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VARYING PARAMETER ESTIMATION OF THE INTEREST ELASTICITY OF M1 DEMAND, 1974:1 — 86:1

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In this paper an analysis of conflicting results from Miller's (1986) and Mehra's (1986) study is that Miller did not properly consider an effect of gradual deregulation of interest-bearing NOWs on the interest coefficient of M1 demand by omitting interest dummies for the subperiod of financial innovation and deregulation. For the case of sequential and systematic parameter variation due to this form of misspecification, this study proposes the varying parameter regression model by Kalman for the period 1974 : 1-86 : 1, allowing the parameters to vary over time. Using Garbade's method along with Chavas's method in determining the process noise variance, our empirical results firmly support the above analysis. It is suggested that over the period of the innovation and deregulation, constant estimation technique seriously biases the estimate of the interest elasticity of M1 demand.

I. INTRODUCTION

Since the event of missing money occurred in the mid-1970s (Goldfeld, 1976), a considerable amount of research has been devoted to solving the puzzle. One approach is to focus on alternative explanatory variables that would explain the change. Many studies searched for a correct scale variable or the appropriate opportunity cost measure to be used in the demand function for money with an assumption that the basic relationship is stable. While this approach somewhat improved performance, the various evidence, as reviewed by Judd and Scadding (1982) indicated that the instability of the post-1973 money demand function may be due to other factors such as financial innovation. Thus, another approach is

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to examine the causes for instability of the conventional demand function for money.

While many studies presented money demand equations capable of explaining the case of missing money, the experience during the early 1980s has once again raised a similar question concerning the conventional demend for money. In 1980 the Federal Reserve adopted the Monetary Control and Depository Institution Deregulation Act. Key parts of the Act include the provision for the Negotiable Orders of Withdrawal (NOW) account and a gradual phase out of the deposit interest rate ceilings. As an attempt to explain the unprecedented decline in M1 velocity after 1979, numerious researchers have analyzed the effects of the monetary control on the conventional demand function for money. In particular, most researchers paid attention to the nature of the shift in money demand. For example, Brayton, Farr, and Porter (1983) argued that the introduction of interest-bearing NOWs into M1 increased the interest elasticity of M1 demand[1]. On the other hand, Gordon (1984) contended that the invention of NOW account changed the level (or intercept) of M1 demand[2]. While the studies concerned with types of shift do not reach a consensus, a comprehensive review of articles by Stone and Thornton (1987)leads to the conclusion that the velocity puzzle was a product of financial innovation and deregulation, and the cyclical variation in measured income.

This paper analyzes a conflicting result from the emprical studies by Miller (1986) and Mehra (1986) who examined the effects of the financial innovation and deregulation after 1973 in the conventional M1 demand by using a time dummy under the fixed estimation methods, and presents an alternative estimation technique that would be desirable for such case, as presented in section II.

It is analyzed that Miller did not properly account for a positive effect of the financial deregulation of NOW account on the interest elasticity by omitting important interest dummies form the estimated equation for the subperiod of the financial innovation and deregulation. As a result, the estimates of the interest coefficients for that subperiod may have been downward biased, therefore leading to a conflicting conclusion between the two studies. By Cooley's and Prescott's (1973) argument, it is shown that the omitted variables of such financial innovation and deregulation would have not only a transitory impact but also a permanent effect, causing the parameters to vary sequentially over time. For the case of sequential and systematic parameter variation due to this form of misspecifica-

[1]Brayton, Farr, and Porter derived a non-linear specification from the inventory-theoretic approach in which the payment on the OCD was explicitly dealt with. Then the specification was estimated for 1977 : 1-81 : 4 and simulated for 1982 : 1-83 : 1. The results indicated that among various specifications, the non-linear specification that fits best over 1982 : 1-83 : 1, has the highest M1 interest elasticity.

[2]Gordon analyzed that the decline in the M1 velocity for the 1981-83 is attributed to the growth in M1 consisting of NOWs types of deposit when financial deregulation transferrs the public from savings to NOWs.

tion, this study proposes the Varying Parameter Regression (VPR) model by Kalman for the period of money demand instability 1974 : 1-86 : 1, allowing the parameters to change over time.

The remainder of the paper is as follows. Section III contains the money demand specification and data. For a statisitical reason, we adopt Spencer's specification. Money is defined as the old M1 until January 1979, and, thereafter, as the old M1 plus the OCDs. The following section describes the Kalman estimation method presented by Harvey (1981). In determining the covariance matrix of the process noise, this study employs Garbade's method for estimating σ^2 along with Chavas's method. In section IV, the empirical results are presented. First, the estimate of the interest coefficient of M1 demand in the VPR model is compared with those in the fixed regression models. Second, this paper analyzes the movements in the varying coefficient of the interest rate by using smoothing procedure. Section V provides conclusion.

II. A STATISTICAL ANALYSIS OF THE TWO EMPIRICAL STUDIES

Statements of inconclusive evidence

Miller applied a standard money demand regression without first-difference form over the sample period of 1960-83 with quarterly data, as replicated in table 1. He arbitrarily selected the subperiod of 74-83 to test the effect of financial changes on the interest elasticity for M1, and carefully chose several subperiods of 74-75, 80-81, 81-83, and 80 to test the effect of financial innovation and deregulation on the level of M1 demand. As shown in the table, his empirical results indicate that sequential coefficients of the intercept dummy variables D1, D2, D3, and D4 were all statistically significant but the cofficients of the interest dummies D5*lnRCP and D5*lnRCB were not. Thus it was concluded that financial innovation and deregulation shifted the intercept, but not the slope coefficients in M1 demand. Below it is shown that this conclusion is inconsistent with Mehra's one.

In a different study, Mehra applied a distributed lag model with first-difference form over the sample period 1961-85 using monthly data. Also, the regression results are duplicated in the table. She selected the subperiod of 81-85 to test the effect of financial deregulation on the level as well as the slope of the interest elasticity of the demand for M1. The empirical results in the table present that the coefficient of the interest rate dummy variable, $D81 * \Delta \ln rR_t$ was statistically significant while the coefficient of the intercept dummy variable D81 was not. Thus she concluded that the financial deregulation in 1980 increased the interest elasticity but did not change the level of M1 demand, which is different from Miller's conclusion.

Its analysis and alternative estimation technique

Such conflicting results are one of the reasons why a number of past studies

[**Table 1**] Money Demand Regression Equations

Miller's regression equation, 1960:1-83:4

$$
\begin{array}{llllll}
\ln(M/P) = 0.9271 & +\,0.2394\ln y & +\,0.0422D5^{*}\ln y & -\,0.0230\ln RCP & -\,0.0132D5^{*}\ln RCP & -\,0.0318\ln RCB \\
\quad (3.07) & (5.03) & (0.62) & (-4.56) & (-1.45) & (-2.09)
\end{array}
$$

$$
\begin{array}{lllllll}
-\,0.1284D5^{*}\ln RCB & +\,0.4934\ln(M/P)_{-1} & -\,0.1312D5^{*}\ln(M/P)_{-1} & -\,0.0083D1 & -\,0.0038D2 & +\,0.0042D3 & +\,0.0162D4 \\
(-1.21) & (4.53) & (-1.06) & (-5.04) & (-3.17) & (3.39) & (2.88) \\
R^2 = 0.9885 & SE = 0.0054 & h\,(m) = 1.35 & & & &
\end{array}
$$

Mehra's regression equation, 1961:01-85:03

$$
\begin{array}{lllllll}
\Delta\ln(M/P) = & -0.002 & +\,0.99\Delta\ln y & -\,0.06\Delta\ln R & -\,0.001D74 & +\,0.001D81 & -\,0.08D81^{*}\Delta\ln R \\
 & (-3.0) & (7.1) & (-6.4) & (-1.7) & (1.6) & (-3.0) \\
 & R^2 = 0.45 & & SE = 0.00366 & DW = 1.93 & &
\end{array}
$$

D1, D2, and D3 are dummy variables corresponding to 1974:1-75:4, 1980:1-81:3, and 1981:4-83:2, respectively. D4 and D5 are the dummies coded"one" in 1980:3 and 1974:1-83:4, respectively, and "zero"otherwise. The Numbers in parentheses are z-statistics. D74 and D81 are dummy variables that equal 1 in the periods 1974:01-80:12 and 1981:01-85:03, respectively and zero otherwise. The figures in parentheses in Mehra's study are t-statistics. The distributed-lag coefficients on the price level constrainted to sum to zero.

have attempted to explain the presence of instability in parameters[3]. Taking a realistic example, if Miller had selected the subperiod of 82-83 (see Brayton, Farr, and Porter) instead of 74-83, his conclusion on stable interest elasticity would have changed. In this paper it is analyzed that Miller did not properly consider a positive effect of the financial innovation and deregulation of interest-bearing NOWs on the slope of the interest rate variable in conventional M1 demand, thereby causing a specification error. In other words, he omitted important dummies of the interest rate from the estimated equation over the subperiod of 74-83[4]. This analysis strongly suggests that the estimate of the coefficient of the interest dummy, for instance, D5*lnRCP in the table must have been downward biased. As a consequence, it is plausible that Miller falsely showed the significance of the coefficient of the interest dummy. In a similar way, Mehra possibly also created a misspecification by not properly accounting for the effect of the financial deregulation on the intercept in M1 demand. If she had chosen the subperiod of 81-83 (see Gordon) instead of 81-85, the coefficient of the intercept dummy D81 would have been significant.

In these cases of misspecification, the omitted variables would have not only a transitory impact but also a permanent effect, causing the parameters to vary sequentially over time, as argued by Cooley and Prescott (1973)[5]. For instance, the omitted variable such as weather will have a temporary effect, but the omitted variables like changes in institutional arrangements or technological development may have permanent effects that persist into the future without decay, therefore causing the parameters to change slowly over time. In relation to the permanent effects of missing variables, it was argued that since the auto-regressive error process assumes that the omitted factors have only transitory changes, such a process is not likely to describe the true distribution of the disturbance. Thus in the case of parameter instability, a constant regression method would have problems. In a survey of stochastic parameter regression, Rosenberg (1973) more specifically argued that in the presence of parameter variation, constant regression techniques

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[3]In treating such kind of problems, Cargill and Meyer (1978) and Lieberman (1979) had explained structural instability with a from of misspecification. Cargill and Meyer anlyzed in their empirical study that the evidence of time variation in the response of income to change in monetary policy indicates that important a prior information is being excluded from the estimation process by confining models via constant coefficient methods. On the other hand, Lieberman viewed that money demand instability in the post 1973 period is due to the omission from the estimated equations of a measure of technological changes. In the analysis of the similar problem from the empirical studies regarding the velocity puzzle, Roley (1985) has simply suggested that the recent financial innovation and deregulation may introduce parameter instability.

[4]For the subperiod of 1974-83, he included several intercept dummies while he did not include any interest dummy, thereby implying that the parameter of the interest coefficient is over time constant for that subperiod.

[5]When sequential parameter variation applies to slope coefficients, parameters are implied to vary systematically also.

have severe problems of inefficiency and even invalidity in estimating individual parameters. Accordingly, the results from the fixed parameter methods in Miller's and Mehra's study for the period of the financial innovation and deregulation, are not reliable.

For the case of sequential and systematic parameter variation due to this form of specification error, this study uses the varying parameter regression model by Kalman for the period of the financial innovation and deregulation 1974:1-86:1. This estimation technique explicitly accounts for the impact of the financial changes in the estimation process by permitting the parameters to vary over time.

III. THE SPECIFICATION AND DATA

For estimation purpose, the demand for money is specified as :

$$\Delta \ln m_t = \sum_{i=0}^{n_1} b_i \Delta \ln Y_{t-i} + \sum_{i=0}^{n_2} c_i \Delta \ln R_{t-i} + \sum_{i=0}^{n_3} d_i \Delta \ln P_{t-i} + E_t \tag{1}$$

where m_t is the real money stock, Y_t is real income, R_t is the short-term interest rate, and P_t is price. Following Mehra (1978), this paper simply chooses $n_1 = n_2 = n_3 = 4$. While we make no ciriticism of first difference form in the specification here, we note that it implies an asymptotically unbounded variance on the demand for money as time goes on.

Regression (1) differs in several ways from a standard money demand regression. First, it considers the Nominal Partial Adjustment Hypothesis (NPAH), which implies the change in the price level affacts the money balances with a lag. As Spencer (1985) suggested, the NPAH would be more important during our sample period of a considerable change in price level. Under the hypothesis, the individual coefficient on the price level d_i should be different from zero. Second, it is freely estimated by simple distributed lags. Under the NPAH, the derived money demand regression using a Koyeck-lag specification contains the income, interest rate, and lagged nominal money stock deflated by the current price level. This regression would have a restriction of the shape of the distributed lags to which the estimates of the coefficients of the variables might be sensitive. In an attempt to avoid such a restriction, Spencer has presented an alternative specification like (1), allowing the shape of the distributed lags to be free with some finite level. Third, more importantly in this study, the regression does not include the lagged dependent variable as a regressor. From the statistical points provided by Ram (1982) and Hetzel and Mehra (1989), the use of a simple distributed lag can avoid the estimation problems that might cccur in the presence of the lagged variable for the period of a substantially unstable economy in the VPR procedure.

All of the data for our study are obtained from the Federal Reserve Bulletin published by the Board of Governors of the FRS, and the Survey of Current

Business (September, 1989 p. 55-58) published by the U. S. Department of Commerce. Y measures real GNP ($ 1982), seasonally adjusted quarterly data while P is the GNP deflator (1982 = 100). The narrow nominal money stock M is measured by a quarterly average of monthly data of M1 until January 1979. From February 1979, it is obtained by a quarterly average of monthly data of the revised M1B series, following Milbourne (1983 p.635). The data are also seasonally adjusted. R is the prime commercial paper rate.

IV. THE VARYING PARAMETER REGRESSION MODEL

The N variables which are observed are defined by $N \times 1$ vector m_t and these are related to the state variables by the measurement equation

$$m_t = z_t \alpha_t + \xi_t, \quad t = 1, \ldots, T \tag{2}$$

where α_t is $m \times 1$ vector, and Z_t is fixed matrix of order $N \times m$. The $N \times 1$ vector of disturbances ξ_t has mean zero and covariance matrix, H_t of order NxN. In case of single observation, the disturbance has mean zero and variance $\sigma^2 h_t$, where h_t is assumed to be one here. Following most applications of Kalman filter, the movements of the state vector α_t are simply assumed by the transition equation,

$$\alpha_t = \alpha_{t-1} + \eta_t, \quad t = 1, \ldots, T \tag{3}$$

where the process noise η_t is a $m \times 1$ vector of disturbances with mean zero and covariance Q_t. If $Q_t = 0$, then equation (2) and (3) become a fixed regression model, where parameters are assumed to be constant over time. Both disturbances are taken to be mutually and serially uncorrelated for all time periods, and with the initial state vector α_0.

Kalman filter is a set of prediction and updating equations that allows an estimator to be updated once a new observation becomes available. The symbol a_t denotes the Minimum Mean Square Linear Estimator (MMSLE) of α_t at time t based on all the information up to the current observation m_t[6] $\cdot$ $a_{t/t-1}$ denotes the MMSLE of α_t at time t-1. Given that a_{t-1} is the MMSLE of α_{t-1} at time t-1 with a form $a_{t-1} - \alpha_{t-1} \sim WS(O, P_{t-1})$, the transition equation (3) suggests that the MMSLE of α_t at time t-1 is given by

$$a_{t/t-1} = a_{t-1}. \tag{4}$$

[6]Consider the model $y = Z\alpha + \xi$. With α random, the GLS estimator, α is best with a form in terms of the estimation error, $(\bar{a} - \alpha) \sim WS[0, \sigma^2(Z'\Omega^{-1}Z)^{-1}]$, where WS stands for "wide sense". The modified Gauss Markov theorem shows that $\bar{a}$, which is the unconditionally unbiased estimator, minimizes the variance of the estimation error, among any other linear estimator, and therefore minimizes the mean square error of the estimation error. It is leading to the statement that $\bar{a}$ is Minimum Mean Square Linear Estimator (MMSLE) of α.

Substracting α_t from both sides of (4) we have the estimation error $a_{t/t-1} - \alpha_t$ with a form,

$$a_{t/t-1} - \alpha_t \sim WS(O, P_{t/t-1}) \text{ where} \tag{5}$$
$$P_{t/t-1} = P_{t-1} + Q_t. \tag{6}$$

Given that $a_{t/t-1}$ is the MMSLE of α_t at time t-1, the MMSLE of m_t at time t-1 is $\overline{m}_{t/t-1} = Z_t a_{t/t-1}$. Then the associated prediction error m_t-$Z_t a_{t/t-1}$ is obtained with mean zero and covariance matrix $z_t p_{t/t-1} Z_t' + H_t$. (4) and (6) are the prediction equations for the state vector and its covariance matrix. The role of the updating equations is to incorporate the prior information in (5) into the sample information in the measurement equation (2) by constructing an augumented model. The MMSLE of α_t in the augumented model, obtained by the GLS estimator is rewritten using a matrix inversion lemma as,

$$a_t = a_{t/t-1} + P_{t/t-1} Z_t' F_t^{-1} (m_t - Z_t a_{t/t-1}) \text{ where} \tag{7}$$
$$F_t = Z_t P_{t/t-1} Z_t' + H_t \text{ and}$$
$$P_t = P_{t/t-1} - P_{t/t-1} Z_t' F_t^{-1} Z_t P_{t/t-1} \tag{8}$$

with a form $a_t - \alpha_t \sim WS(O, P_t)$. (7) and (8) are the derived updating equations. Thus the VPR model by Kalman (4), (6), (7), and (8) is derived from the above linear dynamic model of the measurement and transition equations, which is known as the state space model. The prediction error contains all the information in m_t and it is used to update $a_{t/t-1}$ via the Kalman gain $P_{t/t-1} Z_t' F_t^{-1}$.

Updating equations use the information available at that time. On the other hand, smoothing equations that will be importantly employed in this paper use all of the information since they begin with the estimator of the state variable in the final period a_T, where the Kalman procedure utilizes all of the sample observations. Therefore smoothing provides the optimal means of extracting estimates of the state variables from the observations. If $a_{t/T}$ and $P_{t/T}$ denote the smoothed estimator and its covariance matrices at time t, the smoothing equations are

$$a_{t/T} = a_t + P_t^* (a_{t-1/T} - a_t) \tag{9}$$
$$P_{t/T} = P_t + P_t^* (P_{t+1/T} - P_{t+1/t}) P_t^{*\prime} \tag{10}$$

where $p_t^* = P_t P_{t+1/t}$, $t = T\text{-}1, \ldots 1$.

Garbade (1977) had before used the Kalman filter to test the stability of money demand for the period of a relatively stable economy. We use this method in order to estimate the demand for money for the period of a substantially unstable economy. Thus unlike Garbade, this study focuses on estimation rather than the stability test.

Before performing the VPR, several estimations should be done. This study proceeds as follows. First, most researchers will not have a prior information on parameters α for initializing the Kalman filter. To obtain the prior estimates of a_0 and its variance P_0, this study applies GLS to the period of stable demand function for money 1960 : 1-73 : 4. Second, in estimating process noise variance Q_t, we adopt Chavas's (1983) assumption $Q_t = k^2 P_{t-1}$. Then, the likelihood function in terms of one-step-ahead prediction errors shown by Judge, Griffiths, Hill, Lutkepohl, and Lee (1985) can be rewritten during a time interval $\{t_0$ to $t_1\}$, disregarding the constants as

$$L = -\sum_{t=t_0}^{t_1} \ln|(\sigma^2 + z_t P_{t-1} z_t' (1+k^2))| - \sum_{t=t_0}^{t_1} \left[\frac{(\bar{m}_t - z_t a_{t-1})^2}{\sigma^2 + z_t P_{t-1} z_t' (1+k^2)} \right] \tag{11}$$

where m_t represents $t-$th scalar observation of m_t and z_t denotes N-dimensional row vector of known exogenous regressors of Z_t. An estimator of k^2 for given σ^2 is derived by maximizing (11):

$$\hat{k}^2 = \frac{1}{N} \sum_{t=t_0}^{t_1} \left[\frac{(\bar{m}_t - z_t a_{t-1})^2 - (\sigma^2 + z_t P_{t-1} z_t')}{z_t P_{t-1} z_t'} \right] \tag{12}$$

Third, this paper employs the VPR technique suggested by Garbade (1977) to properly estimate the variance of the measurement error σ^2 [7]. He has strongly pointed out in his empirical analysis that in the case of the significant parameter changes with the large sample length, the stationary coefficient regression model upward biases the estimate of σ^2 severely[8]. In this regard, the technique would be desirable

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[7]In eventually obtaining such an idea to make lower the standard errors of varying coefficients, Chavas's personal suggestion (March, 1991) was greatly helpful. It was suggested that the use of the recursive residuals in estimating σ^2 and therefore k^2 would improve the varying parameter estimatioin results.

[8]We estimated σ^2 using GLS technique for the 1960:1-73:4 period, following Chavas. And then, this study estimated (1) over the 1974:1-86:1 period (and therefore the period of severe parameter changes with large sample length)by OLS, GLS, and the VPR by Kalman with the estimated value of k, 2. 5235. The estimation results indicate that the interest elasticity of the sum of c_i -0.4193 in the VPR is higher than that -0.1034 in the GLS which is higher than that -0.0505 in the OLS, therefore obtaining the expected results. However, the standard error of the varying coefficient of the interest rate is extremely large, thus failing to obtain satisfactory results. As an attempt to lower such a huse standard error, we employed the annual data (and therefore the small sample length of 11 observations) instead, and the equation (1) that does not include the lag variables and first differences. Especially, the annual data of M1 were measured by the end of monthly data here. Also, using GLS for estimating σ^2 for the 1950-73 period, this paper obtained the estimated value of k, 2.0110. For such cases, the estimation results present that for the 1974-85 period, the coefficient of the interest rate -0.0508 in the OLS is in absolute smaller than that -0.1101 in the GLS, which in turn is smaller than that, -0.5064 in the VPR, thus producing the necessary results. Also the plots of the smoothed estimates present that the coefficient was over time increasing after 1980 through 1985. However, although the standard

for our study. The VPR estimate of the error variance for given k^2 is available for a fixed sample length T by analytical maximization of (11):

$$\hat{\sigma}^2_{VPR} = \frac{1}{N}\sum_{t=1}^{T}\left[\frac{(\overline{m}_t - z_t a_{t-1})^2}{1 + z_t P_{t-1} z_t'(1+k^2)}\right] \tag{13}$$

Note that the variance of the state variable estimator is redefined as $a_{t-1} - \alpha_{t-1} \sim WS(O, \sigma^2 P_{t-1})$ for a simplicity here.

V. EMPIRICAL RESULTS[9]

Money demand equation (1) was estimated by GLS based on data from 1960 : 1 to 1973 : 4. The estimation results in table 2 indicate that the sums of coefficients of the income and interest rate 0.3690 and -0.0398, respectively have expected signs and magnitudes. Also they are statistically significant at 10 percent level. Concerning the estimated coefficient of the price variable, unexpectedly, it has a positive sign but it is statistically insignificant. In general, the elasticity estimates and their variances obtained from the 1960 : 1-73 : 4 period can be taken as the prior information a_0 and P_0 in the Kalman filter. Next task is to determine the variances of the measurement error σ^2 and process noise Q_t.

When k^2 is assumed to be zero, the whole term of the big parenthesis in (13) leads to the recursive residuals defined by Brown, Durbin, and Evans (1975). Thus the recursive algorithm is identical to the prediction and filtering components of VPR when k^2 is set to zero. For such a case, the varying parameter estimate of σ^2 is $(0.0087)^2$ [10].

If $Q_t = 0$, then, the VPR results are asymptotically equilevent to the OLS results. For such a case, over the 1974 : 1-86 : 1 period, the estimated money demand elasticities in the only final observation 86 : 1 are presented in table 3. The corresponding entire estimation results are reported in appendix 1. Indeed, this seems to happen for the interest elasticities. The estimated elasticity from OLS for the period, shown in table 2 is nearly equal to that from the VPR. The role of Kalman filtering is to incorporate new information and to revise the estimator

error of varying coefficient (0.4621) is much more lower compared to that of varying coefficient from quarterly data, the varying coefficient is not yet sufficiently significant. The above estimation results imply that for the sample period of the serious instability in money demand parameters, constant estimation technique such as GLS for estimating σ^2 produces a largerly spurious standard error of varying coefficient.

[9]The Kalman filter estimation in this paper was performed by PCRATS (Doan, 1990)

[10]Setting $k^2 = 0$ and $\sigma^2 = 1$, this study regressed equation (1)by Kalman filter for the 1974:1-85:4 to obtain the Kalman estimates a_{t-1} and P_{t-1} in estimator (13). Then over the fixed sample length 1974:1-86:1, $\hat{\sigma}^2_{VPR}$ was computed.

[**Table 2**] The fixed parameter estimation results of equation (1), 1960:1-1986:1

Estimation period	Estimation technique	The sums of coefficients[a]			Summary Statistics[b]		
		$\Delta\ln Y_t$	$\Delta\ln R_t$	$\Delta\ln P_t$	R	DW	SE
1960:1-73:4	GLS	0.3690	−0.0398	0.1098	0.3613	1.9087	0.0065
		(0.1725)	(0.0252)	(0.1680)			
1974:1-86:1	OLS	0.7848	−0.0489	−0.3132	0.4394	0.8811	0.0105
		(0.2288)	(0.0346)	(0.1271)			
	GLS	0.4772	−0.0992	−0.0235	0.6720	2.4284	0.0078
		(0.3419)	(0.0451)	(0.2393)			

[a]The numbers shown in parentheses are the standard errors computed by the method of Spencer.
[b]DW is Durbin-Watson statistic and SE is the standard error of regression.

in the previous period in each filter step, thus yielding a more efficient estimator over time. This can be clearly seen from the entire estimation results in case of stable coefficient although it is ambiguous in case of varying coefficient. From the appendix it is easily observed that the standard errors of stable coeffieients were continuously declining over each time interval from 1975 : 1 through 1986 : 1, as also straightforwardly shown in equation (8).

The maximum likelihood estimation of process noise variance using the $\hat{\sigma}^2_{VPR}$ gives $\hat{k}^2 = 0.3914$[11]. This magnitude seems much more reasonable than those obtained when using $\hat{\sigma}^2$ from GLS technique (see footnote 8). Such a large value of k^2 suggests a significant parameter change while $k^2 = 0$ indicates no parameter process noise.

When $k^2 = 0.3914$, the varying parameter estimate of the error variance is $(0.0014)^2$ [12]. It is remarkably lower than that, $(0.0087)^2$ obtained from the VPR technique when k^2 is set to zero. Thus when the parameter changes are severe, the stable coefficient regression model that does ignore parameter variations, largely overestimates the error variance, and would create a high standard error of varying coefficient. As (13) shows, $\hat{\sigma}^2_{VPR}$depends inversely on the magnitude of k^2.

With all the starting values, a_0, P_0, $\hat{Q}_t$, and $\hat{\sigma}^2_{VPR}$, this paper applied the VPR model by Kalman to M1 demand equation (1) over the period of unstable demand function for money 1974 : 1-86 : 1. The end data of 1986 : 1 were selected based on the suggestion by Simpson (1985)[13]. The fixed estimation results for the period

[11]Initially, the value of log-likelihood function (11) was in each time interval computed. The value was maximized at 347.4850 when the time interval was 1974:1-86:1. And then, for that time interval, $\hat{k}^2$ was computed. The terms of a_{t-1} and P_{t-1} in (12) denote the Kalman estimates (under $\hat{\sigma}^2 = 0.0087$ and $k^2 = 0$) obtained based on observation to time t-1, 1985:4.

[12]The same procedure as footnote 10 was performed under $\hat{k}^2 = 0.3914$ and $\sigma^2 = 1$.

[13]Simpson has suggested that once the deregulation of M1 deposit is complete, the elasticity of M1 with respect to the interest rate is likely to be lower, mentioning that the deregulation of the remaining NOWs will be completed in early 1986.

[**Table 3**] The varying parameter estimation results of equation (1), 1974:1-86:1

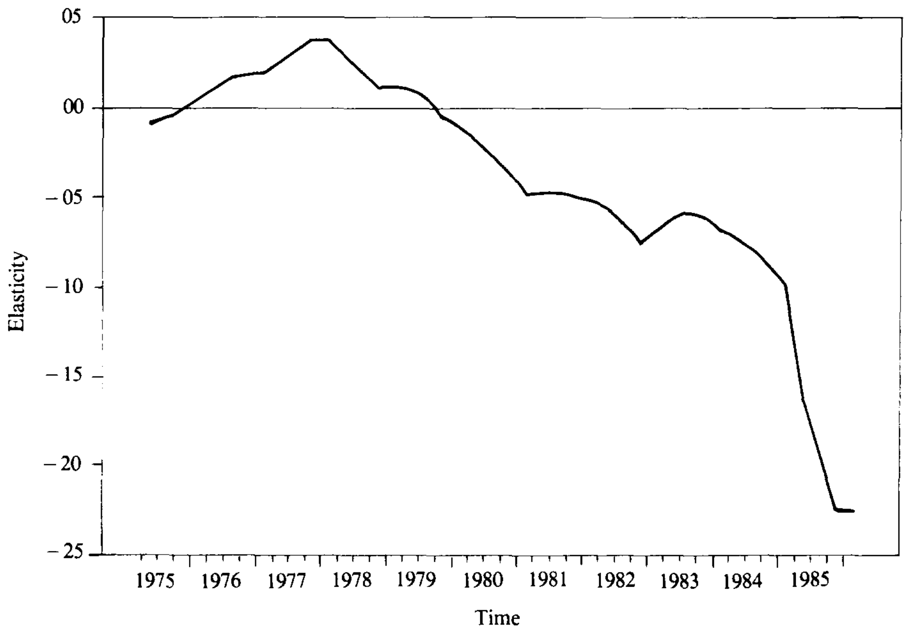
Estimation technique	The sums of coefficients[a]			σ_{VPR}	k^2
	$\Delta \ln Y_t$	$\Delta \ln R_t$	$\Delta \ln P_t$		
VPR	0.6327	-0.0504	-0.2113	0.0087	0.0000
(Kalman, $Q_t=0$)	(0.1131)	(0.0176)	(0.0793)		
VPR	0.8212	-0.2263	-0.0290	0.0014	0.3914
(Kalman, $Q_t \neq 0$)	(0.2656)	(0.0393)	(0.4183)		

[a]The numbers shown in parentheses are the standard errors computed by the method of Spencer.

are given in table 2, and the varying parameter estimation results in the only last observation 1986 : 1 are presented in table 3, also. Likewise, the related entire estimation results are reported in appendix e.

As presented in section II, it is analyzed on the basis of the argument by Brayton, Farr, and Porter that Miller did not take into account the positive effects of gradual deregulation of interest-bearing NOWs on the interest coefficients in M1 demand by using the fixed regression method for the subperiod of the financial innovation and deregulation. From this analysis it is to be expected that for the sample period of the innovation and deregulation, the estimated coefficient of the interest rate variable from the ordinary least squares technique is smaller than that from the fixed regression method in corporating auto correlated disturbance which in turn is smaller than that from the VPR model.

The expected and satisfactory results were successfully obtained, as will be shown in this paragraph. From table 3 it is pointed out that the sum of the coefficient of the interest rate, -0.2263 has a correct sign and a more reasonable magnitude (see footnote 8). Furthermore, the standard error of the coefficient, 0.0393 undoubtedly indicates that the interest coefficient estimated from the varying parameter regression is highly statistically significant. Thus over the sample period of money demand instability (and therefore, the period of the substantial parameter changes with the large sample length of 45 observations), Garbade's method for estimating σ^2 ultimately and dramatically lowered the standard error of the time-varying coefficient of the interest rate. The following comparisons of the estimated interest elasticities from the fixed and varying parameter regressions fairly well confirm the statistical analysis of Miller's study presented in section II. Table 2 and 3 show that the highly significant coefficient of the interest rate of M1 demand equation (1), estimated by the VPR, -0.2263 is in absolute larger than the coefficient by GLS, -0.0992, which is larger than the coefficient by OLS, -0.0489. One may compare this way : the interest elasticity, -0.2263 in case $Q_t \neq 0$ (VPR) is higher than the elasticity, -0.0504 in case $Q_t=0$ (OLS). Thus for the period of the financial innovation and deregulation, conventional regression methods downward bias the estimates of the interest coefficients in M1 demand. In the following, the regression results of other variables, income and price, are briefly



[**Figure 2**] Smoothed Estimates of Interest Elasticity of M1 Demand

discussed. The varying coefficient of the income variable, 0.8212 also has an expected sign and plausible magnitude, and it is statistically significant with its standard error, 0.2656. Concerning the price elasticity, it has a correct sign but unexpectedly, each coefficient d_i was all insignificant, thus rejecting the NPAH. Here, we do not intend to discuss this result since the price elasticity is not a main concern in this study,

To analyze the movements of the varying coefficients over time, smoothing equations (9) and (10) started with the estimated varying coefficients in the final period 1986 : 1 and worked backward until 1975 : 2. The smoothed estimates of the only interest coefficient for the money demand equation were plotted across the sample period in figure 2, due to limitation of space. Cooley and Prescott (1976) argued that a systematic movement in the coefficient may suggest a particular kind of specification error. From this point of view, the analysis of Miller's study expects that during the period of the financial deregulation, the coefficient of the interest rate will have an increasing movement over time. The reason is that dynamic and positive effects of gradual deregulation of NOW account on the coefficient, which are omitted in the fixed regression models, will be explicitly accounted for in varying parameter regression models. As expected, the evidence in the figure obviously demonstrates that for the period of the deregulation 1981-85, the interest rate elasticity was continuously rising over time, thus again confirming the analysis of Miller's conclusion.

Therefore, the presented empirical results of both estimations and plots of the smoothed estimates, support the statistical analysis of Miller's study. Based on the empirical evidence, it is suggested that for the period of the financial deregulation, fixed regression methods largely underestimate the coefficient of the interest rate in M1 demand due to disregarding the effects of the gradual deregulation of interest-bearing NOW account on the coefficient. This is in line with the argument by Cargill and Mayer (1979) that estimating money demand via constant estimation methods amounts to specification error. Thus, it turns out that Miller's conclusion from the constant regression method for the subperiod of the financial innovation and deregulation, is misleading.

VI. CONCLUSION

In this paper, the inconclusive evidence between the two empirical studies has been analyzed to be due to a specification error. Miller did not appropriately consider an effect of the deregulation of interest bearing NOWs on the interest coefficients in M1 demand by omitting interest dummies for the subperiod of the financial innovation and deregulation, thereby causing the parameters to vary sequentially over time. For such case, this study has presented the VPR model by Kalman for the period of the innovation and deregulation, allowing the parameters to change over time.

Using Chavas's method with Garbade's method for estimating σ^2 in deterining the process noise variance, our emprical results strongly support the analysis of Miller's study. The highly significant coefficient of the interest rate of the money demand estimated by the VPR was substantially larger than those by OLS and GLS. Also, for the period of the deregulation, the coefficient of the smoothed estimates was continually increasing over time.

It is implied that for the period of the financial innovation and deregulation, using constant regression method leads to the downward bias of the estimate of the interest rate coefficient of M1 demand since the regression method fails to consider the effects of the gradual deregulation of interest bearing NOWs on the coefficient. The policy implication of the empirical evidence is that the monetary policy formulated based on the information obtained from the fixed regression methods for the period of the financial changes would not be desirable due to understating the consequences of the change in interest rate on M1 velocity.

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[**Appendix 1**] The varying parameter estimation results of equation (1) under $\sigma_{VPR} = 0.0087$ and $k^2 = 0.0000$ (1974:1-86:1)

with respect to	The sums of coefficients[a]			with respect to	The sums of coefficients[a]		
	$\Delta \ln y_t$	$\Delta \ln R_t$	$\Delta \ln P_t$		$\Delta \ln Y_t$	$\Delta \ln R_t$	$\Delta \ln P_t$
75:1	0.3690	-0.0398	0.1098	78:1	0.5862	-0.0152	-0.2841
	(0.1683)	(0.0245)	(0.1600)		(0.1496)	(0.0228)	(0.1175)
2	0.4811	-0.0256	-0.0539	2	0.5804	-0.0105	-0.2710
	(0.1656)	(0.0244)	(0.1511)		(0.1495)	(0.0222)	(0.2371)
3	0.5569	-0.0285	-0.1540	3	0.5820	-0.0119	-0.2763
	(0.1570)	(0.0244)	(0.1341)		(0.1494)	(0.0219)	(0.1156)
4	0.6236	-0.0272	-0.2585	4	0.5999	-0.0170	-0.2955
	(0.1538)	(0.0244)	(0.1247)		(0.1482)	(0.0219)	(0.1138)
76:1	0.6316	-0.0264	-0.2845	79:1	0.6058	-0.0190	-0.3126
	(0.1536)	(0.0243)	(0.1224)		(0.1481)	(0.0211)	(0.1129)
2	0.6226	-0.0244	-0.2900	2	0.6047	-0.0189	-0.3084
	(0.1530)	(0.0241)	(0.1221)		(0.1480)	(0.0211)	(0.1108)
3	0.5980	-0.0198	-0.2975	3	0.5785	-0.0149	-0.2794
	(0.1513)	(0.0238)	(0.1219)		(0.1467)	(0.0209)	(0.1088)
4	0.5876	-0.0167	-0.3012	4	0.5886	-0.0165	-0.2901
	(0.1502)	(0.0232)	(0.1217)		(0.1448)	(0.0206)	(0.1058)
77:1	0.5884	-0.0171	-0.3002	80:1	0.5989	-0.0180	-0.2985
	(0.1501)	(0.0229)	(0.1214)		(0.1411)	(0.0200)	(0.1024)
2	0.5874	-0.0169	-0.2969	2	0.6275	-0.0182	-0.3374
	(0.1500)	(0.0229)	(0.1207)		(0.1400)	(0.0200)	(0.0997)
3	0.5893	-0.0162	-0.2912	3	0.5970	-0.0199	-0.2983
	(0.1500)	(0.0229)	(0.1203)		(0.1391)	(0.0200)	(0.0975)
4	0.5900	-0.0163	-0.2969	4	0.5782	-0.0188	-0.2840
	(0.1498).	(0.0229)	(0.1186)		(0.1351)	(0.0199)	(0.0942)

[a]The numbers shown in parentheses are the standard errors computed by the method of Spencer.

[**Appendix 1**] (Continued)

with respect to	The sums of coefficients[a]			with respect to	The sums of coefficients[a]		
	$\Delta \ln y_t$	$\Delta \ln R_t$	$\Delta \ln P_t$		$\Delta \ln Y_t$	$\Delta \ln R_t$	$\Delta \ln P_t$
81:1	0.5781	−0.0188	−0.2839	84:1	0.5339	−0.0328	−0.2016
	(0.1338)	(0.0198)	(0.0887)		(0.1234)	(0.0186)	(0.0800)
2	0.5226	−0.0116	−0.2397	2	0.5267	−0.0327	−0.1999
	(0.1302)	(0.0194)	(0.0868)		(0.1214)	(0.0186)	(0.0797)
3	0.5636	−0.0174	−0.2721	3	0.5301	−0.0328	−0.2006
	(0.1281)	(0.0191)	(0.0868)		(0.1199)	(0.0186)	(0.0796)
4	0.5602	−0.0176	−0.2775	4	0.5168	−0.0315	−0.1981
	(0.1279)	(0.0191)	(0.0861)		(0.1180)	(0.0185)	(0.0795)
82:1	0.5528	−0.0191	−0.2655	85:1	0.5537	−0.0346	−0.2056
	(0.1274)	(0.0190)	(0.0841)		(0.1163)	(0.0184)	(0.0794)
2	0.5509	−0.0192	−0.2636	2	0.5750	−0.0378	−0.2087
	(0.1266)	(0.0190)	(0.0829)		(0.1152)	(0.0183)	(0.0794)
3	0.5554	−0.0190	−0.2693	3	0.6080	−0.0444	−0.2104
	(0.1261)	(0.0190)	(0.0817)		(0.1144)	(0.0180)	(0.0794)
4	0.5068	−0.0257	−0.2183	4	0.6182	−0.0487	−0.2081
	(0.1254)	(0.0189)	(0.0804)		(0.1141)	(0.0177)	(0.0794)
83:1	0.5094	−0.0290	−0.2064	86:1	0.6327	−0.0504	−0.2113
	(0.1254)	(0.0188)	(0.0802)		(0.1131)	(0.0176)	(0.0793)
2	0.5214	−0.0321	−0.2026				
	(0.1251)	(0.0187)	(0.0801)				
3	0.5247	−0.0330	−0.1990				
	(0.1250)	(0.0186)	(0.0800)				
4	0.5234	−0.0327	−0.1993				
	(0.1248)	(0.0186)	(0.0800)				

[**Appendix 2**] The varying parameter estimation results of equation (1) under $\sigma_{VPR}=0.0014$ and $k^2=0.3913542$ (1974:1-86:1)

with respect to	The sums of coefficients[a]			with respect to	The sums of coefficients[a]		
	$\Delta \ln y_t$	$\Delta \ln R_t$	$\Delta \ln P_t$		$\Delta \ln Y_t$	$\Delta \ln R_t$	$\Delta \ln P_t$
75:1				78:1	0.6762	0.0630	−0.3878
	—	—	—		(0.6020)	(0.0768)	(0.3511)
2	0.5662	−0.0143	−0.1685	2	0.5832	0.0501	−0.3666
	(0.2154)	(0.0429)	(0.1843)		(0.5051)	(0.0583)	(0.3982)
3	0.7645	−0.0222	−0.4157	3	0.6085	0.0367	−0.4214
	(0.2230)	(0.0377)	(0.1582)		(0.5840)	(0.0288)	(0.3940)
4	0.8388	−0.0132	−0.5726	4	0.5740	0.0107	−0,3816
	(0.2587)	(0.0440)	(0.1578)		(0.6884)	(0.0282)	(0.4636)
76:1	0.8386	−0.0132	−0.5730	79:1	1.0995	0.0158	−0.8740
	(0.3045)	(0.0519)	(0.1836)		(0.7194)	(0.0315)	(0.4179)
2	0.7991	−0.0058	−0.5771	2	0.8055	0.0209	−0.6290
	(0.3479)	(0.0590)	(0.2164)		(0.7771)	(0.0367)	(0.4027)
3	0.6809	0.0147	−0.5898	3	0.2273	0.0350	−0.3147
	(0.3818)	(0.0645)	(0.2547)		(0.6099)	(0.0404)	(0.2955)
4	0.6921	0.0116	−0.5892	4	0.7601	0.0061	−0.5128
	(0.4211)	(0.0617)	(0.3003)		(0.5731)	(0.0408)	(0.3088)
77:1	0.7459	0.0067	−0.5716	80:1	0.8367	0.0029	−0.5621
	(0.4874)	(0.0723)	(0.3529)		(0.6277)	(0.0470)	(0.3265)
2	0.7556	0.0067	−0.5674	2	0.7658	0.0025	−0.5022
	(0.5572)	(0.0853)	(0.4117)		(0.7080)	(0.0554)	(0.3386)
3	0.9023	0.0195	−0.6013	3	0.2836	−0.0066	−0.1973
	(0.6045)	(0.0980)	(0.4819)		(0.7002)	(0.0648)	(0.2770)
4	0.8092	0.0371	−0.5011	4	0.5881	−0.0023	−0.3164
	(0.6182)	(0.0941)	(0.4205)		(0.4038)	(0.0757)	(0.1661)

[a]The numbers shown in parentheses are the standard errors computed by method of Spencer.

[**Appendix 2**](Continued)

with respect to	The sums of coefficients[a]			with respect to	The sums of coefficients[a]		
	$\Delta \ln y_t$	$\Delta \ln R_t$	$\Delta \ln P_t$		$\Delta \ln Y_t$	$\Delta \ln R_t$	$\Delta \ln P_t$
81:1	0.6424	−0.0519	−0.2432	84:1	0.2130	−0.0579	−0.1280
	(0.4703)	(0.0626)	(0.1666)		(0.2884)	(0.0590)	(0.1878)
2	0.4167	−0.0511	−0.1517	2	0.2626	−0.0597	−0.1362
	(0.4614)	(0.0657)	(0.1518)		(0.2068)	(0.0689)	(0.2171)
3	0.4615	0.0374	−0.2168	3	0.5801	−0.0610	−0.1823
	(0.5404)	(0.0749)	(0.1524)		(0.1995)	(0.0813)	(0.2553)
4	0.5688	−0.0456	−0.1880	4	0.5464	−0.0560	−0.1549
	(0.5149)	0.0836)	(0.1488)		(0.1654)	(0.0925)	(0.2689)
82:1	0.5676	−0.0436	−0.1926	85:1	0.8748	−0.0592	−0.4971
	(0.6065)	(0.0817)	(0.1191)		(1.0399)	(0.1011)	(0.2925)
2	0.6795	−0.0337	−0.2089	2	0.9024	−0.0886	−0.2043
	(0.9736)	(0.0953)	(0.1384)		(0.1831)	(0.0857)	(0.3035)
3	0.6496	−0.0272	−0.2194	3	0.8977	−0.1031	−0.1823
	(0.9541)	(0.1035)	(0.1467)		(0.2143)	(0.0572)	(0.3348)
4	0.8621	−0.0993	−0.1019	4	0.7795	−0.2213	−0.0298
	(0.9262)	(0.1105)	(0.1519)		(0.2461)	(0.0354)	(0.3812)
83:1	0.7312	−0.0862	−0.1068	86:1	0.8212	−0.2263	−0.0290
	(0.8098)	(0.1076)	(0.1771)		(0.2656)	(0.0393)	(0.4183)
2	0.5775	−0.0649	−0.1286				
	(0.5877)	(0.0721)	(0.1796)				
3	0.5677	−0.0521	−0.1617				
	(0.6925)	(0.0744)	(0.1834)				
4	0.3557	−0.0384	−0.1750				
	(0.4924)	(0.0770)	(0.2124)				