information dissemination. When information is disseminated and signal-contingent markets are created, there is an isomor-

[11] It might be expected that information will be usually valuable in a production economy in the sense that it makes no one worse off even without additional trading opportunities contingent on signals. This is what Kunkel (1982) has pointed out. But, his argument is very much restrictive because he assumed that endowments of consumption and sharing holdings are equilibrium positions with respect to prior beliefs.

phism between C.C.C.M. without information and the standard regime regardless of the property of information structures.

When the markets are large in an economy under uncertainty, it might well be expected that there is individual uncertainty, but no aggregate uncertainty. One of the interesting problems in this case is the welfare implication of public information and this has been analyzed in the context of a pure exchange and a production economy. It has been confirmed that information will be more harmful in a pure exchange economy than in a production economy unless signal-contingent markets are created.

The most important and intriguing question related to social value of information would arise when the original markets are incomplete. This is a very difficult problem to be tackled because of the complex welfare property of equilibria in economies with incomplete markets. But, the case of incomplete markets under no or small aggregate uncertainty could be utilized as a benchmark economic situation when we intend to evaluate social value of information in a general equilibrium framework.

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