

RATIONAL EXPECTATIONS IN A STANDARD KEYNESIAN MODEL

JUN YOUNG KIM*

In the supply side analysis based on the Lucas' supply function, most remarkable result of rational expectations is expected monetary policy is neutral to real economy. Our analytic point is completely different from the previous one in the sense that rational expectations are introduced into demand side analysis reflecting a standard Keynesian model. According to the results in the demand side model expected monetary policy under rational expectations is not neutral to real output, while the neutrality of monetary policy under the Lucas' supply function could hold only in a limited case. Further, the effect of monetary policy is in general dependent on the elasticity of the demand for money. Hence, in this work we reconstruct rational expectations into a standard Keynesian model and the principle of effective demand.

I. INTRODUCTION

In both economic theory and economic policy there have been wide differences between Keynesian analysis and rational expectations. Most remarkable result of rational expectations on monetary policy is only unexpected money supply would affect real economy through increasing forecasting error on price expectations, whose analysis was originated from Lucas (1972, 1975), Sargent (1976) and Barro (1976). They had supply side analysis based on the Lucas' supply function, i.e., economic fluctuation around the natural output comes from an unexpected price change. Since expected monetary policy under rational expectations results in on average correct price expectations, such a monetary policy has been considered as neutral to real economy.

In this work our interest is in how rational expectations modify the Keynesian analysis based on demand side model. Our analytic point is completely different from the previous one in the sense that rational expectations are introduced into

*Department of Economics, Sungkyunkwan University.

I appreciate prof. Young Goo Lee for his comments on my paper, which was presented at the annual conference of the Korean Economic Association held in Seoul, Korea, 1990. I also thank the anonymous referees for their comments. But any remaining errors are mine.

demand side analysis reflecting a standard Keynesian model instead of a supply side analysis under the Lucas' supply function. Here, we apply rational expectations on price to the Keynesian IS-LM model and the principle of effective demand.

According to the analytic results of this work, the neutrality of monetary policy under the Lucas' supply function could survive only in a limited case, and the effect of monetary policy is in general dependent on the elasticity of the demand for money. It has been pointed out that the elasticity of the demand for money is important analyzing monetary policy in both Keynesian and monetarists. Rational expectations in the Keynesian philosophy can be reinterpreted as modifying the traditional Keynesian results rather than erasing them. In this respect the compatibility of the rational expectations theory to the Keynesian and monetarist theory would be reconsidered according to the model specific aspect.

Although McCallum (1977) and Phelps-Taylor (1977) demonstrated the non-neutrality of monetary policy under rational expectations, their models could not consider the main stream of Keynesian effective demand theory, but assumed price or wage stickiness. However, the theoretical model in this paper is accommodating the rational expectations theory with the principle of effective demand in a standard Keynesian theory.

In the next section the theoretical model with rational expectations on price is explained, and monetary policy between rational expectations and fixed expectations is made in section 4.

II. RATIONAL EXPECTATIONS AND THE STANDARD KEYNESIAN MODEL

Let's assume price expectations are formed rationally and introduce rational expectations into the standard Keynesian model consisting of the equations (1) through (4). Then the characteristics of the Keynesian model including rational expectations are summarized as:

(i) real output depends on expectations of future demand, and expectations of future demand are rational; (ii) wage is not perfectly flexible and factors are in general underemployed; (iii) expectations of inflation affecting real interest are rational; (iv) real income and employment are determined by the effective demand and are not directly related to labor market.

The standard Keynesian model reflecting these characteristics can be constructed as

- (1) $Y_t = \alpha(i_{t-1}D_t^e - Y_{t-1}) + Y_{t-1} + \xi_t \quad (0 \leq \alpha \leq 1)$
- (2) $M_t = i_{t-1}P_t^e - a_1 i_t + a_2 Y_t + \eta_t \quad (a_1, a_2 > 0)$
- (3) $D_t = b_1 Y_t + b_2 \{i_t - (i_{t-1}P_t^e - P_{t-1})\} + b_3 Z_t + \varepsilon_t$
 $(0 < b_1 < 1, b_2 < 0, b_3 > 0)$

$$(4) \quad P_t - P_{t-1} = c(Y_t - Y^*) + \sum_{i=0}^{\infty} \beta_i (P_{t-1-i} - P_{t-2-i}) + V_t$$

$$(c > 0, \sum_{i=0}^{\infty} \beta_i = 1)$$

where Y is real output, D effective demand, P price level, M money supply, i interest rate, Z the variable affecting aggregate demand, Y^* natural output, ${}_{t-1}X_t^e$ expectations on X_t at $(t-1)$, and ξ_t , η_t , ε_t , V_t are serially uncorrelated random variables with mean zero.

Equation (1) says the principle of effective demand that real output depends on expectations of future demand, and increases when expected demand is greater than present output. Introducing the equation (1) rather than Lucas' supply function in a rational expectations model is quite different from other models. Equation (2) is LM curve, and the effective demand is defined in equation (3), and it is equation (4) that price changes according to the information of real output and past price levels, and $\sum_{i=0}^{\infty} \beta_i = 1$ implies real output reaches natural output under constant inflation rate in the long run. Here price expectations change according to economic states, while price level was assumed fixed in the standard IS-LM model.

To analyze expected inflation take a rational expectations of equation (4), then we have

$$(5) \quad {}_{t-1}P_t^e - P_{t-1} = c({}_{t-1}Y_t^e - Y^*) + \sum_{i=0}^{\infty} \beta_i (P_{t-1-i} - P_{t-2-i})$$

Put equation (2) and (5) into equation (3), and then take a rational expectations.

$$(6) \quad {}_{t-1}D_t^e = b_1 {}_{t-1}Y_t^e + b_2 \left\{ -\frac{1}{a_1} {}_{t-1}M_t^e + \frac{1}{a_1} P_t^e + \frac{a_2}{a_1} Y_t^e \right\}$$

$$- b_2 c({}_{t-1}Y_t^e - Y^*) - b_2 \sum_{i=0}^{\infty} \beta_i (P_{t-1-i} - P_{t-2-i})$$

$$+ b_3 {}_{t-1}Z_t^e$$

$$= b_1 {}_{t-1}Y_t^e + b_2 \left[-\frac{1}{a_1} {}_{t-1}M_t^e + \frac{1}{a_1} \{ P_{t-1} + c({}_{t-1}Y_t^e - Y^*) \right.$$

$$+ \sum_{i=0}^{\infty} \beta_i (P_{t-1-i} - P_{t-2-i}) \} + \frac{a_2}{a_1} Y_t^e \left. \right] - b_2 c({}_{t-1}Y_t^e - Y^*)$$

$$- b_2 \sum_{i=0}^{\infty} \beta_i (P_{t-1-i} - P_{t-2-i}) + b_3 {}_{t-1}Z_t^e$$

Take a rational expectation of equation (1) and rearrange it.

$$(7) \quad {}_{t-1}D_t^e = \frac{1}{\alpha} {}_{t-1}Y_t^e + Y_{t-1}(1 - \frac{1}{\alpha})$$

From (7)

$$(7)' \quad {}_{t-1}Y_t^e = \alpha {}_{t-1}D_t^e + Y_{t-1}(1 - \alpha)$$

Substitute (7)' into (6), and expected demand is

$$(8) \quad {}_{t-1}D_t^e = \{(\frac{1}{\alpha} - b_1) - b_2(\frac{c}{a_1}) - b_2(\frac{a_2}{a_1}) + b_2c\}^{-1}\alpha^{-1}[-\frac{b_2}{a_1} {}_{t-1}M_t^e \\ + \frac{b_2}{a_1} P_{t-1} - b_2c(\frac{1}{a_1} - 1)Y^* + \{b_2(\frac{1}{a_1}) - b_2\} \sum_{i=0}^{\infty} \beta_i (P_{t-1-i} \\ - P_{t-2-i}) + (1 - \alpha)Y_{t-1}\{b_1 + b_2(\frac{c}{a_1}) + b_2(\frac{a_2}{a_1}) - b_2c\}]$$

From (6) and (7) expected output is

$$(9) \quad {}_{t-1}Y_t^e = \{(\frac{1}{\alpha} - b_1) - b_2(\frac{c}{a_1}) - b_2(\frac{a_2}{a_1}) + b_2c\}^{-1}[-\frac{b_2}{a_1} {}_{t-1}M_t^e \\ + \frac{b_2}{a_1} P_{t-1} - b_2c(\frac{1}{a_1} - 1)Y^* + b_2 {}_{t-1}Z_t^e - Y_{t-1}(1 - \frac{1}{\alpha}) \\ + \{b_2(\frac{1}{a_1}) - b_2\} \sum_{i=0}^{\infty} \beta_i (P_{t-1-i} - P_{t-2-i})]$$

Substituting equation (9) into equation (5) rational expectations on price depend on ${}_{t-1}M_t^e$, p_{t-1} , ${}_{t-1}Z_t^e$, Y_{t-1} , Y^* and past inflations, i.e.,

$$(10) \quad {}_{t-1}P_t^e = f({}_{t-1}M_t^e, p_{t-1}, {}_{t-1}Z_t^e, Y_{t-1}, Y^*, P_{t-1-i} - P_{t-2-i}, i \geq 0)$$

When we introduce rational expectations into the standard Keynesian model, the variables on the right hand side of (10) affect expected price, which changes expected aggregate demand by (6) and thereby real output by (1).

III. NONNEUTRALITY OF MONETARY POLICY UNDER RATIONAL EXPECTATIONS

The analytic point of this section is how rational expectations on price revises the traditional Keynesian results of monetary policy in IS-LM model. From equation (9) the effect of expected money supply on expected output under rational expectations is

$$\begin{aligned}
 (11) \quad d_{t-1}Y_t^e/d_{t-1}M_t^e &= \frac{1}{-\left(\frac{1}{\alpha} - b_1\right)\frac{a_1}{b_2} + c + a_2 - a_1c} \\
 &= \frac{1}{-a_1\left[\left\{\left(\frac{1}{\alpha} - b_1\right)\frac{1}{b_2} + c\right\} - \left\{\frac{a_2}{a_1} + \frac{c}{a_1}\right\}\right]}
 \end{aligned}$$

In the denominator $\left(\frac{1}{\alpha} - b_1\right)\frac{1}{b_2} + c$ is the slope of IS curve, while $\frac{a_2}{a_1} + \frac{c}{a_1}$ is the slope of LM curve on (i, Y) plane. However, if price expectations are fixed (${}_{t-1}P_t^e = P_{t-1} = \bar{P}$) as in the Keynesian IS - LM model, the expected aggregate demand in equation (6) is rewritten as

$$(12) \quad {}_{t-1}D_t^e = b_{1t-1}Y_t^e + b_2\left\{-\frac{1}{a_1}{}_{t-1}M_t^e + \frac{1}{a_1}\bar{P} + \frac{a_2}{a_1}Y_t^e\right\} + b_{3t-1}Z_t^e$$

From (7), (8) and (9) expected output under fixed expectations is

$$\begin{aligned}
 (13) \quad {}_{t-1}Y_t^e &= \left\{\left(\frac{1}{\alpha} - b_1\right) - b_2\left(\frac{a_2}{a_1}\right)\right\}^{-1}\left[-\frac{b_2}{a_1}{}_{t-1}M_t^e + b_2\left(\frac{1}{a_1}\right)\bar{P}\right. \\
 &\quad \left.+ b_{3t-1}Z_t^e - Y_{t-1}\left(1 - \frac{1}{\alpha}\right)\right]
 \end{aligned}$$

Hence, the effect of expected money supply on expected output under fixed expectations is

$$\begin{aligned}
 (14) \quad d_{t-1}Y_t^e / d_{t-1}M_t^e &= \frac{1}{-\left(\frac{1}{\alpha} - b_1\right)\frac{a_1}{b_2} + a_2} \\
 &= \frac{1}{-a_1\left[\left\{\left(\frac{1}{\alpha} - b_1\right)\frac{1}{b_2}\right\} - \frac{a_2}{a_1}\right]}
 \end{aligned}$$

In the denominator $\left(\frac{1}{\alpha} - b_1\right)\frac{1}{b_2}$ and $\frac{a_2}{a_1}$ are the slope of IS curve and the slope

of LM curve on (i, Y) plane, respectively. Let's compare the slopes of IS & LM curves under rational expectations with them under fixed expectations in Table 1.

For the downward sloping IS curve, IS curve under rational expectations is more elastic (flatter) than that under fixed expectations. Because expected price under rational expectations affect real interest rate and output.

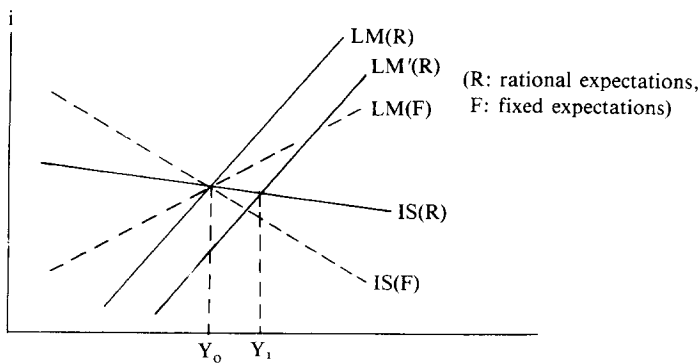
For the upward sloping LM curve, LM curve under rational expectations is more

[Table 1] The Slope of IS & LM Curve

	rational expectations		fixed expectations
slope of IS curve	$(\frac{1}{\alpha} - b_1)\frac{1}{b_2} + c$	>	$(\frac{1}{\alpha} - b_1)\frac{1}{b_2}$
slope of LM curve	$\frac{a_2}{a_1} + \frac{c}{a_1}$	>	$\frac{a_2}{a_1}$

inelastic (steeper) than that under fixed expectations. Because expected price under rational expectations affect real money supply.

Under rational expectations in which IS curve is downward sloping and LM curve is upward sloping, increase in money supply shifts LM curve down and thereby raises output as in Figure 1.



[Figure 1] The effect of increase in money supply

The result which monetary policy under rational expectations affects real output in the standard Keynesian model is contradictory to the neutrality of monetary policy to real output analyzed in the Lucas' supply function. The analytic difference between two models comes from the fact that expected aggregate demand increases from expanding money supply by equation (8), and thereby real output increases from equation (1). That is, from equation (8)

(15)
$$\frac{d_{t-1}D_t^e}{d_{t-1}M_t^e} = \frac{1}{-\alpha a_1[\{(\frac{1}{\alpha} - b_1)\frac{1}{b_2} + c\} - \{\frac{a_2}{a_1} + \frac{c}{a_1}\}]}$$

which is positive, where IS curve is downward sloping, $(\frac{1}{\alpha} - b_1)\frac{1}{b_2} + c < 0$ and

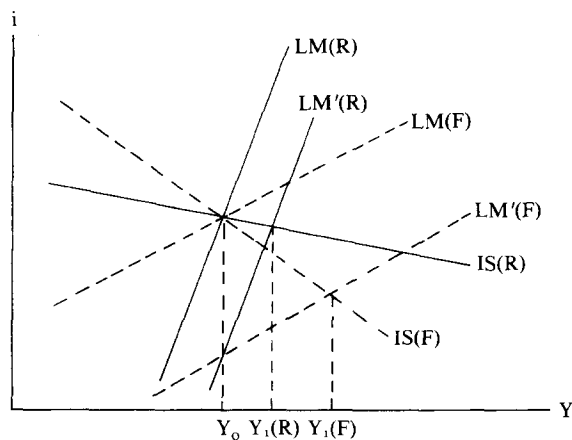
LM curve is upward sloping, $\frac{a_2}{a_1} + \frac{c}{a_1} > 0$.

However, from (15) when IS curve with positive slope is steeper than LM curve, expanding money supply even reduces expected aggregate demand and thereby real output. The neutral effect of monetary policy on real output makes a sense in this Keynesian model only when the slope of IS curve or LM curve is infinite.

IV. COMPARISON OF MONETARY POLICY BETWEEN RATIONAL EXPECTATIONS AND FIXED EXPECTATIONS

In the standard Keynesian model it is interesting to compare the effect of monetary policy on real output between rational expectations and fixed expectations, whose results are summarized as equation (11) and equation (14), respectively. The relative size of both results depends on the elasticity of money demand w.r.t. interest (a_1).

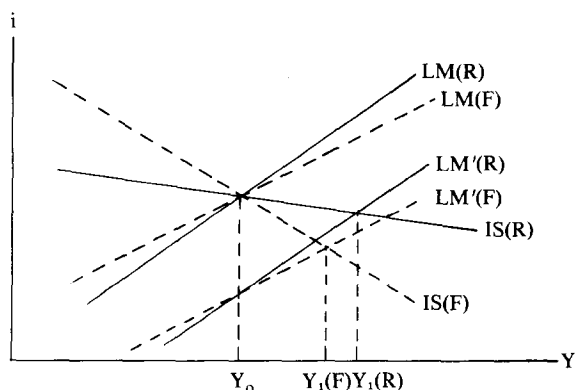
From (11) and (12) if the demand for money is inelastic ($0 < a_1 < 1$), the effect of increase in money supply on real output under rational expectations is weaker than that under fixed expectations, i.e., (11) < (14), as in Figure 2.



[Figure 2] $0 < a_1 < 1$

According to this result, for inelastic money demand where the demand for money is stable, expected money supply under rational expectations contributes to economic stability.

From (11) and (14) if the demand for money is however elastic ($a_1 > 1$), the effect of increase in money supply on real output under rational expectations is stronger than that under fixed expectations, i.e., (11) > (14), as in Figure 3.

[Figure 3] $a_1 > 1$

For elastic money demand where the demand for money is unstable, expected money supply under rational expectations increases real output more than that under fixed expectations, implying expected money supply doesn't help economic stability.

Therefore, in the aspect of economic stability expected monetary policy would be reevaluated on the elasticity of the demand for money. When the demand for money is inelastic, expected monetary policy, for example $k\%$ rule of monetarists, is better than unexpected monetary policy for economic stability even in the standard Keynesian model.

V. SOME IMPLICATIONS OF RATIONAL EXPECTATIONS IN THE STANDARD KEYNESIAN MODEL

Lots of rational expectations model have been developed and have opened new version of economic theory. One of key results in their analyses based on the supply side model of the Lucas' supply function is that expected monetary policy is neutral to real variables.

Here, rational expectations in the standard Keynesian model based on the demand side model have different insights on past results derived from rational expectations. Some results in this work can be summarized as:

First, expected monetary policy under rational expectations is not neutral to real output in the standard Keynesian model, since expected monetary policy affects expected aggregate demand and thereby real output.

Second, comparing monetary policy between rational expectations and fixed expectations depends on the elasticity of the demand for money.

Third, when the demand for money is inelastic, expected monetary policy under rational expectations contributes to economic stability.

Fourth, the downward sloping IS curve under rational expectations is flatter than that under fixed expectations, while the upward sloping LM curve under rational expectations is steeper than that under fixed expectations.

Therefore, in the standard Keynesian model based on the principle of effective demand the analysis of monetary policy under rational expectations would have different results from well-known neutrality ones of monetary policy.

REFERENCES

- BARRO, R.J. (1976), "Rational Expectations and the Role of Monetary Policy," *Journal of Monetary Economics*, 2, pp. 1-33.
- and RUSH, M. (1980), *Unanticipated Money and Economic Activity*, in *Rational Expectations and Economic Policy* (Ed. S. Fischer), University of Chicago Press for National Bureau of Economic Research, Chicago.
- LUCAS, R.E.Jr. (1972), "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 4, pp. 103-24.
- (1975), "An Equilibrium Model of the Business Cycle," *Journal of Political Economy*, 83, pp. 1113-44.
- MCCALLUM, B.T. (1977), "Price Level Stickiness and the Feasibility of Monetary Stability Policy under Rational Expectations," *Journal of Political Economy*, 85, pp. 627-34.
- PHELPS, E.S. and TAYLOR, J.B. (1977), "The Stabilizing Powers of Monetary Policy under Rational Expectations," *Journal of Political Economy*, 85, pp. 165-90.
- SARGENT, T.J. (1976), "A Classical Macroeconometric Model of the United States," *Journal of Political Economy*, 84, pp. 207-38.
- (1987), *Macroeconomic Theory*, Academic Press, New York.