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OPTIMAL LABOR CONTRACTS UNDER WAGE AND SHARE SYSTEMS*

JOO MIN PARK**

This paper examines optimal labor contracts in a traditional wage system and Weitzman's share system incorporated into the implicit contract model, under symmetric and asymmetric information. In addition, this paper examines whether a share system yields a more efficient solution in terms of employment level than a wage system. When the information is symmetric, the implicit contract theory shows that the maximization problem under both compensation systems yield the same solution as the auction spot market. In the meanwhile, when information is asymmetric incentive compatibility provides the same solutions in both compensation systems, however, they are no longer Pareto optimal, i.e. specifying the inefficient underemployment or overemployment.

I. INTRODUCTION

Labor contracts between firms and workers specify rules which determine optimal levels of employment and compensation based on the assumption that firms maximize their profits and workers their utilities. Most labor contracts in the real world are mediated by verbal or tacit agreements rather than by written contractual forms. Such verbal or tacit agreements are termed 'implicit contracts', while written agreements are termed 'explicit contracts'.

Hart (1983) provides explanation as to why implicit contracts appear. He argues that there may not exist complete contracts between firms and workers in the real world. Contracts tend to be in force for limited periods of time, and are then renegotiated. He explains that the reason why we do not see explicit and long-term contracts in reality is because individuals simply can not conceive of all the possible eventualities that may occur, and so prefer to adopt a 'wait and see' approach. Thus, people have appealed to the idea of implicit, rather than explicit contracts.

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** Planning Department, Coryo-CM Life Insurance Corporation, Coryo Daeyungak Center Building, 25-5, 1-ka, Chungmu-ro, Chung-ku, Seoul

Implicit contract literature shows that informational asymmetries yield inefficient underemployment and/or overemployment in the labor market. Informational symmetry predicts exactly the same levels of employment as a Walrasian auction market. Under symmetric information all available information is given to both contracting parties so that they behave in a manner consistent with rational expectations. However, when information is given to only one party, the other will prevent it from cheating by implementing rules to insure enforcement of the contract. Such enforcement causes models of implicit contracts to yield inefficient employment level under asymmetric information. To show this, most implicit contract literature employs a traditional fixed wage system. In this paper, we will adopt two different types of compensation system; a wage system and a share system.

The share system has been suggested by Weitzman (1983). He examines the compensation system which contains a base wage and a certain share of output, revenue, profit, etc., per worker to be paid to each worker. He also argues that a share system makes the average worker, as well as the economy as a whole, better off because of a built-in bias toward eliminating unemployment, expanding production and lowering prices.

Cooper (1983) suggests a methodology for solutions to problems in the economics of asymmetric information taking the same form as the contracting problem. He studies implicit contracts in two states of nature. He also argues that models of labor contracts under asymmetric information predict either overemployment or underemployment.

This paper surveys the implicit contract theory and finds solutions of the maximization problem for both a share system and a wage system incorporated into the implicit contract model, under symmetric and asymmetric information. In addition, this paper examines whether a share system yields a more efficient solution in terms of employment level than would a wage system.

The rest of the paper is organized as follows. Section II describes the economy in question. The model of symmetric information is developed in Section III. Section IV explains an idea of incentive compatibility. Section V presents the optimal contract under asymmetric information for both wage and share systems. Finally Section VI offers conclusions.

II. THE ECONOMY

Consider the contract designed between a single firm and a group of m workers before any real economic activity occurs. Such an agreement must specify rules which determine the levels of employment and compensation over some future period, and, therefore, specify the levels of profit and utility received by the firm and workers respectively, in different states.

1. The Firm's Profit Function

Assume that the firm is risk neutral and has the following profit function:

(1) $\pi = sf(\boldsymbol{L}(s)) - \boldsymbol{W}(s)$

Where s represents a state of nature reflecting the firm's technological uncertainty drawn from a set of possible states $S = \{s | s = s_1, s_2, ..., s_n \ ; \ s_i > s_j$ for any $i > j\}$ with associated probability set $Q = \{q(s) \,|\, q(s) = q(s_1), q(s_2), ..., q(s_n)\}$; $sf(\boldsymbol{L}(s))$, the firm's gross revenue; $\boldsymbol{L}(s)$, the workers total labor supply in the state of nature s; and, $\boldsymbol{W}(s)$, the total amount of compensation paid by the firm to workers in state s. We assume that labor, $L(s)$ is the only variable entering the firm's production function, where $L(s)$ is an increasing function in s, i.e., $dL/ds > 0$.[1] We also assume that the m workers are homogeneous, and, therefore, it follows $\boldsymbol{L}(s) = \sum L_i(s) = m \cdot L(s)$ and $\boldsymbol{W}(s) = \sum W_i(s) = m \cdot W(s)$.[2] The production function is assumed to be increasing and concave in $L(s)$ so that $f'(\boldsymbol{L}(s)) > 0$ and $f''(\boldsymbol{L}(s)) < 0$.

2. Preferences of Workers

Each worker is assumed to have a von Neumann-Morgenstern utility function represented by the following:

(2) $U = U(W(s), L(s))$

Given that workers are homogeneous, we can assume that each worker's consumption equals the compensation paid by the firm. We also assume that $U_L < 0$, $U_{LL} < 0$. Notice that the worker is strictly risk averse over consumption i.e., $U_W > 0$ and $U_{WW} < 0$, and that $U_{WL} = U_{LW}$.

3. Compensation System

In this paper, we examine two different types of contracts between the firm and workers. One is associated with wage system and the other associated with share system.

a. Wage system

[1] An interpretation of $dL/ds > 0$ would be that with a positive shock, an increase in s, the firm will hire more labor.

[2] See Hart (1983).

In the wage system, any compensation function obeys the following.

(3) $\partial W(s)/\partial L(s)=0$ for all s

which implies that under wage system the compensation rate in each state of nature is fixed for any level of employment, i.e. the compensation depends only on the state of nature. Thus, the compensation function under the wage system is :

(4) $W=W(s)$

Equation (4) represents the traditional fixed wage system. This compensation form is also used in most implicit contract literature.

b. Share System

Consider next a compensation scheme suggested by Weitzman, called a share system.[3] As we will see, the compensation under the share system depends upon the state of nature as well as the level of employment.

Generally a contract between the firm and workers specifies workers compensation as a function of the firm's performance, so that

(5) $W=F(\tau, z)$

where τ stands for a contract parameter, fixed in the short run, but fully flexible in the long run. z is some index of the firm's performance. z will be determined indirectly by the amount of labor supply by workers, L, via some transformation function,

(6) $z=M(L(s), s)$.

Substituting (6) into (5) determines the reduced form compensation function:

(7) $W=F(\tau; M(L(s), s))=W(\tau;L(s), s)$

The reduced form compensation function, (7), can be expressed by a simple wage-cum-bonus formula[4]:

(8) $W=\Phi+\tau \cdot (G(L(s), s))$

[3] See the discussion in Weitzman (1983).

[4] See Ellis and Park (1985).

where Φ stands for the base wage and is fixed in the short run but fully flexible in the long run. The function G(·) is a transform of the performance index which satisfies $\tau G_L = F_Z M_L$, $\tau G_S = F_Z M_S$. In a share system, any compensation function satisfies:

$$(9)\ dW/dL = (dW/dL) \cdot (dL/dL) = m\tau \cdot G_L < 0.$$

With such a system, the firm will choose L so as to maximize:

$$(10)\ \pi = sf(\boldsymbol{L}(s)) - \boldsymbol{W}(\tau; \boldsymbol{L}(s), s)$$
$$\text{s.t. } \boldsymbol{L}(s) = mL(s)$$
$$\boldsymbol{W}(s) = mW(s).$$

Maximization problem (10) can be rewritten:

$$(11)\ \max_{L(s)} \pi = sf(mL(s)) - mW(\tau;\ mL(s), s).$$

Differentiating (11) we obtain the following:

$$(12)\ \partial\pi/\partial L = msf'(mL(s)) - m^2\tau \cdot G_L.$$

Based on equations (9) and (12), a share system can generally be characterized by the following:

$$(13)\ \partial\pi/\partial L > 0.$$

This shows that an increase in labor supply will raise the firm's profits. The firm will, therefore, wish workers to work more. However, no extra employment can be sustained in the long run, since $\partial W/\partial L < 0$. In the long run, workers will simply accept the market clearing compensation rate elsewhere in the economy.

III. SYMMETRIC INFORMATION

In the section, we assume that the firm and workers know the probabilities $q(s_1)$, $q(s_2)$, ..., $q(s_n)$; the profit function, $sf(\boldsymbol{L}(s) - \boldsymbol{W}(s))$; and the utility function, $U(W(s), L(s))$. Thus, there is symmetric information in the economy, which, in other words, implies that the realization of s is the public information.

1. Wage System

When s is publicly informed under the wage system, the optimal contract $\Omega^* = \{L^*(s), W^*(s)\}$ solves the following problem:

$$(14)\ \max\ E_s[\pi]$$

$$\text{s.t.}\quad E_s[U(W(s), L(s))] \geq \underline{U}$$

where $\underline{U}$ is the reservation value of worker's expected utility.

The associated Lagrangian is:

$$(15)\ H = \Sigma q(s) \cdot [sf(mL(s)) - mW(s)] + \mu \cdot [\Sigma q(s) \cdot U(W(s), L(s)) - \underline{U}]$$

The first order conditions for a maximum are:

$$(16)\ \partial H/\partial L = mq(s) \cdot sf'(mL(s)) + \mu q(s) \cdot U_L = 0$$
$$(17)\ \partial H/\partial W = -mq(s) + \mu q(s) \cdot U_W = 0$$
$$(18)\ U_W = m/\mu = k \text{ (constant) for } \forall s$$
$$(19)\ U_L = -k \cdot sf'(mL(s)) \text{ for } \forall s.$$

Equation (18) represents an optimal risk sharing (or coinsurance) and equation (19) ensures productive efficiency. From (18) and (19) we obtain:

$$(20)\ (-U_L/U_W) = sf'(mL(s)).$$

Equation (20) implies that worker's marginal rate of subsititution between the compensation and labor is equated with the marginal revenue product of labor for any state of nature. Therefore, the wage system yields the Pareto optimal contract under symmetric information.

2. Share System

Now consider the optimal contract with a share system under symmetric information, which maximizes the firm's expected profit subject to the worker's expected utility :

$$(21)\ \max\ Es[W]$$
$$\text{s.t.}\quad E_S[U\{W(\tau;L(s),s),L(s)\}] \geq \underline{U}.$$

Using Lagrangian, (21) can be rewritten:

$$(22)\ H=\Sigma q(s)\cdot\{sf(mL(s))-m\cdot[\Phi+\tau G(mL(s),s)]\}$$
$$+\mu\cdot\{\Sigma q(s)\cdot U(\Phi+\tau G(mL(s),s),L(s))-\underline{U}\}.$$

The necessary conditions for a maximization problem (21) are:

$$(23)\ \partial H/\partial\Phi=-mq(s)+\mu q(s)\cdot U_W=0$$
$$(24)\ \partial H/\partial\tau=-mq(s)\cdot G(mL(s))+\mu q(s)\cdot G(mL(s))\cdot U_W=0$$
$$(25)\ \partial H/\partial L=mq(s)[sf'(mL(s))-m\tau G_L]+\mu q(s)[m\tau G_L U_W+U_L]=0.$$

From equation (23), (24), and (25) we obtain:

$$(26)\ U_W=m/\mu=k\quad\text{for }\forall s$$
$$(27)\ U_L=-k\cdot sf'(mL(s))\quad\text{for }\forall s.$$

Equations (26) and (27) immediately lead to:

$$(28)\ (-U_L/U_W)=sf'(mL(s))\quad\text{for }\forall s.$$

As in the case of a wage system, a share system specifies an optimal risk sharing and productive efficiency, {(26), (27)}, and, therefore, yields a Pareto optimal contract.

PROPOSITION 1

Under symmetric information, a Pareto optimal contract is obtained regardless of the compensation system and the state of nature.

PROOF:

Equations (20) and (28) show that the Pareto optimum condition holds for any state of nature under both the wage and share systems. Q.E.D.

Proposition 1 implies that when the realization of s is public information, the implicit contract model predicts exactly the same employment level in each state of nature as the spot auction market, reagardless of the compensation system. Under symmetric information, both parties behaviors are based on rational expectations. Therefore, there is no incentive or scope for either the firm or the worker to cheat.

Ⅳ. INCENTIVE COMPATIBILITY

Under symmetric information, since s is public information, the contract designed *ex ante* need not contain any provisos against the firm's cheating. However, if s is private information to the firm, the firm may have an incentive to lie. At the time of negotiating the contract, the worker knows that the firm will lie if it has an incentive to do so. The contract designed between the firm and the worker will, therefore, be such that the firm will not have an incentive to do so. The contract designed between the firm and the worker will not have an incentive to lie. The optimal contract must satisfy the following incentive compatibility or self-selection constraints:

$$(29)\quad \pi(s_i|s_i)=s_if(mL(s_i))-mW(s_i)\geq\pi(s_j|s_i)=s_if(mL(s_j))-mW(s_j)\quad \text{for } i, j=1, 2,\dots,\bar{n}$$

which means that firm's profits, when the announced state of nature is the same as the true state of nature, are at least as large as the firm's profits when it announces that a state of nature is different from the true one.

1. Wage System

Under wage system in every incentive compatible contract, the compensation and employment functions are both non-decreasing functions of the state of nature, which implies that $L(s_i)\geq L(s_j)$ and $W(s_i)\geq W(s_j)$, for $s_i\geq s_j$.[5] When the firm realizes that it can make more profits by requiring employees to work more, it will have an incentive to announce s_i when s_j is true. On the other hand, when the firm can increase profits by wage cuts, it will have an incentive to announce s_j when s_i has occurred. However, the incentive compatibility constraints, (29), ensure that the firm will always announce the true state of nature.

2. Share System

We now consider incentive compatibility under a share system. Before

[5] See Azariadis (1983) for the proof.

doing this, we need to examine the compensation function, showing how the wage rate changes in response to a change in the state of nature. Differentiating the compensation function with respect to s provides:

$$(30)\quad dW/ds=(dL/ds)\cdot m\tau G_L+\partial W/\partial s$$

Equation (30) shows two effects of a change in s on the wage rate. The first term represents indirect effects of a change in s on W via a change in L, that is, if a positive shock occurs, the firm will hire more labor and, therefore, the share per each worker will fall. The second term shows direct effects of a change in s on W. We reasonably assume that $\partial W/\partial s>0$, implying that when there is a positive shock in the economy, the firm will pay a higher wage. The sign of dW/ds, however, is indeterminate since the first term in (30) is negative, and the second term is positive. The sign of dW/ds depends on which term dominates.

Therefore, i) when indirect effects are greater than direct effects, formally, $|(dL/ds)\cdot m\tau G_L|>|\partial W/\partial s|$, W is decreasing in s, i.e. $dW/ds<0$. Even if the firm pays a higher wage as s increases, the indirectly decreased amount in W due to more labor is greater than the directly decreased amount of wage. ii) If $|(dL/ds)\cdot m\tau G_L|<|\partial W/\partial s|$, then $dW/ds>0$. This is the opposite to i). In this case, as in a wage system, incentive compatibility constraints hold in both states of nature. In other words, incentive compatibility ensures $dL/ds>0$ and $dW/ds>0$, and the converse is also true.

PROPOSITION 2.a (The case $dL/ds>0$ and $dW/ds<0$)

Under a share system, when $dL/ds>0$ and $dW/ds<0$, the incentive compatibility condition is satisfied only in the good state of nature.

PROOF:

$dL/ds>0$ and $dW/ds<0$ implies:

$f(mL(s_i))>f(mL(s_j))$ and $W(s_i)<W(s_j)$ for $s_i>s_j$. Thus,
$\pi(s_i|s_i)-\pi(s_j|s_i)=s_i[f(mL(s_i))-f(mL(s_j))]+[mW(s_j)-mW(s_i)]>0$.
Therefore, $\pi(s_i|s_i)>\pi(s_j|s_i)$. And
$\pi(s_j|s_j)-\pi(s_i|s_j)=s_j[f(mL(s_j))-f(mL(s_i))]+[mW(s_i)-mW(s_j)]<0$
implying $\pi(s_j|s_j)<\pi(s_i|s_j)$. Q. E. D.

Proposition 2.a implies that if the wage is decreasing in s (i.e. $dW/ds<0$), the incentive compatibility condition is satisfied in the good state of nature but not in the bad state of nature. Therefore, the firm will always announce the good state of nature no matter what the true state of nature is because it is profitable.

COROLLARY 2.a

Proposition 2.a also implies that a share system in which $dL/ds>0$ and $dW/ds<0$ can not constitute an incentive compatible share system.

PROPOSITION 2.b (The case $dL/ds>0$ and $dW/ds>0$)

When $dL/ds>0$ and $dW/ds>0$ incentive compatibility constraints will be satisfied in both states of nature if and only if

$s_i \geq [mW(s_i)-mW(s_j)]/[f(mL(s_i))-f(mL(s_j))] \geq s_j$

PROOF:

Suppose that $\pi(s_j|s_j) \geq \pi(s_i|s_j)$ and $\pi(s_i|s_i) \geq \pi(s_j|s_i)$, implying

$s_j[f(mL(s_i))-f(mL(s_j))] \leq mW(s_i)-mW(s_j)$ and

$s_i[f(mL(s_i))-f(mL(s_j))] \geq mW(s_i)-mW(s_j)$.

Thus, $s_j[f(mL(s_i))-f(mL(s_j))] \leq mW(s_i)-mW(s_j)$

$\leq s_i[f(mL(s_i))-f(mL(s_j))]$

which leads to $s_i \geq [mW(s_i)-mW(s_j)]/[f(mL(s_i))-f(mL(s_j))] \geq s_j$. We now can easily derive incentive compatibility constraints from the above. Q.E.D.

COROLLARY 2.b

The share system in which $dL/ds>0$ and $dW/ds>0$ can be compatible with incentive compatibility constraints.

COROLLARY 3

In seeking an incentive compatible share scheme, one only needs to consider the case for which $dL/ds>0$ and $dW/ds>0$.

Proposition 2.a, corollary 2.a, proposition 2.b, corollary 2.b, and corollary 3 together imply that incentive compatible contracts can be obtained only when $dL/ds>0$ and $dW/ds>0$.

Now based on propositions and corollaries above, we examine incentive compatibility under specific forms of a share contract. There are numerous potential share contract forms, these include:[6]

(a) Revenue sharing : $W=\Phi+\tau(R/L)$ (R=Total Revenue)
(b) Wage Fund : $W=\Phi+(A/L)$ (A=Predetermined fund)
(c) Product Wage : $W=\Phi+\tau P$ (P=Product price)

Now we examine each form of a share contract whether or not it specifies incentive compatibility.

[6] See the discussion in Ellis (1985).

a) Revenue Sharing

Under this form, the compensation function is $W=\Phi+\tau \cdot sf(mL(s))/L(s)$. By definition of a share system;

$$dW/dL=(\tau/L)\cdot\{msf'(mL(s))-[sf(mL(s))/L(s)]\}<0.$$

Differentiating the compensation function with respect to s provides:

$$dW/ds=(\tau/L)\cdot\{f(mL(s))+[msf'(mL(s))-sf(mL(s))/L(s)]\cdot(dL/ds)\}$$

which can be rewritten as:

$$dW/ds=(\tau/s)\cdot\{\varepsilon\cdot msf'(mL(s))-(\varepsilon-1)\cdot[sf(mL(s))/L(s)]\}$$
where $\varepsilon=(dL/ds)\cdot(s/L)>0$; the state elasticity of labor demand.

Now to see incentive compatibility condition to be satisfied, we consider some cases depending on the value of ε.

i) The case in which $0<\varepsilon<1$

If L is inelastic in s, then $dW/ds>0$. Therefore, by proposition 2.b, corollary 2.b, and corollary 3 revenue sharing form always specifies incentive compatibility in this case.

ii) The case $\varepsilon>1$

In this case, incentive compatibility condition to hold (i.e. $dW/ds>0$), the following, must be satisfied, i.e. $msf'(mL(s))>[(\varepsilon-1)/\varepsilon]\cdot[sf(mL(s))/L(s)]$. Now together with a definition of a share system, we obtain the following incentive compatibility condition in this case:

$$[(\varepsilon-1)/\varepsilon]\cdot[sf(mL(s))/L(s)]<msf'(mL(s))<sf(mL(s))/L(s).$$

As above cases show, under the revenue sharing form, incentive compatibility depends on the value of ε.

b) Wage Fund

If we employ a share scheme of the wage fund form, then

$$dW/ds=(-A/L^2)\cdot(dL/ds)<0.$$

Therefore, a wage fund form always yields non-incentive compatibility.

c) Product Wage

When a share contract employs the product wage form, the compensation function is : $W = \Phi + \tau P(L(s), s)$. By definition of a share system;

$$dW / dL = \tau P_L < 0.$$

Now differentiating W with respect to s gives:

$$dW/ds = \tau(L/s) \cdot [(P/L) \cdot \beta + P_L \cdot \varepsilon], \text{ where } \beta = (\partial P/\partial s) \cdot (s/P).$$

Notice that the incentive compati-bility condition is $dW/ds > 0$. Therefore, the product wage form to be compatible with incentive compatibility constraints, the following must hold: $-[P_L/(P/L)] < (\beta / \varepsilon)$.

V. ASYMMETRIC INFORMATION

Consider the situation of asymmetric information under which s is observed only by the firm. There may be employment distortion relative to the full information solution due to asymmetric information.

To examine the impact of asymmetric information we now assume that there are only two states of nature in the economy. Let s_g be the good state of nature with the probability q, and s_b, the bad state of nature which occurs with the probability $1-q$, where $0<q<1$ and $s_g > s_b$. We also define underemployment as occurring when $(-U_L/U_W) < sf'(mL)$ and overemployment when $(-U_L/U_W) > sf'(mL)$.

1. Wage System

The firm will choose the optimal contract under a wage system with asymmetric information to maximize its expected profits. Notice that when the firm always tells the truth or lies the expected profits are as follows:

$$E[\pi|\text{always truth-telling}] = q \cdot \pi(s_g|s_g) + (1-q) \cdot \pi(s_b|s_b)$$
$$E[\pi|\text{always lying}] = q \cdot \pi(s_b|s_g) + (1-q) \cdot \pi(s_g|s_b).$$

However, $E[\pi \mid \text{always truth-telling}] \geq E[\pi \mid \text{always lying}]$ since $\pi(s_g \mid s_g) \geq \pi(s_b \mid s_g)$ and $\pi(s_b \mid s_b) \geq \pi(s_g \mid s_b)$ by incentive compatibility. Similarly, $E[\pi \mid \text{truth-telling about } s_g, \text{ lying about } s_b] \leq E[\pi \mid \text{always truth-telling}]$ and, $E[\pi \mid \text{truth-telling about } s_b, \text{ lying about } s_g] \leq E[\pi|\text{always truth-telling}]$.

Given that the contract is incentive compatible, the expected profits when telling the truth dominate those when announcing any state alternative, implying that the firm will maximize its expected profits of telling the truth subject to utility constant and incentive compatibility constraints.

The firm, therefore, will face the following maximization problem to choose the optimal contract.

$$(31)\ \max E[\pi]$$

$$\text{s.t. } E[U(W(s), L(s))] \geq \underline{U}$$
$$\pi(s_g|s_g) \geq \pi(s_b|s_g)$$
$$\pi(s_b|s_b) \geq \pi(s_g|s_b).$$

The Lagrangian incorporating the probabilities is:

$$(32)\ H = q \cdot [s_g f(mL_g) - mW_g] + (1-q) \cdot [s_b f(mL_b) - mW_b]$$
$$+ \mu \cdot [q \cdot U(W_g, L_g) + (1-q) \cdot U(W_b, L_b) - \underline{U}]$$
$$+ \sigma \cdot [s_g f(mL_g) - mW_g - s_g f(mL_b) + mW_b]$$
$$+ \Theta \cdot [s_b f(mL_b) - mW_b - s_b f(mL_g) + mW_g].$$

The necessary conditions for a maximum are:

$$(33)\ \partial H/\partial L_g = q \cdot s_g f'(mL_g) + \mu q \cdot U_{Lg} + m\sigma \cdot s_g f'(mL_g) - m\Theta \cdot s_b f'(mL_g) = 0$$
$$(34)\ \partial H/\partial W_g = -mq + \mu q \cdot U_{wg} - m\sigma + m\Theta = 0$$
$$(35)\ \partial H/\partial L_b = m(1-q) \cdot s_b f'(mL_b) + \mu(1-q) \cdot U_{Lb} - m\sigma \cdot s_g f'(mL_b) + m\Theta \cdot s_b f'(mL_b) = 0$$
$$(36)\ \partial H/\partial W_b = -m(1-q) + \mu(1-q) \cdot U_{wb} + m\sigma - m\Theta = 0.$$

Using (33) and (34), we obtain:

$$(37)\ (-U_{Lg}/U_{wg}) = s_g f'(mL_g) + [\Theta/(q+\sigma-\Theta)] \cdot (s_g - s_b) f'(mL_g).$$

Since $q+\sigma-\Theta > 0$ by (34) and $s_g > s_b$, (37) can be written as:

$$(38)\ (-U_{Lg}/U_{wg}) \geq s_g f'(mL_g) \quad \text{for } \Theta \geq 0.$$

Similarly, equations (35) and (36) yield:

$$(39)\ (-U_{Lb}/U_{wb}) \leq s_b f'(mL_b) \quad \text{for } \sigma \geq 0.$$

We now examine the binding condition, i.e. either Θ or σ is zero, and then the other positive. i) Suppose that both incentive compatibility constraints are binding so that $\sigma>0$ and $\Theta>0$. In this case, we would have $\pi(s_b|s_b)=\pi(s_g|s_b)$, implying that: $W_g - W_b = s_g[f(mL_g) - f(mL_b)] = s_b[f(mL_g) - f(mL_b)]$. Hence, $\sigma>0$ and $\Theta>0$ follow that $W_g = W_b$ and $L_g = L_b$, which lead to a contradiction. Therefore, both cannot be binding simultaneously. ii) We now consider the case in which none of constraints is binding, implying that $\sigma=0$ and $\Theta=0$. If this holds true, the second-best contract (asymmetric information) coincides with the first-best contract (symmetric information). If the later is not incentive compatible, then it is a contradiction. We can see it from the following example.

EXAMPLE[7]:

The first-best contract is not incentive compatible for any s. Suppose the firm is risk neutral[8] and utility function is additively separable,

$$U(W, L)=h(W)-g(L).$$

By the equation (18), $U_W=h_W=k$ which implies that $W(s) = W_0$, independent of s. Hence,

$$\pi(s_b|s_b)=s_b f(mL(s_b))-mW_0<s_b f(mL(s_g))-mW_0=\pi(s_g\,|\,s_b).$$

Therefore, the first-best contract is not incentive compatible for $s_g>s_b$, which leads to a contradiction. In this example, both $\sigma=0$ and $\Theta=0$ cannot happen simultaneously. Furthermore, Chari (1983), Green and Kahn (1983), and Hart (1983) argue that in general, when the firm is risk neutral and the worker's utility function takes the general form $U(W, L)$ the first-best contract is not consistent with the incentive compatibility. We now reduce the cases to ($\sigma=0$, $\Theta>0$) and ($\sigma>0$, $\Theta=0$).

In the maximization problem (31), $\sigma=0$, $\Theta>0$ and $\sigma>0$, $\Theta=0$. From (37) $\Theta>0$ implies overemployment in the good state of nature, and similarly $\sigma>0$ implies underemployment in the bad state of nature. In other words, which one of them is positive determines whether we have overemployment or underemployment.

2. Share System

Under a share system, by proposition 2.a., corollary 2.a, and corollary 3,

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7 See Azariadis (1983).

8 We assume risk neutral firm in this paper.

only when $dL/ds>0$ and $dW/ds>0$, incentive compatibility conditions are satisfied in both states of nature, and, therefore, we only consider the case of $dL/ds>0$ and $dW/ds>0$. As in the case of a wage system, under a share system, the expected value of profits when telling the truth is at least as much as those when the firm announces the wrong state of nature, i.e.,
$E[\pi|\text{always truth-telling}] \geq E[\pi|\text{lying about at least one state of nature}]$.

Thus, the firm will face the following maximization problem.

$$(40)\quad \max E[\pi]$$

$$\text{s.t.}\quad E[U\{W(\tau;L(s),s),L(s)\}] \geq \underline{U}$$

$$\pi(s_g|s_g) \geq \pi(s_b|s_g)$$
$$\pi(s_b|s_b) \geq \pi(s_g|s_b).$$

The Lagrangian associated with the problem (40) is:

$$\begin{aligned}(41)\quad H=&\,q\cdot\{s_g f(mL_g)-m\cdot[\Phi+\tau G(mL_g)]\}\\ &+(1-q)\cdot\{s_b f(mL_b)-m\cdot[\Phi+\tau G(mL_b)]\}\\ &+\mu\cdot\{q\cdot U(\Phi+\tau G(mL_g),L_g)\\ &\qquad+(1-q)\cdot U(\Phi+\tau G(mL_b),L_b)-\underline{U}\}\\ &+\sigma\cdot\{s_g f(mL_g)-m\cdot[\Phi+\tau G(mL_g)]-s_g f(mL_b)\\ &\qquad+m\cdot[\Phi+\tau G(mL_b)]\}\\ &+\Theta\cdot\{s_b f(mL_b)-m\cdot[\Phi+\tau G(mL_b)]-s_b f(mL_g)\\ &\qquad+m\cdot[\Phi+\tau G(mL_g)]\}.\end{aligned}$$

The necessary conditions for a maximum are:

$$\begin{aligned}(42)\quad \partial H/\partial L_g=&\,mq\cdot[s_g f'(mL_g)-m\tau G_{Lg}]+\mu q\cdot[U_{wg}\cdot m\tau G_{Lg}+U_{Lg}]\\ &+m\sigma\cdot[s_g f'(mL_g)-m\tau G_{Lg}]\\ &+m\Theta\cdot[-s_b f'(mL_g)+m\tau G_{Lg}]=0\\ (43)\quad \partial H/\partial L_b=&\,m(1-q)\cdot[s_b f'(mL_b)-m\tau G_{Lb}]\\ &+\mu(1-q)\cdot[U_{wb}\cdot m\tau G_{Lb}+U_{Lb}]\\ &+m\sigma\cdot[s_g f'(mL_b)-m\tau G_{Lb}]\\ &+m\Theta\cdot[s_g f'(mL_b)-m\tau G_{Lb}]=0\\ (44)\quad \partial H/\partial\Phi=&-mq-m(1-q)+\mu q\cdot U_{Wg}+\mu(1-q)\cdot U_{Wb}=0\\ (45)\quad \partial H/\partial\tau=&-mq\cdot G(mL_g)-m(1-q)\cdot G(mL_b)\\ &+\mu q\cdot U_{Wg}\cdot G(mL_g)+\mu(1-q)\cdot U_{Wb}\cdot G(mL_b)\\ &-m\sigma\cdot G(mL_g)+m\sigma\cdot G(mL_b)-m\Theta\cdot G(mL_b)\\ &+m\Theta\cdot G(mL_g)=0.\end{aligned}$$

From equations (42), (44), and (45), we obtain

$$(46)\ (-U_{Lg}/U_{Wg}) \geq s_g f'(mL_g) \quad \text{for } \Theta \geq 0.$$

Now equations (43), (44),and (45) lead to:

$$(47)\ (-U_{Lb}/U_{Wb}) \leq s_b f'(mL_b) \quad \text{for } \sigma \geq 0.$$

Equations (46) and (47) are the same with (38) and (39), respectively. Therefore, when $dL/ds > 0$ and $dW/ds > 0$, a share system yields overemployment in the good state of nature and underemployment in the bad state of nature as does a wage system.

THEOREM

Incentive compatibility always specifies the same solution, thus, the same level of employment regardless of the compensation system.

PROOF:

Equations (20) and (28) show that both systems yield the same solutions under symmetric information, and under asymmetric information, a solution to the wage system {(38), (39)} is the same with a solution to the share system {(46), (47)}. Therefore, theorem holds true. Q.E.D.

The above theorem summarizes results we have obtained under the different types of information and compensation system. Regardless of compensation systems, asymmetric information specifies employment distortion by the same amount in each system with incentive compatibility conditions being satisfied. This demonstrates that a share system can not yield a more efficient solution in terms of employment level than a wage system. Given the incentive compatible contract, the share system coincides with the wage system.

VI. CONCLUSION

In this paper, we have analyzed optimal labor contracts in a traditional wage system and Weitzman's share system under alternative cases of symmetric and asymmetric information. We also have shown that both compensation systems yield the same solution, implying that both systems employ the same amount of labor.

When information is symmetric, the implicit contract theory shows that the maximization problem under both compensation systems yields the same solution as the auction spot market. This implies that rational expectations of both contracting parties always guarantee a Pareto optimal solution.

When information is asymmetric incentive compatibility provides the same solutions in both compensation systems. However, they are no longer Pareto optimal. Since incentive compatibility constraints are binding either in the good state or in the bad state of nature, but not in both the solution specifies the inefficient underemployment or overemployment, depending on which state incentive compatibility constraints are binding in.

Regardless of compensation systems, symmetric information provides the optimal distribution of employment, and asymmetric information specifies employment distortion by the same amount in each system with incentive compatibility conditions being satisfied.

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