

## INTRA-INDUSTRY TRADE THEORY IN VERTICALLY DIFFERENTIATED PRODUCTS

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*This paper presents a theory of international trade in a two sector, one factor economy in which one sector is vertically differentiated. The paper shows that trade arises from the cost differences in goods in this sector between countries. Furthermore, this trade is characterized as inter-industry trade when cost differences are uniform and intra-industry trade when cost differences are biased. Uniform cost differences occur when there is a difference in labor productivity in the homogenous goods or a difference in the fixed cost required for the differentiated goods between countries. Biased cost differences result from changes in the parameter of the cost function representing the rate of change in cost in relation to quality. In both cases, an economy with either of these types of trade is more efficient than an autarkic economy because production is increased.*

### I. INTRODUCTION

A careful observation of differentiated products in international trade reveals that vertically differentiated goods are at least as popular as horizontally differentiated goods. Grubel and Lloyd (1975, ch.6), for example, showed that there is a significant intra-industry trade in both vertically and horizontally differentiated products.<sup>1</sup> Therefore, intra-industry trade theory in horizontally differentiated goods can be a partial explanation of the total amount of trade in differentiated products. However, contrary to abundance of well developed literature on intra-industry trade in horizontally differentiated products, the literature on that of vertically differentiated products are scarce.

The lack of literature on vertically differentiated products in international trade

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<sup>1</sup>For other empirical studies, see Balassa (1976), and Kravis (1971). These studies confirm that intra-industry trade is prevalent in quality differentiated goods.

results from the fact that there has not been any micro economic theory for vertically differentiated products. This contrasts with the extensive research on intra-industry trade theory in horizontally differentiated products following the development of monopolistic competition theory by Dixit and Stiglitz (1977) and Lancaster (1979).

Theoretical attempts to explain patterns of trade in vertically differentiated products date back to Linder (1961). He envisioned trade in quality differentiated products on the assumption that income is the dominant determinant of tastes. Therefore, the quality of products which are well developed within a country is the quality that is demanded by the population of average income level of that country. From this assumption, Linder's hypothesis says that a country tends to specialize in the production and export of that quality of products which is demanded by the majority of its population, while it imports the qualities demanded by both the richest and the poorest segments of its population.

Donnenfeld and Ethier (1984) combined the demand structure of Linder with the factor endowment model of trade to explain inter-industry trade as well as intra-industry trade in vertically differentiated products. They showed that if trade in commodities does not lead to factor price equalization, then a country will export the range of qualities which are relatively intensive in its abundant factor and import the range of qualities which are intensive in its relatively scarce factor.

Donnenfeld (1986) extended Donnenfeld and Ethier's model to include imperfect information about quality and explain the pattern of trade.

In a separate development, Grubel and Lloyd (1975) suggested the life cycle theory of Vernon (1966) as a possible explanation of intra-industry trade in vertically differentiated products. They showed that trade resulting from the life-cycle theory is intra-industry trade if goods are differentiated by quality. In such trade, a country at a higher technological state produces and exports higher quality goods and imports lower quality goods for a country in a lagging technological state.

This paper emphasizes technological factors in the determination of patterns of trade along with the demand structure of the Hedonic price model. Presenting an explicit model based on the idea of Grubel and Lloyd, this paper studies causes of trade and its resulting gains when there are differences in technological factors and consumer types between countries.

This paper shows that intra-trade arises from the cost differences in goods in the vertically differentiated sector between countries. This paper also shows that the gains from intra-industry trade of vertically differentiated products more closely resemble the gains resulting from inter-industry trade rather than those based on intra-industry trade of horizontally differentiated products.

The present paper uses a utility function of Rosen (1974), and assumes there is a competitive market in the differentiated sector with free entry with the usual U-shaped cost function. In every quality, there is perfect competition and free entry which reduces each firm's profits to zero. It is assumed that there exists a suf-

ficiently large number of firms producing the same quality in every quality.<sup>2</sup>

The situation described in this paper is an economy in which (1) no firm ever has any market power, and (2) no horizontal differentiation (varieties) exists within qualities.

In the next section, the model is presented. In Section 3, autarkic equilibrium is derived, and its nature is explained. In Section 4, implications of the model on international trade are presented. This section is concerned with an open economy compared to the autarkic equilibrium discussed in Section 3. In the final section, summaries and brief conclusions will be stated.

## II. THE MODEL

Consider an economy made up of two sectors, one consisting of vertically (quality) differentiated goods, and the other of composite (outside) goods. Labor is the only factor in the production of both goods. Outside goods will be used as a numeraire. In the market for differentiated products, there are many qualities of goods available. The quality level of these differentiated products is represented by a one dimensional hedonic attribute  $q$ , which is referred to as product quality. A larger value in the subscript of  $q$  indicates higher quality products.

### A. Production

The production function for the outside goods (composite goods) is:

$$(1) M = a_m L_m$$

where  $M$  represents the outside goods, and  $L_m$  is the labor employed in the outside sector. Each worker produces  $a_m$  units of  $M$ . Thus, the wage rate simply equals  $a_m$  if the outside goods are produced, because  $M$  is the numeraire.

The cost function in the differentiated goods sector is assumed to be similar to the cost function of the one-factor model of Krugman (1979)<sup>3</sup> modified to give a U-shaped AC curve. Furthermore, quality is added to both fixed and variable costs.<sup>4</sup> Because labor is the only factor of production, total costs are always equal to wage costs. The labor used in producing each quality is:

<sup>2</sup>Chiang and Masson (1988) presents a model on quality differentiation which operates on a similar situation. They explain how this economic situation fits well to major export industries of NICs. For example, in textile industry numerous firms are competing for the market divided by quality bracket.

<sup>3</sup>The cost function used in Krugman (1979) could be written as:  $C = w[Q + F]$ . It should be clear that both Krugman's and my model specify the sunk cost,  $F$  in the one-factor Ricardian model. Thus, our models assume other factors of the economy such as capital embodied in the sunk cost as fixed.

<sup>4</sup>Other formulas regarding how quality enters the cost function are discussed in my Dissertation. Quality can be added only to either fixed or variable costs. These different formulas does not affect the qualitative results of the paper.

$$(2) l(Q, q) = h(q) [Q^2 + F]$$

where  $Q$  represents the total quantity of quality goods produced, and  $Q^2$  and  $F$  are variable and fixed costs respectively.  $l$  is the labor used in producing goods of quality  $q$ . Total costs are the product of labor requirement (2) multiplied by wage rate,  $w$ :

$$(3) C(Q, q) = w h(q) [Q^2 + F]$$

This cost function has the same minimum point of average costs,  $AC$  for every quality produced at  $Q^* = \sqrt{F}$ .<sup>5</sup>

In the long run, with free entry and a competitive market for each quality, each firm will earn zero profit and produce at a minimum average cost level:

$$(4) C^* = C(Q_i^*)/Q_i^*$$

If the demand for the output of this industry is some integral multiple of  $Q_i^*$ , then each firm will produce at  $Q_i^*$ , and the equilibrium price will be  $p^* = c^*$ . Thus, profit will be zero.

By substituting  $\sqrt{F}$  for  $Q^*$  for in  $AC$  derived from eq. (3), the prices of qualities are derived as:

$$(5) p_i = \min. \text{ of } AC = 2wh(q)\sqrt{F} = 2a_m h(q)\sqrt{F}$$

For the unique solution of the quality demanded by consumers, a necessary condition on the price schedule (5) are:

$$(6) p(q) \geq 0 \quad p'(q) > 0 \quad p''(q) > 0$$

Intuitively, condition (6) implies that price should increase at an increasing rate as quality level rises. If price increases at a constant rate with the rate of increase of quality ( $dp = dq$ ; a case of  $p''(q) = 0$ ), any consumer who chooses to buy a quality product will be indifferent to the level of quality, because every quality yields the same consumer surplus (= utility-price) for consumers. Therefore, an infinite number of consumer quality choices exists (indeterminate solution). If price increases at a decreasing rate as quality level rises ( $dp < dq$ ; a case of  $p''(q) < 0$ ), any consumer who chooses to buy quality goods will be better off by upgrading quali-

<sup>5</sup>This functional form of the optimum quantity results from the specific form of the cost function. If we use other formulas about cost function, the optimum quantity is shown to be a function of quality too. But the qualitative results of the paper does not affected. This specific form also implies that an increase in quality due to an increase in  $F$  raises the optimum quantity.

ty, because higher quality will provide him with more quality for the money spent. Thus, the consumer's decision problem in this case yields a corner solution consisting only of zero or the highest quality level. This intuition will be clear in the next section.

### B. Consumers and Utility Function

Consumers are assumed to differ in their preferences for qualities, each buying either one unit of a differentiated product or none. A particularly useful form of the utility function,  $U(M, q, \Theta, X)$ , originated by Rosen (1974), can be represented by:

$$(7) \quad U(M, q, X, \Theta) = M + \Theta qX \\ X = 1 \text{ if buy quality, } X = 0 \text{ if not.}$$

where  $M$  is the composite goods.  $X$  denotes the total units of quality goods bought by consumers, ( $X$  takes binary values of 0, 1 because each consumer either buys zero or one unit), and  $\Theta$  indexes consumer types.  $\Theta$  is proportional to the amount of money that each consumer is willing to pay for one unit of output of quality  $q$  of the differentiated products. Thus, consumers valuations of quality vary in proportion to  $\Theta$ , so that the taste patterns of consumers are characterized by a distribution of parameter  $\Theta$  among consumers.  $\Theta$  is assumed to be a distribution on the interval of real numbers  $[0, R]$  with the density  $f(\Theta)$ .

The utility function (7) has convenient properties useful for the study of quality differentiated goods. First, it ignores the income effects because it is defined only by price, quality and parameter  $\Theta$ , space. Second, it assumes a strong separability between the composite goods and the differentiated products in question. Third, each consumer has a constant marginal utility with regard to quality which depends on his preference  $\Theta$ .

Consumer income is only from labor with the wage rate  $w$  in this one-factor economy. The budget constraint is:

$$(8) \quad M + p(q) = Tw$$

where  $T$  and  $p$  are the total amount of labor time supplied by consumers/workers and the price of the quality differentiated good respectively, and  $p$  is dependent on  $q$ .

The first and second order conditions of the utility maximization require:

$$(9) \quad p'(q) = \Theta \quad p''(q) > 0$$

The restriction (9) corresponding to a unique solution can be expressed as a

restriction of the cost function. By rewriting the cost function as:

$$(10) C(Q, q) = h(q) [V(Q) + F]$$

where,  $Q$  represents the quantity of the differentiated goods, and  $V(Q)$  and  $F$  are variable and fixed costs respectively. Quality is added to both variable and fixed costs proportionately. The price schedule at some quality level must equal the marginal change of cost as quantity changes at the same quality, that is:

$$(11) \frac{\partial C(Q^*, q)}{\partial Q} = h(q) V'(Q^*) \\ p(q) = h(q) \left[ \frac{\partial V(Q^*)}{\partial Q} \right]$$

where  $Q^*$  represents the optimum level of production of the competitive firm. Therefore,  $p''(q) > 0$  will require the following restriction:

$$(12) h''(q) > 0$$

This will be satisfied for the specific functional form  $h(q)$  of  $h(q) = q^r$ , if  $r > 1$ . This restriction will be used in the specification of the cost function in later sections.

### III. AUTARKIC EQUILIBRIUM

Now consider an economy consisting of  $L$  workers/consumers with the same type of preferences.  $\Theta$ . For the solution of the model, we will use a specific functional form for  $h(q)$ :

$$(13) h(q) = q^r, r > 1$$

From the price schedule (eq. (5)), and F-O-C of the utility maximization (eq. (9)), the quality produced at equilibrium, and its price can be solved as:

$$(14) q = (\Theta/2ra_m\sqrt{F})^{1/(r-1)} \quad p = 2a_m\sqrt{F} (\Theta/2ra_m\sqrt{F})^{r/(r-1)}$$

The equilibrium quantity of the differentiated good produced with  $L$  workers/consumers is equal to the total number of consumers  $L$ . This is because each consumer demands one unit of the group goods according to his utility function. For this model, the utility attainable from the consumption of quality goods  $q (= \Theta q)$  is always larger than that from consuming composite goods ( $= p$ ), because  $\Theta q - p > 0$  is satisfied at equilibrium for any  $\Theta > 0$ , i.e.:

$$(15) \Theta q - p = (1/r)^{1/(r-1)} > (1/r)^{r/(r-1)}, \text{ for any } \Theta > 0$$

Given prices of quality goods, wage rate and total labor time, the demand for the composite goods from the budget constraints of the economy can be derived:

$$(16) \quad TwL = p(q)L + M$$

By substituting the equilibrium price into the budget constraint, the income spent for the composite goods  $M$  is:

$$(17) \quad M = TwL - 2a_m q^r \sqrt{F} L = a_m L (T - 2q^r \sqrt{F})$$

If the composite goods consumed is positive, assume  $T > 2q^r \sqrt{F}$ .

The total number of firms existing in the differentiated goods sector can be derived by dividing the total number of market demand  $L$  by the optimum production of each firm  $Q^* = \sqrt{F}$  which corresponds to the minimum AC in a perfectly competitive economy. The number of firms in the differentiated sector is:

$$(18) \quad L/Q^* = L/\sqrt{F}$$

Equilibrium values of the quality goods and their prices are depicted graphically in Figure 1 for the differentiated goods sector. From the F-O-C of the market demand, and the equilibrium price of quality goods, the following equilibrium condition for quality and price is derived:

$$(19) \quad 2ra_m \sqrt{F} q^{r-1} = \Theta$$

The LHS of eq. (19) is derived from  $p(q) = 2a_m \sqrt{F} q^r$ ,  $r > 1$ , which in turn depends on the cost conditions. Therefore, it is called the “supply factors”. The RHS of eq. (19) is derived from the utility function,  $U = M + \Theta q = (Tw - p) + \Theta q$ . Thus, call this the “demand factors” from now on.

For a given utility level  $U$ , the “demand factors” is derived by substituting  $M$  in the budget constraint,  $M + P = w$ , into the utility function,  $U = M + \Theta q$ . Therefore, the “demand factors” in the graph is the same as in the indifference curve, and utility rises as one moves in a southeast direction. This implies a tangency solution of the consumers’ choice given the “supply factors”, the price equation.

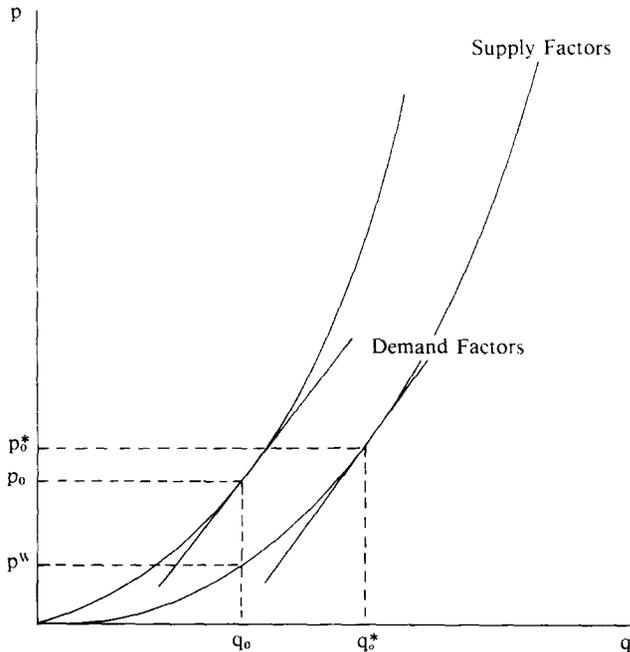
The closed economy equilibrium is represented by  $L$  production of  $q$  by a tangent solution of  $(q, p)$  for each worker/consumer.

## IV. TECHNOLOGICAL DIFFERENCES AND INTERNATIONAL TRADE

A. *Uniform Cost Differences*

Suppose changes are introduced in the parameters of the price schedule shifting the “supply factors” of the economy. By innovating the production processes of its quality products, through R&D investments for example, a country can reduce its production costs, which may be expressed in lowered fixed cost  $F$ , or it can maintain a higher wage rate because of its higher productivity in the outside goods sector. These changes in the parameters  $a_m$  and  $F$  shift the price schedule uniformly. This is depicted in Figure 1.

In Figure 1, the shift-out of the price schedule from the original state ( $p$ , the home country) to the starred state ( $p^*$ , the foreign country) corresponds to a lowering of either the value  $a_m$  or  $F$ . The shift-out occurs because each quality product can be supplied at a lower price with the new state. Therefore, the consumer’s tangency solution for each quality good will force the consumer to choose higher quality at a new equilibrium ( $q < q^*$ ). This can be shown by the partial derivatives of eq. (14) with regard to  $a_m$  and  $F$ , keeping other exogenous variables ( $\Theta$ ,  $r$ ) constant:



[Figure 1]

Uniform Cost Differences

$$(20) \quad \partial q / \partial a_m < 0 \quad \partial q / \partial F < 0$$

From the partial derivatives of the equilibrium price eq. (14), the effects of change of  $a_m$  and  $F$  on equilibrium price can be derived. The result is:

$$(21) \quad \partial p / \partial a_m > 0 \quad \partial p / \partial F < 0$$

This result shows that when there are uniform changes in price schedule (shift-out) resulting from the lowered values of  $a_m$  and  $F$ , the equilibrium quality consumed is raised, and its price is raised in both cases. Thus, consumers can get higher quality at higher prices in this new equilibrium.

Now consider two countries, one with the original "supply factors" and the other with new "supply factors" in the differentiated products represented by the shifted-out "supply factor" in Figure 1. In this one factor economy, the country with a shifted-out price schedule has a comparative advantage over the other in the production of the differentiated products. In addition, the country with lagged technology in the production of quality goods will have a relative comparative advantage in the production of composite goods which are assumed to require the same labor per unit of production in both countries. Once trade opens between the two countries, each country will specialize in the products of its relative comparative advantage. This specialization of production after trade will increase total world wide production to the benefit of both countries.

The trade resulting from the technological innovations in the production of the differentiated products is characterized as inter-industry trade between countries in which one country specializes in differentiated products and the other in composite products.

In fact, at free trade equilibrium there exists at least one country completely specializing in the production of either composite or differentiated goods in this two sector, one factor Ricardian economy. The exact determination of the specialization depends on the parameter values of the model. The total world demand for  $q_0^*$  (\* = the foreign country) with free trade is:

$$(22) \quad D = 2L$$

The total labor required ( $L$ ) to produce the amount of quality goods demanded can be derived by dividing the total income spent on quality goods by income  $T_w$  ( $= Ta_m$ ), because every worker/consumer earns the same income.<sup>6</sup>

<sup>6</sup>This is true in a closed economy of diversified production, but not necessarily true in an open economy. Generally, wage ( $w$ ) equals  $a_m$  (productivity in the outside sector) as discussed before. If specialization exists, wage rate used in this discussion refers to that of the foreign country.

$$(23) \bar{L} = 2Lp^*/Ta_m = 4L\sqrt{F} q^*/T$$

Assuming the total labor force of the two countries is the same, if the labor required for quality goods demanded by both countries is matched by the exact labor force of the foreign country, both countries will completely specialize in the sector of their relative advantage. The home (foreign) country will produce only composite (differentiated) goods if:

$$(24) 4\sqrt{F} lq^*/T = L (= L^*)$$

Incomplete specialization in one country occurs if the total labor demand for quality goods production is not equal to the labor force of one country. If the former is greater than the latter, the foreign country will specialize in quality goods, but the home country will diversify by producing both quality goods and composite goods.<sup>7</sup> The condition for this is:

$$(25) 4\sqrt{F} Lq^*/T > L$$

Foreign country diversification and home country specialization in composite goods also occurs if inequality is reversed in the above equation.

For either of the above situations, free trade can be shown to be Pareto efficient than an autarkic economy, because total world wide production increases.

Total production gain from the trade between the two countries can be shown in terms of composite goods. To calculate production gain the production of quality is held constant at autarkic level ( $L = q_0$ ,  $L = q^*$ ) rather than allowing it to change as expected under trade ( $2L = q^*$ ). In this case, the production in the trade between the two countries decreases the price of  $q_0$ .

The price of  $q$  can be lowered from  $p_0$  to  $p^w$  by the reallocation of the production between the two countries. (See figure 1.)

Now, the total production of composite goods after trade ( $M^w$ ), keeping the qualities produced constant at autarkic level, can be derived from eq. (16), and  $M^w$  is greater than  $(M + M^*)$

$$(26) M^w = (wTL - p^wL) + (wTL - p^*L) \\ > (wTL - p_0L) + (wTL - p^*L) \\ = M + M^* (* = \text{Foreign}), \text{ because } p^w < p$$

<sup>7</sup>Even though this is possible, there remains a question about how the home country can be competitive in the quality differentiated products. A pricing scheme will be required for this. If less efficient home quality goods are still preferred by consumers over the outside goods, then consumers will buy them. However, if foreign quality goods are preferred over those of the home, consumers are willing to pay a premium for foreign quality goods. A pricing scheme must solve these questions.

Furthermore, the increase of production with free trade is:

$$(27) \Delta M = M^W - (M + M^*) = (P_o - p^W)L$$

Thus, the increase of production or gains from trade depend on the price differentials of the two countries resulting from the difference in "supply factors."

### B. Biased Cost Differences

Now consider the effects of changes of parameter "r" on the p (q) schedule. An increase in r raises the production cost of the differentiated products if  $q > 1$ , but lowers the cost if  $q < 1$ . This is clearly seen by looking at the cost function.

$$(28) C(Q, q) = q^r \{ Q^2 + F \} w$$

If  $q < 1$ ,  $q^r$  falls as r increases  
 If  $q = 1$ ,  $q^r$  unchanged as r increases  
 If  $q > 1$ ,  $q^r$  increases as r increases

Thus, changes in "r" cause the twist in the p (q) schedule which is drawn in Figure 2.<sup>8</sup>

The effects of changes in r on the equilibrium values of quality and price can be derived by the partial differentiation of eq. (14) with respect to r keeping all other exogenous variables ( $a_m$ , F, and  $\Theta$ ) constant. Since r appears in the exponents of q and p, derivatives can be found by logarithmic differentiation.

$$(29) \ln q = [1/(r-1)] \ln(\Theta/2ra_m\sqrt{F}) \\ = [1/(r-1)] [\ln(\Theta/2ra_m\sqrt{F}) - \ln r]$$

$$(30) (1/q) (dq/dr) = - [1/(r-1)^2] [\ln(\Theta/2ra_m\sqrt{F}) - \ln r] \\ + [1/(r-1)] (-1/r) = - [1/(r-1)] [\ln q + 1/r]$$

$$(31) \text{Sign } (dq/dr) = - \text{Sign } (\ln q + 1/r)$$

$$(32) dq/dr \geq 0 \text{ iff } q \leq q^1 < 1 \\ \text{for } q^1 = e^{-1/r} \text{ (from } \ln q + 1/r = 0)$$

Similar derivatives can be attained for the equilibrium price.

<sup>8</sup>The parameter r represents technological states of the country. In this model "high" r implies "high" R&D investment of the country. An increase in R&D lowers the production cost at the early state of innovation (when  $q < 1$ ) when innovation leads to mass production, but it increases the production cost in the later high technological state (when  $q > 1$ ) when R&D rises astronomically.

$$(33) \ln p = \ln(2a_m \sqrt{F}) + r \ln q$$

$$(34) \begin{aligned} (1/p) (dp/dr) &= \ln q + r(1/q)(dq/dr) \\ &= \ln q + (r/q) [ - [q/(r-1)] [\ln q + 1/r] ] \\ &\quad \text{by substituting } (1/q) (dq/dr) \text{ from eq. (30)} \\ &= -1/(r-1) [\ln q + 1] \end{aligned}$$

$$(35) \text{Sign } dp/dr = -\text{Sign} [\ln q + 1]$$

$$(36) \begin{aligned} dp/dr \leq 0 &\text{ iff } q \leq q^2 < q^1 < 1 \\ &\text{ for } q^2 = e^{-1} \text{ (from } \ln q + 1 = 0) \end{aligned}$$

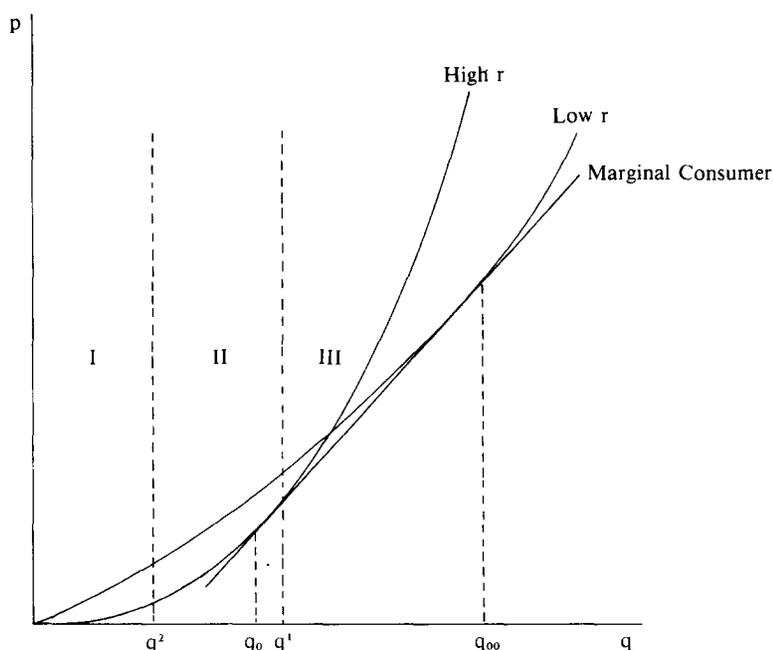
From eq. (32) and (36), we can divide the  $p(q)$  schedule into three zones according to the signs of  $dq/dr$  and  $dp/dr$ :

$$(37) \begin{aligned} \text{Zone I (for } q \leq q^2); & dq/dr > 0, dp/dr \geq 0 \\ \text{Zone II (for } q^2 < q \leq q^1); & dq/dr \geq 0, dp/dr < 0 \\ \text{Zone III (for } q < q^1); & dq/dr < 0, dp/dr < 0 \end{aligned}$$

This is drawn in Figure 2. In zone I, the increase in  $r$  causes both quantity and its price to rise. Therefore, the quality consumed rises, and the price paid for this higher quality also rises. For lower quality with low  $r$ . In zone II, the increase in  $r$  causes the quality consumed to rise but its price falls. Therefore, consumers pay less for higher quality than lower quality. In zone III, the increase in  $r$  causes both the quality consumed and its price to fall. Therefore, consumers buy lower quality and pay a lower price.

Even though consumers' choice of quality and price is affected differently depending on the zone, they are better off as they move to the outer frontier of the price schedule. Let's call this outer envelop of the price schedule the "technological frontier." It is represented by a thick line in Figure 2. In Figure 2, "marginal consumers" who can be satisfied by the qualities produced in both countries,  $q_0$  and  $q_{00}$  are represented. These consumers are equally well off with high quality high price ( $q_{00}$ ) or low quality low price ( $q_0$ ). People with  $\Theta$ 's which are greater than those of marginal consumers buy higher quality goods from the country with low  $r$ , and people with lower  $\Theta$ 's than those of marginal consumers buy lower quality from the country with high  $r$ .

Therefore, the pattern of trade depends on the labor type which is assumed to be the same between countries. If the labor type  $\Theta$  is lower than that of marginal consumers, then the country with high  $r$  will export differentiated goods in exchange for outside goods imported from the country with low  $r$ . On the other hand, if the labor type  $\Theta$  is greater than that of marginal consumers, the country with low  $r$  will export differentiated goods in exchange for imports of outside goods



[Figure 2]

Quality zones, Technological Frontier and Marginal Consumer

from the country with high  $r$ . If labor type  $\Theta$  is equal to that of marginal consumers, there will be no trade between the two countries. Thus, if consumers buy higher quality goods, the country with low  $r$  has a relative comparative advantage in the production of differentiated goods and the other country has a relative comparative advantage in the outside goods. Similarly, if lower quality is consumed by consumers, the country with high  $r$  has a relative comparative advantage in differentiated goods with the other country having a comparative advantage in outside goods.

For each case, once trade opens, the two countries will engage in inter-industry trade in which one country exports the goods of its relative advantage. The exact pattern of specialization is dependent upon the parameter values by the same reasoning as in the last section, and it can be shown that trade is better than no trade because total world wide production is increased.<sup>9</sup>

### C. Intra-Industry trade

Now suppose the economy of the home country consists of  $L$  workers/consumers

<sup>9</sup>For the gains from the trade, see the proof of the next section. The same method of proof can be used with slight modification.

with three different types of preferences  $\Theta_i$  denoted by numeric subscripts on labor  $L$ :

$$(38) L = L_0 + L_1 + L_2$$

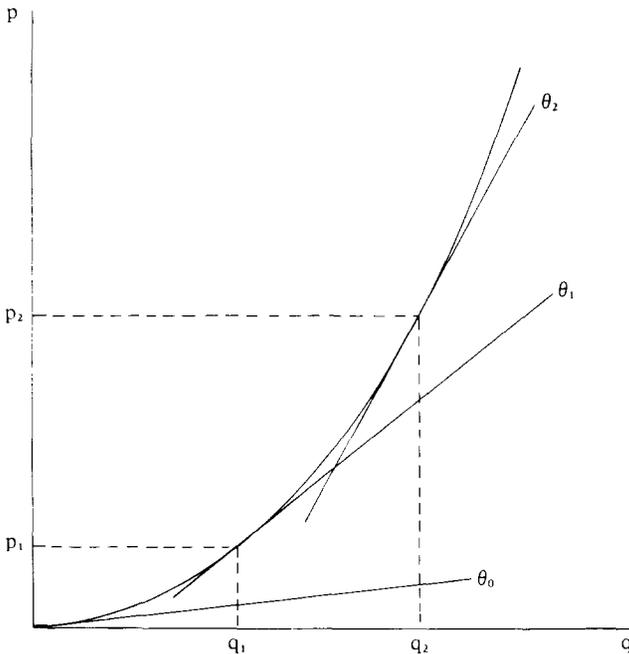
where  $L_i$  represents workers/consumers with preferences  $\Theta_i$ , and assume that:

$$(39) 0 = \Theta_0 < \Theta_1 < \Theta_2$$

The equilibrium in the closed economy will produce  $L_1$  and  $L_2$  units of quality  $q_1$  and  $q_2$  respectively because each consumer of type 1 and 2 will demand one unit of quality goods of type of 1 and 2 respectively. Given prices of quality goods and the total income  $T_w$ , we can derive the demand for composite goods from the budget constraint of the economy.

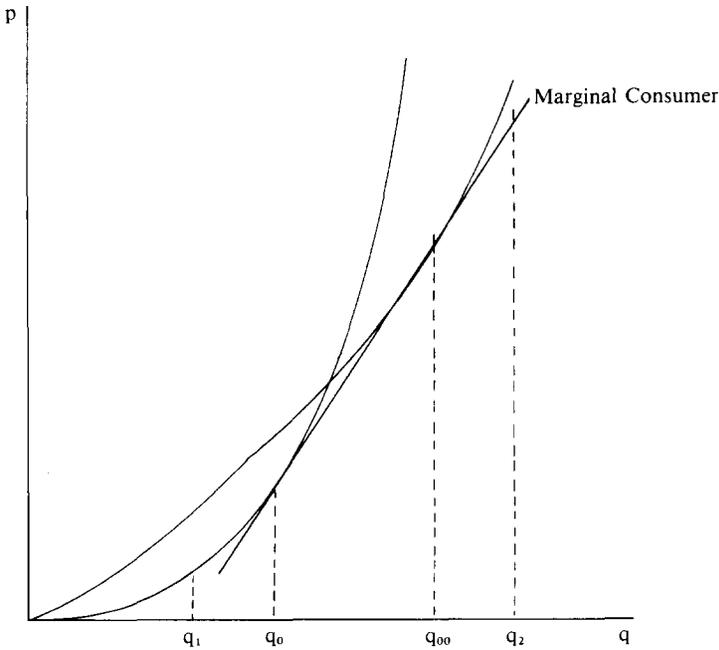
$$(40) T_w L_0 + T_w L_1 + T_L L_2 = p(q_1)L_1 + p(q_2)L_2 + M$$

By substituting  $p_i = 2w\sqrt{F}q_i^r$  into the budget constraint, we get the income spent for the demand of the composite goods  $M$ :



[Figure 3]

Autarkic Equilibrium with Multiple Consumer Types



[Figure 4]

Free Trade Equilibrium

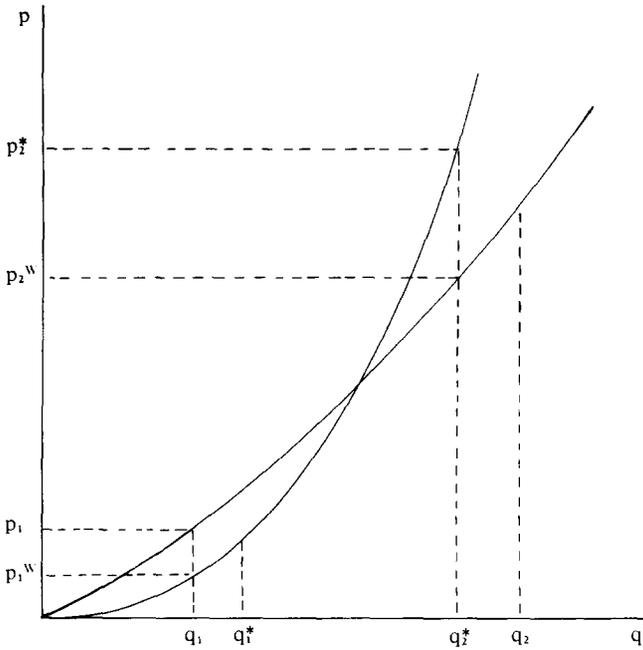
$$\begin{aligned}
 (41) \quad M &= wTL_0 + (wTL_1 - p_1L_1) + (wTL_2 - p_2L_2) \\
 &= wTL_0 + wL_1(T - 2\sqrt{F}q_1r) + wL_2(T - 2\sqrt{F}q_2r) \\
 &= \sum_i a_m L_i (T - 2\sqrt{F}q_i r), \quad q_0 = 0
 \end{aligned}$$

This autarkic equilibrium is depicted graphically in Figure 3.

Suppose the foreign country has a higher “r” than the home country with the same labor types. The price schedules for the two countries are drawn in Figure 4.

In Figure 5,  $q_1^*$  and  $q_2$  are tangency solutions of the “technological frontier” of the two countries of consumers with labor types of  $\Theta_1$  and  $\Theta_2$  respectively. Consumers with  $\Theta_1$  ( $\Theta_2$ ) will be better off consuming  $q_1^*$  ( $q_2$ ) from the foreign (home) country after trade, assuming  $\Theta_1 < \Theta < \Theta_2$ . The exact changes in (q, p) with trade will depend on the initial position of q (zone I, II, and III) as we discussed in the last section.

The trade resulting from biased cost differences (differences in r) between countries is intra-industry trade in which each country specializes in one part of the differentiated products and then trades with the other country. The home country has a relative comparative advantage in high quality goods and the foreign country has a relative comparative advantage in low quality goods. Both countries will gain from the trade because they can consume quality products at lower prices



[Figure 5]

Gains from the Trade

after trade.

In fact, the total production gain from the trade between the two countries can be shown in terms of composite goods. To calculate production gain the production of quality is held constant at autarkic level ( $L_1 = Q_1$ ,  $L_2 = Q_2$ ,  $L_1 = Q_1^*$ , and  $L_2 = Q_2^*$ ) rather than allowing it to change as expected under trade ( $Q_1^* = 2L_1$ , and  $Q_2 = 2L_2$ ). In this case, the ‘‘technological frontier’’ in the trade between the two countries decreases the prices of  $q_1$  and  $q_2^*$ . This is drawn in Figure 5.

The price of  $q_1$  ( $q_2^*$ ) can be lowered from  $p_1$  ( $p_2^*$ ) to  $p_1^w$  ( $p_2^w$ ), by the reallocation of the production between the two countries.

Now, the total production of composite goods after trade ( $M^w$ ), keeping the qualities produced constant at autarkic level, can be derived from eq. (16), and  $M^w$  is greater than  $(M + M^*)$ .

$$\begin{aligned}
 (42) \quad M^w &= 2wTL_0 + L_1(2wT - p_1^* - p_1^w) + L_2(2wT - p_2 - p_2^w) \\
 &> 2wTL_0 + L_1(2wT - p_1 - p_1^*) + L_2(2wT - p_2 - p_2^*) \\
 &= M + M^* \text{ because } p_1 > p_1^w, \text{ and } p_2^* > p_2^w
 \end{aligned}$$

In addition, the production expansion in terms of composite goods ( $\Delta M$ ) depends on the differentials of  $(p_1$  and  $p_1^w)$  and  $(p_2^*$  and  $p_2^w)$  which in turn depend on

the technological differences between the two countries.

$$(43) \Delta M = M^W - (M + M^*) = L_1(p_1 - p_1^W) + L_2(p_2^* - p_2^W)$$

Thus, we have shown that free trade is better than no trade. This also shows that the gains from intra-industry trade of vertically differentiated products more closely resemble the gains resulting from inter-industry trade rather than those based on intra-industry trade of horizontally differentiated products.

## V. SUMMARY AND CONCLUSIONS

This paper presents a model for a two sector one factor Ricardian economy in which one sector is vertically differentiated. Perfect competition along with free entry with a U-shaped cost function is assumed in the differentiated sector.

The discussion of the cost function with quality, shows that the quantity level of differentiated goods at the minimum average cost depends on how quality is factored into the cost function. There are three ways quality may be factored in. It may be multiplied with the variable cost, the fixed cost, or both. The minimum AC is the same in the first two ways, and only depends on the fixed cost and the quality. However, the minimum AC of the third way is different from that of the first two ways and only depends on the quality. Therefore, since the cost function is not affected by quantity in any of the three ways, the utility maximization can be used to determine optimum quality. The optimum quantity for each firm is determined irrespective of the quality goods produced in the market. Therefore, the results of this paper, which proceeds on the assumption that quality enters both variable and fixed cost, would remain the same if the cost function were defined using the first two methods.

The closed economy equilibrium of the model shows the quality produced and its price given labor type. The equilibrium quantity of outside goods demanded is derived from the budget constraint of the economy. In the equilibrium, “supply factors” representing cost conditions and “demand factors” representing the consumers’ problem played major roles.

Changes in parameters  $a_m$  and  $F$  lead to shifts in the price schedule of the differentiated products. Unbiased cost differences resulting from decreases in  $a_m$  and  $F$  raise the equilibrium level of quality and cause its price to fall. Thus, two countries which have a difference in productivity in the outside goods sector ( $a_m$ ) or a difference fixed costs in the differentiated goods sector ( $F$ ) engage in inter-industry trade in which one country with high values of  $a_m$  and  $F$  has a relative comparative advantage in the production of composite goods, and the other country with low values has a relative comparative advantage in the production of differentiated goods.

The change in value of parameter  $r$  leads to “twist” in the price schedule. This biased cost difference changes the equilibrium quality and its price depending on the initial value of quality. Thus, countries with different values of “ $r$ ” engage in intra-industry trade when the two countries have more than one type of labor. In this trade, the country with higher  $r$  has a relative comparative advantage in the production of low quality goods, and the country with low  $r$  has a relative comparative advantage in high quality goods.

Furthermore, it is shown that if either of the above types of trade happens, total production efficiency is increased.

This paper also shows that the gains from intra-industry trade of vertically differentiated products more closely resemble the gains resulting from inter-industry trade rather than those based on intra-industry trade of horizontally differentiated products.

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