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RATIONALITY vs. ADAPTIVITY : An Empirical Test Using the ASA–NBER Survey Data

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I. INTRODUCTION

The tendency toward underprediction of the change in economic variables is frequently observed property of forecasts. This can arise in unbiased as well as biased predictions.[1] Underprediction of one variable is of the same sign but of smaller size than the actual variable. When we consider the expectations of the implicit price deflator (IPD), the gross national product (GNP), and the unemployment rate (UNP) from the ASA–NBER survey, the first two variables show the tendency toward underprediction (APPENDIX D and E). However, for the unemployment rate it is not clear whether forecasters make overpredictions or underpredictions (APPENDIX F).

Muth (1961) has shown that a tendency toward underprediction is generally consistent with the rational expectations hypothesis following Theil's (1961) definition of underprediction. Theil defined underprediction as the size of the estimated coefficient, $\hat{b}$, being less than 1 in the equation:[2]

$$E_{t-1}\Pi_t = a + b\Pi_t + u_t \tag{1}$$

where $E_{t-1}\Pi_t$ is the expectations on each variable for time t based on all information at $t-1$ and u_t is a random disturbance. The rational expectations hypothesis states that, in the aggregate, the expected value is an unbiased predictor of the actual value. That is,

$$\Pi_t = E_{t-1}\Pi_t + u'_t,\ E(u'_t E_{t-1}\Pi_t) = 0,\ Eu'_t = 0 \tag{2}$$

Then in equation (1) $b = \mathrm{Var}(E_{t-1}\Pi_t)/\mathrm{Var}(\Pi_t) < 1$

This tendency is also consistent with the following simple adaptive expectations

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[1]Theil (1961, especially Chapter V) and Mincer and Zarnowitz (1968) discuss this phenomenon very extensively. More recently Zarnowitz (1985) shows the general tendency toward underprediction in the ASA–NBER survey data set.

[2]Among three definitions that he has given, we have chosen the first which is most appropriate. See pp. 158–60, and for an empirical result, pp. 173–79 in Theil (1961).

model:[3]

$$E_{t-1}\Pi_t - E_{t-2}\Pi_{t-1} = \beta(\Pi_{t-1} - E_{t-2}\Pi_{t-1}) \tag{3}$$

which first appears in Cagan (1956) and is based on Hicks' definition of elasticity of expectations. Positive forecast error causes increase in foreacst. Despite of some limitations, like a lack of theoretical justification for the model and an identification problem, adaptive expectations have been popular for their simplicity because maximum-likelihood estimates for β can be easily obtained and because such models appear to work well in a number of empirical studies. Above all, the most persuasive argument for the model is that there is some supporting empirical evidence: "the respondents tended to overestimate sales for periods in which sales actually fell and to underestimate them for periods in which sales rose"(Modigliani and Sauerlender, 1955).[4] This is consistent with equation (3) as long as β is between zero and one, i.e., only a fraction of the forecasting error is corrected in any one period.

We, therefore, need to determine which model best describes the process of expectations formation by using the directly observed ASA–NBER survey data set. Of course, there is a fairly large literature on the test for the rationality of expectations using the survey data set (e.g., Pesando (1975), Su and Su (1975), Mullineaux (1978), and so on). Recently Zarnowitz (1985) has shown most expensive empirical studies of expectations using not only the aggregated but the disaggregated data from surveys.[5]

Unlike the analyses in the time domain, when the appropriate techniques in the frequency domain are taken, they lend a conceptual simplicity to the theoretical interpretation of time–varying behaviour.

In this article we construct some criteria for adaptive expectations model in terms of frequency response function and for rational expectations model in terms of causality test in the frequency domain respectively.

II. SOME CRITERIA FOR DISCRIMINATION

1. Previous Test Criteria

There are basically four different tests that have been adopted in the econometric literature on testing for rationality. As the first test, the mean of the forecast error should be zero where the forecast error is the difference between

[3]We will be concerned with only the one–period ahead forecast because this span is most widely used for discriminating among the various alternative expectations formation processes.

[4]For a further discussion of the limitations and reasons for popularity, see Nerlove, Grether and Carvalho (1979).

[5]Chun (1987) gathered and reviewed the related works on this topic. For further, see Chapter II.

the survey expectation and the actual realization of the variable. To put it other way, the survey expectation should be an unbiased predictor of the variable. In the regression form

$$\Pi_t = \alpha + \beta E_{t-1}\Pi_t + u_t \tag{4}$$

a joint test ($\alpha = 0$, $\beta = 1$) should not be rejected. The second one is, so called, efficiency test. This indicates that the survey expectation should use information about the past history of the variable in the same way that the variable actually evolves through time. Empirically, in the regression,

$$\Pi_t = \sum_{i=0}^{m} \alpha_i \Pi_{t-i-1} + u_{1t} \tag{5}$$

$$E_{t-1}\Pi_t = \sum_{i=0}^{m} \beta_i \Pi_{t-i-1} + u_{2t} \tag{6}$$

the null hypothesis of efficiency is H_0: $\alpha_i = \beta_i$ for all i which can be tested by Chow's F–test.

The third requirement for rationality is the consistent application of the available information to generate multi–span forecasts. When forecasts are given for the same variables at different times in the future, they should be consistent with one another. And the last one is the forecast error orthogonality test. Under REH the forecast error should be uncorrelated with any information available at the time the forecast is made.

Pesando (1975) was the first person who adopted these tests for rationality of the Livingston series over the sample period 1959–69. And many works thereafter (as mentioned earlier) attempted similar tests or a slight modification of them. Since all these attempts made use of the time series technique in the time domain, we could not infer the time–varying behaviour, that is, the behaviour in the long run or in the short run. For this Lahiri and Chun (1987) and Chun (1987) reexamine the first three tests in the frequency domain using the ASA–NBER and Livingston survey data. Next are the new test criteria on the basis of the fourth test.

2. Shape of Gain Function of the Simple Adaptive Expectations

Figlewski and Wachtel (1981) change equation (3) into the testable form:

$$E_{t-1}\Pi_t - E_{t-2}\Pi_{t-1} = \alpha + \beta(\Pi_{t-1} - E_{t-2}\Pi_{t-1}) + e_t \tag{7}$$

Applying OLS to this using the Livingston data set, they obtain regression results and conclude that the adaptive expectations model is more appropriate. However their analysis is invalid if there is a proxy–error problem. The disturbance e_t is then serially correlated.

When we use cross–spectral analysis or, more exactly, least squares analysis in the frequency domain, we can make a stronger argument. For an empirical test we reformulate (3) into the geometric distributed lag structure. Rewriting using lag operator notation, we have

$$(1 - \gamma L)E_{t-1}\Pi_t = \beta L \Pi_t,\ \gamma = 1 - \beta \tag{8}$$

Solving we find

$$E_{t-1}\Pi_t = \frac{\beta L}{1-\gamma L}\Pi_t = \beta \sum_{i=0}^{\infty} \gamma^i \Pi_{t-i-1} \tag{9}$$

which is the geometric distributed lag structure. Opposed to this, the efficiency condition of REH posits a more general polynomial lag structure assuming that $S_{t-1} \in \Omega_{t-1}$ where S_{t-1} includes the past history of Π_t:

$$E(\Pi_t \mid S_{t-1}) = \sum_{i=0}^{m} a_i \Pi_{t-i-1} \tag{10}$$

and the actual value is generated by

$$\Pi_t = \sum_{i=0}^{m} b_i \Pi_{t-i-1} + u_{3t} \tag{11}$$

for which we give the following interpretation. The value of the autoregression at any time t consists of a linear function of the values of the process for the past in observation times plus a random shock, which is unrelated to the past values of the process. This may be interpreted as the simplest possible model for a stochastic process with a memory of fixed length. The testable condition here is $a_i = b_i$ for all i. Following Mullineaux (1978) we set up a new equation by subtracting (10) from (11):

$$\Pi_t - E_{t-1}\Pi_t = \sum_{i=0}^{m} c_i \Pi_{t-i-1} + e_t \tag{12}$$

where $c_i = b_i - a_i$ and $e_t = u_{3t}$. This shows that the forecast error is a distributed lag function of the actual value. The comparable hypothesis for REH in the frequency domain is that the gain function of (12) is not significantly different from zero at each frequency, at least, over the low frquency band.

The question of which model is appropriate can be investigated by examining the shape of the gain estimates compared with theoretical Bode plots.[6] For the geometrically distributed lag model (9), the frequency response function is given by

$$\beta(\omega) = \beta \sum_{j=0}^{\infty} \gamma e^{-i(j+1)\omega} = \frac{\beta e^{-i\omega}}{1-\gamma e^{-i\omega}} \tag{13}$$

[6]This technique was used by Engle (1976) to specify the relationship between housing and interest rates, and by Raj and Siklos (1986) to discriminate the exponentially declining lag structure for Phillips curve from the general distributed lag structure.

which is just replacement of the lag operator L by $z = e^{-i\omega}$. The gain for this model is therefore

$$G(\omega) = \frac{\beta}{\sqrt{1 + \gamma^2 - 2\gamma\cos\omega}} \tag{14}$$

which is largest at low frequencies and then falls off at higher frequencies. At zero frequencies, the gain is the sum of lag coefficients. $G(0) = \frac{\beta}{1-(1-\beta)} = 1$. The gain at frequency π is $\frac{\beta}{2-\beta}$. And since the lag weights are positive in this model, the peak of the gain function will always occur at zero frequency. Therefore the theoretical shape for the gain function of the geometrically distributed lag model will have a smoothing filter with characteristics selectable to a degree by varying β. The gain function of this model is plotted for several values of β in the Appendix G. When the adjustment coefficient (β) is close to unity, the frequency response function is approximately horizontal at a height of 1.0. As β becomes smaller, the frequency response function becomes steeper.

3. Granger Causality and Rationality

For the rational expectations hypothesis, an important property is $E[\Pi_t - E_{t-1}\Pi_t \mid \Omega_{t-1}] = 0$. This shows that the prediction error should be uncorrelated with any information or linear combination of information in Ω_{t-1}. And according to the definition of REH, the expected value may embody all the available information at time $t-1$, which means that past forecast error is uncorrelated with the future and present expected value $E_{t-1}\Pi_t$. From this, one can construct several possible testable hypotheses.

First, the integrated sample co–spectrum and the sample phase spectrum may be a candidate for testing the REH analogous to the correlation test in the time domain. If the two series are uncorrelated, then the co–spectrum at each frequency is identically zero and hence the integrated co–spectrum is zero. Then the sample phase spectrum will be approximately uniformly distributed in the range $-\pi/2$, $\pi/2$, so that the cumulative distribution function of the phase angle will be a straight line in this range.[7] However, this is not a good test criterion because REH does not exclude correlation between the future values of forecast error and present expectations.

A second candidate is to check the shape of the frequency response function of equation (13). The frequency response function at each frequency should be identically zero under REH. But this does not give us a powerful test in the sense that S_{t-1} includes the past history of Π_t only.

A third candidate is a causality test using the spectral decompostion of Geweke's measure in the frequency domain.[8] In the time domain there were

[7]See Watts and Jenkins (1968).

[8]See Geweke (1982, 1986).

some related studies. For example Brown and Maital (1981) used this concept to test for "full rationality" of Livingston survey data. They regressed the current forecast error on lagged policy and state variables whose values were known when the forecast was made, and tested whether all the coefficients of these variables were zero or not.

Another example is Noble (1982). Using the same approach as Brown and Maital, he tested for rationality of the inflation rate using the University of Michigan Survey Research Center survey data on CPI. In contrast to this approach, we note the following proposition. In the equation $\Pi_t = {}_{t-1}\Pi_t + u_t$, REH implies that the forecast error should theoretically not be Granger–caused by the expectations series, ${}_{t-1}\Pi_t$. But when we consider the problem of error–contaminated data, the linear feedback from the forecast series to forecast errors should not be detected, at least, over the low frequency band. Fortunately Geweke (1982, 1986) gives the full basis for what we need. To get a result of the causality test we will briefly introduce some basic concepts on causality and Geweke's idea in the next section.

III. COMPUTATION AND RESULTS

First, let's consider the possibility of adaptive expectations. The problem of estimating a frequency response function is formally equivalent to a least squares regression analysis conducted at each frequency. To apply spectral methods to the estimation of frequency response functions, it is convenient to change (9) into a form with conventional notation:

$$y_t = A(L)X_t + u_t \tag{15}$$

where $y_t = E_{t-t}\Pi_t$, $X_t = \Pi_t$ for equation (9). Then, taking the Fourier transform of (15) yields:

$$y(\omega) = A(\omega)X(\omega) + u(\omega) \tag{16}$$

Now this may be written

$$y(\omega)\overline{X(\omega)} = A(\omega)X(\omega)\overline{X(\omega)} + u(\omega)\overline{X(\omega)} \tag{17}$$

where the bar above $X(\omega)$ denotes the transpose of the complex conjugate. If we assume that the disturbance is uncorrelated with the past value of X, or $u(\omega)\overline{X(\omega)}$ is negligibly small, then the frequency response function $A(\omega)$ is

$$A(\omega) = \frac{f_{yx}(\omega)}{f_x(\omega)}\,. \tag{18}$$

But since it is well known that the raw periodogram, f, at each frequency should be smoothed to get efficient estimators of the gain, we choose the Daniell window

$$W(\omega_m) = \begin{cases} \frac{1}{m}, & \frac{-(n-1)}{2} < m < \frac{n}{2} \\ 0, \text{ otherwise} \end{cases} \tag{19}$$

where n is the width of the rectangular smoothing band. Because the window we have chosen has a very sharp cutoff a taper we need to apply tapering to the unpadded part of a series prior to taking the Fourier transform. The taper reduced the window leakage by scaling the ends of the data so that they merge smoothly with the zeros on either side. The type of taper is trapezoidal, which multiplies the input series by:

$$b(t) = \begin{cases} \frac{t}{m}, & 1 < t < m \\ 1, & m+1 < t < N-m \\ \frac{(N-t+1)}{m}, & N-m+1 < t \leq N \end{cases} \tag{20}$$

where m is the number of observations to be tapered on each end.[9]

However, in applying smoothing to a raw periodogram it is assumed that the true frequency response function $A(\omega)$ remains approximately constant over the bandwidth of the spectral window. This assumption will not be valid unless the two series have been aligned so that the cross correlation function (ccf) has its peak at zero lag. If this is not done, Jenkins and Watts (1968) have demonstrated that the estimates will be biased if the phase changes rapidly. If the sample ccf has its largest value at lag L, the two series are aligned by translating one of series a distance L, so that the aligned series has a peak at zero lag. But as is shown in Tables 1 through 3 for the cross correlations, the peak appears at zero lag for all the cases. Therefore data alignment is not necessary in our case.

The gains of the expected value on the actual value for the inflation rate, growth rate of GNP, and unemployment rate, are plotted in APPNEDIX H through J. This summary measure is the frequency domain analog of the regression coefficient in the time domain. The three figures show that the frequency response function does not appear to be one characterized by geometrically distributed declining weights when we compare them with APPENDIX G. They have two peaks, one in a low frequency band and the other in a high frequency band. Apparently the adaptive expectations model is not appropriate for UNP since A–J shows two peaks at low and high frequencies and the gain at each frequency is generally greater than 1. This story also holds for GNP though it is not as apparent as in the UNP case. Without a peak in the high frequency range, A–I

[9]The Daniell window combined with trapezoidal tapering is suggested in the RATS Manual (1986) and by Koopmans (1974).

Table 1. Cross–Correlations of Series Expected IPD and Actual IPD

Number of Observations 72
FROM 68: 4 UNTIL 86: 3

−18:	−0.230385	−0.208710	−0.205654	−0.167384	−0.150532	−0.137732
−12:	−0.144533	−0.119939	−0.052787	−0.045148	0.023988	0.105767
−6:	0.196674	0.320610	0.453435	0.551554	0.634424	0.712431
0:	0.850008	0.837777	0.701050	0.657703	0.596503	0.452822
6:	0.364227	0.302249	0.182909	0.129824	0.091010	0.049995
12:	0.029463	−0.059709	−0.062985	−0.052356	−0.081394	−0.087153
18:	−0.103066					

Table 2. Cross–Correlations of Series Expected GNP and Actual GNP

Number of Observations 72
FROM 68: 4 UNTIL 86: 3

−18:	−0.046524	0.066514	0.001349	−0.082873	−0.148681	−0.037159
−12:	−0.166936	0.058718	0.110374	0.004365	−0.076662	0.042868
−6:	0.105587	0.064230	0.177850	0.185234	0.194483	0.276306
0:	0.677358	0.542547	0.249792	0.063479	0.044018	−0.078727
6:	−0.121601	−0.047230	−0.109970	−0.163619	0.011709	0.079920
12:	−0.031280	−0.104357	0.011515	0.035387	0.008627	0.069082
18:	0.100089					

Table 3. Cross–Correlations of Series Expected UNP and Actual UNP

Number of Observations 72
FROM 68: 4 UNTIL 86: 3

−18:	0.075731	0.071618	0.060973	0.053572	0.053208	0.060001
−12:	0.072973	0.095211	0.126821	0.162087	0.207754	0.276985
−6:	0.361813	0.462515	0.579702	0.701949	0.820419	0.924426
0:	0.994651	0.952001	0.853949	0.730466	0.602195	0.480962
6:	0.374793	0.287633	0.213130	0.156308	0.120164	0.088940
12:	0.062806	0.047477	0.044230	0.047375	0.055373	0.065859
18:	0.073625					

looks like a combination of adjustment speed of 1 in the long run and with negligible speed in the short run–if we are to argue for the adaptive expectations model.

To some degree A–H is similar to the theoretical shape of gain for adaptivite expectations with β small, except for the fact that a peak appears at high frequency. If one, therefore, works with only IPD for empirical tests, one is apt to believe that adaptivity can be an appropriate model of expectations formation.[10]

Next comes the possibility of rational expectations. The concept of causality itself, as well as causal inference from empirical relationships, have long attracted the attention of econometricians.[11] When the observed data are time series, the possibility of causal inference does arise, if one accepts the axiom that the future can not cause the present or past. This idea led Granger (1969), based to some extent on ideas in Wiener (1956), to introduce a definition of causality which opened the way of empirical testing on the basis of time series data. The Wiener–Granger definition of causality is as follows: A time series X 'causes' another time series Y if present Y can be predicted better by using past value of X(denoted X^-) than by not doing so: the past values of Y (Y^-) being used in either case. Meanwhile Sims (1972) defined causality as X does not cause Y if and only if coefficients on future Y ($Y - Y^+$) are all zero where Y^+ represents the past and present of Y. He also shows this is equivalent to Granger causality for linear models.

Geweke (1982) develops and interprets new measures of unidirectional and two–way feedback for vector processes by deriving an interesting decomposition of the linear relationships between two multiple time series, X and Y. That is, the linear relationship between X and Y is the sum of the linear causality from X to Y, the linear causality from Y to X, and the instantaneous linear causality between X and Y. Furthermore he takes note of the fact that measures of feedback from X to Y and, Y to X can, under circumstances often satisfied by economic time series, can be additively decomposed by frequency so that one may speak of "feedback from Y to X (or X to Y) at frequency ω".

To understand the meaning of the various definitions of feedback, and the formulas for them given by Geweke's Theorem 1 (1982, p. 307), we will construct the prediction errors and their covariance matrices, corresponding to five sets of explanatory variables:

[10]In this sense, it is understandable that Figlewski and Wachtel (1981) argue for the adaptive model of inflationary expectations.

[11]See, for example, Simon (1953), Wold (1954), Basmann (1963), Zellner (1979), and the references therein.

Prediction error	Covariance Matrix
$v_{1t} = Y_t - E(Y_t \mid Y^-)$	$\Sigma(Y \mid Y^-)$
$v_{2t} = Y_t - E(Y_t \mid X^-, Y^-)$	$\Sigma(Y \mid X^-, Y^-)$
$v_{3t} = Y_t - E(Y_t \mid X^+, Y^-)$	$\Sigma(Y \mid X^+, Y^-)$ (21)
$v_{4t} = Y_t - E(Y_t \mid X)$	$\Sigma(Y \mid X)$
$v_{5t} = Y_t - E(Y_t \mid X, Y^-)$	$\Sigma(Y \mid X, Y^-)$

Then we can define

i) the measure of linear dependence

$$\begin{aligned} F_{X,Y} &= \ln \det\Sigma(X \mid X^-) - \ln \det\Sigma(X \mid X^-, Y) \\ &= \ln \det\Sigma(Y \mid Y^-) - \ln \det\Sigma(Y \mid Y^-, X), \end{aligned}$$

ii) the measure of linear feedback from Y to X

$$\begin{aligned} F_{Y\to X} &= \ln \det\Sigma(X \mid X^-) - \ln \det\Sigma(X \mid X^-, Y^-) \\ &= \ln \det\Sigma(Y \mid X^+, Y^-) - \ln \det\Sigma(Y \mid X, Y^-), \end{aligned}$$

iii) the measure of linear feedback from X to Y

$$\begin{aligned} F_{X\to Y} &= \ln \det\Sigma(Y \mid Y^-) - \ln \det\Sigma(Y \mid X^-, Y) \\ &= \ln \det\Sigma(X \mid X^-, Y^+) - \ln \det\Sigma(X \mid X^-, Y), \end{aligned}$$

iv) and the measure of instantaneous linear feedback

$$\begin{aligned} F_{X\cdot Y} &= \ln \det\Sigma(X \mid X^-, Y^-) - \ln \det\Sigma(X \mid X^-, Y^+) \\ &= \ln \det\Sigma(Y \mid X^-, Y^-) - \ln \det\Sigma(Y \mid X^+, Y^-).[12] \end{aligned}$$

Hence, $F_{X,Y} = F_{Y\to X} + F_{X\to Y} + F_{X\cdot Y}$ and the measure of linear dependence is the sum of the measures of the three types of linear feedback.

Now let's consider the frequency decomposition of $F_{X\to Y}$ and $F_{Y\to X}$ given by Geweke. The model is

$$x_t = \sum_{i=1}^{\infty} A_i x_{t-i} + \sum_{i=1}^{\infty} B_i y_{t-i} + u_{1t} \tag{22}$$

$$y_t = \sum_{i=0}^{\infty} C_i x_{t-i} + \sum_{i=1}^{\infty} D_i y_{t-i} + u_{2t}. \tag{23}$$

which can be rearranged as

$$\begin{bmatrix} 1-A(L) & -B(L) \\ -C(L) & 1-D(L) \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \tag{24}$$

The matrix of lag operators in euqation (24) can be inverted to express x_t and y_t as one–sided distributed lags of u_{1t} and u_{2t}:

12 See the proof in Geweke (1982) in p. 307.

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} 1-A(L) & -B(L) \\ -C(L) & 1-D(L) \end{bmatrix}^{-1} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

$$= \begin{bmatrix} E(L) & F(L) \\ G(L) & H(L) \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}. \tag{25}$$

x_t may be represented as distributed lags of the orthogonal, serially uncorrelated process u_{1t} and u_{2t}.

$$x_t = E(L)u_{1t} + F(L)u_{2t} \tag{26}$$

and the spectral density $S_x(\omega)$ can be decomposed into the sum of two positive semidefinite matrices:

$$S_x(\omega) = E(\omega)\Sigma(X \mid X^-, Y^-)\overline{E(\omega)} + F(\omega)\Sigma(Y \mid X^+, Y^-)\overline{F(\omega)}. \tag{27}$$

Here $F(\omega)\Sigma(Y \mid X^+,Y^-)\overline{F(\omega)}/S_x(\omega)$ is the fraction of $S_x(\omega)$ due to u_{2t}. If $F(\omega) = 0$ in equation (27), x_t is not explained by the unexplained part of y_t. This suggests the measure of linear feedback from Y to X at frequency ω:

$$f_{Y\to X}(\omega) = \ln(\mid S_x(\omega) \mid /E(\omega)\Sigma(X \mid X,^-Y^-)\overline{E(\omega)}). \tag{28}$$

Geweke also shows that $1/(2\pi)\int_{-\pi}^{\pi} f_{Y\to X}(\omega)d\omega < F_{Y\to X}$ and $1/(2\pi)\int_{-\pi}^{\pi} f_{X\to Y}(\omega) d\omega < F_{X\to Y}$ and that we will have $1/(2\pi)\int_{-\pi}^{\pi} f_{Y\to X}(\omega)d\omega = F_{Y\to X}$ and $1/(2\pi)\int_{-\pi}^{\pi} f_{X\to Y}(\omega)d\omega = F_{X\to Y}$, given side conditions usually satisfied by point estimates. Hence, the measures of feedback $F_{Y\to X}$ and $F_{X\to Y}$ can be decomposed by frequency. For practical inference let's suppose all distributed lags have been truncated at length p. Then the equations are estimated and the conditional maximum-likelihood estimates $\hat{F}$ of the various measures F are constructed in the obvious way. Under the null hypothesis of no feedback from Y to X, $n\hat{F}_{Y\to X}$ has asymtotically a chi-square distribution with p degrees of freedom. That is, under $F_{Y\to X} = 0$. $n\hat{F}_{Y\to X} \approx \chi^2(p)$ where n is the number of observations. Inference about other measures of feedback can be undertaken in similar fashion.

However inference about $f_{Y\to X}(\omega)$ is somewhat more difficult, because non-linear combinations of parameters are involved. However, following Geweke's (1982) argument that the test statistics for the hypothesis $f_{Y\to X}(2\pi j/p) = 0$, $j = 0, \ldots, \frac{p}{2}$, are asymptotically independent, the limiting distributions of the test statistics under the null hypothesis are $\chi^2(1)$ at frequencies 0 and π, and $\chi^2(2)$ elsewhere.

We estimate three vector autoregressions with 6 lags: a vector autoregression for the expected inflation rate and its forecast errors, another for the GNP growth rate and its forecast errors, and a third for the unemployment rate and its forecast errors over the period 1968 : 4 to 1986 : 2. Estimated measures of feedback and their decomposition by frequency were computed following Geweke's *multiple time series manipulator* modified by B. Diefferbach (1983). These estimates are presented in APPENDIX A through C, and APPENDIX K through M.

The estimated measures of feedback from the expectations to the corresponding forecast errors are in accordance with the proposition of REH that past history of errors should be uncorrelated with the present forecasts. For IPD, $n\hat{F}_{Y\rightarrow X} = 4.26$, for GNP, $n\hat{F}_{Y\rightarrow X} = 7.45$, and for UNP, $n\hat{F}_{Y\rightarrow X} = 3.08$, none of which can be rejected at the 10 percent significance level. Further properties of these relationships are suggested by the decomposition of feedback by frequency in the columns of F(Y to X) for each Table (in Appendix A through C). For IPD, whose information at each frequency is in Appendix A, the estimated measures of feedback from expected inflation to its forecast error are sharply increasing at cycles of five years and longer. Over the frequencies higher than 0.1π (corresponding to 5 years) all the estimates $\hat{f}_{Y\rightarrow X}$ are close to zero.

The column of F(Y to X) in Appendix B show the actual estimates for GNP. Over the range of frequencies corresponding to periods around four quarters, $\hat{f}_{Y\rightarrow X}$ exceeds 20, implying that 20 percent or more of the variation in the forecast errors is explained by the forecast for GNP at these frequencies. This is evidence of feedback from Y to X around a one year cycle. For UNP in APPENDIX C, the actual estimate $\hat{f}_{Y\rightarrow X}$ increases at cycles of 10 years and longer, but not significantly different from zero. Except over low frequencies for IPD and around four quarters for GNP, generally all variables are in accordance with the REH proposition that $\hat{f}_{Y\rightarrow X}$ is close to zero.

IV. CONCLUSION

As a way of discrimination between rational expectations and adaptive expectations, we have proposed tests based on the gain function of expectations on actual realization, and the linear feedback from these expectations to their expectational error.

On the basis of the first criterion, UNP series is definitely far from the adaptive expectations, and IPD and GNP series give the mixed results previously mentioned.

As for the rationality test on the basis of the second criterion, all the three series are consistent with the null hypothesis that the expectational errors do not cause the expected variables over all frequencies. This argument is well supported when a careful look is taken at the adjusted values of F(Y to X). When we investigate the hypothesis at each frequency, the evidence is far from our anticipation that the test statistics will become more insignificant as we approach zero frequencies. Instead, the feedback measures become greater.

Nevertheless,.the general evidence is that the past history of forecast errors is not Granger–caused by the forecasts at each frequency.

APPENDIX A Estimated Measures of Feedback between the Expexted inflation Rate and the Forecasting Errors, 1968: 4–1986: 2

X Vector: 1 Expectational Error

Y Vector: 2 Forecasts for IPD

Equation 1 (X is a dependent variable.)

Lag	1	2	3	4	5	6
X	0.041	0.129	0.221	−0.157	0.048	0.150
Y	0.047	−0.071	0.001	0.111	0.074	−0.261

Equation 2 (Y is a dependent varible.)

Lag	1	2	3	4	5	6
X	−0.149	0.102	0.023	−0.138	−0.033	0.016
Y	0.915	−0.113	0.224	0.001	−0.036	−0.137

Measures of Feedback

F(X.Y)	F(Y to X)	F(X to Y)
0.097 (9.3%)	0.060 (5.8%)	0.096 (9.1%)

Frequency	Period	F(Y to X)	F(X to Y)
0.000PI		0.273 (23.9%)	0.045 (4.4%)
0.050PI	40.000	0.272 (23.8%)	0.041 (4.0%)
0.100PI	20.000	0.135 (12.6%)	0.036 (3.5%)
0.150PI	13.333	0.061 (5.9%)	0.029 (2.9%)
0.200PI	10.000	0.035 (3.4%)	0.020 (2.0%)
0.250PI	8.000	0.027 (2.6%)	0.010 (1.0%)
0.300PI	6.667	0.028 (2.7%)	0.007 (0.7%)
0.350PI	5.714	0.037 (3.6%)	0.027 (2.6%)
0.400PI	5.000	0.054 (5.3%)	0.063 (6.1%)
0.450PI	4.444	0.078 (7.5%)	0.103 (9.8%)
0.500PI	4.000	0.092 (8.8%)	0.145 (13.5%)
0.550PI	3.636	0.082 (7.8%)	0.198 (17.9%)
0.600PI	3.333	0.058 (5.7%)	0.268 (23.5%)
0.650PI	3.077	0.036 (3.5%)	0.341 (28.9%)
0.700PI	2.857	0.019 (1.9%)	0.308 (26.5%)
0.750PI	2.667	0.009 (0.9%)	0.140 (13.1%)
0.800PI	2.500	0.005 (0.5%)	0.039 (3.8%)
0.850PI	2.353	0.006 (0.6%)	0.018 (1.8%)
0.900PI	2.222	0.008 (0.8%)	0.028 (2.7%)
0.950PI	2.105	0.011 (1.1%)	0.043 (4.2%)
1.000PI	2.000	0.011 (1.1%)	0.050 (4.9%)

	Estimate	Mean	25.00%	75.00%	Adjusted
F(Y to X)	0.06	0.13	0.08	0.15	0.03
F(X to Y)	0.10	0.13	0.08	0.17	0.07
F(X, Y)	0.10	0.10	0.05	0.15	0.10

APPENDIX B Estimated Measures of Feedback between the Expected Growth Rate of GNP and the Forecasting Errors, 1968 : 4–1986 : 2

X Vector: 1 Expectational Error for GNP
Y Vector: 2 Forecasts for GNP

Equation 1 (X is a dependent variable.)

Lag	1	2	3	4	5	6
X	−0.184	−0.053	−0.024	−0.061	−0.115	−0.058
Y	0.488	−0.425	−0.072	0.166	−0.008	0.088

Equation 2 (Y is a dependent variable.)

Lag	1	2	3	4	5	6
X	−0.012	−0.126	−0.157	−0.178	−0.084	0.029
Y	0.561	0.018	−0.032	−0.284	−0.091	−0.076

Measures of Feedback

F(X,Y)	F(Y to X)	F(X to Y)
0.017 (1.7%)	0.105 (10.0%)	0.164 (15.1%)

Frequency	Period	F(Y to X)	F(X to Y)
0.000PI		0.142 (13.3%)	0.464 (37.1%)
0.050PI	40.000	0.103 (9.8%)	0.481 (38.2%)
0.100PI	20.000	0.027 (2.7%)	0.525 (40.8%)
0.150PI	13.333	0.002 (0.2%)	0.560 (42.9%)
0.200PI	10.000	0.014 (1.4%)	0.504 (39.6%)
0.250PI	8.000	0.035 (3.4%)	0.345 (29.2%)
0.300PI	6.667	0.061 (5.9%)	0.189 (17.3%)
0.350PI	5.714	0.098 (9.3%)	0.090 (8.6%)
0.400PI	5.000	0.159 (14.7%)	0.037 (3.6%)
0.450PI	4.444	0.236 (21.0%)	0.016 (1.6%)
0.500PI	4.000	0.242 (21.5%)	0.022 (2.2%)
0.550PI	3.636	0.175 (16.1%)	0.048 (4.7%)
0.600PI	3.333	0.129 (12.1%)	0.068 (6.6%)
0.650PI	2.077	0.109 (10.3%)	0.064 (6.2%)
0.700PI	2.857	0.103 (9.8%)	0.045 (4.4%)
0.750PI	2.667	0.104 (9.9%)	0.024 (2.3%)
0.800PI	2.500	0.107 (10.1%)	0.007 (0.7%)
0.850PI	2.353	0.107 (10.1%)	0.000 (0.0%)
0.900PI	2.222	0.100 (9.5%)	0.005 (0.5%)
0.950PI	2.105	0.087 (8.4%)	0.014 (1.4%)
1.000PI	2.000	0.080 (7.7%)	0.018 (1.8%)

	Estimate	Mean	25.00%	75.00%	Adjusted
F(Y to X)	0.11	0.22	0.14	0.30	0.05
F(X to Y)	0.16	0.22	0.16	0.26	0.12
F(X,Y)	0.02	0.02	0.00	0.04	0.01

APPENDIX C Estimated Measures of Feedback between the Expected Unemployment Rate and the Forecasting Errors, 1968 : 4–1986 : 2

X Vector: 1 Expectational Error for UNP
Y Vector: 2 Forecasts for UNP

Equation 1 (X is a dependent variable.)

Lag	1	2	3	4	5	6
X	0.280	0.117	0.022	0.052	0.001	−0.022
Y	−0.131	0.057	−0.006	0.024	0.011	−0.078

Equation 2 (Y is a dependent variable.)

Lag	1	2	3	4	5	6
X	0.111	0.189	0.772	−0.058	−0.079	−0.151
Y	0.522	0.023	0.175	0.099	−0.255	0.035

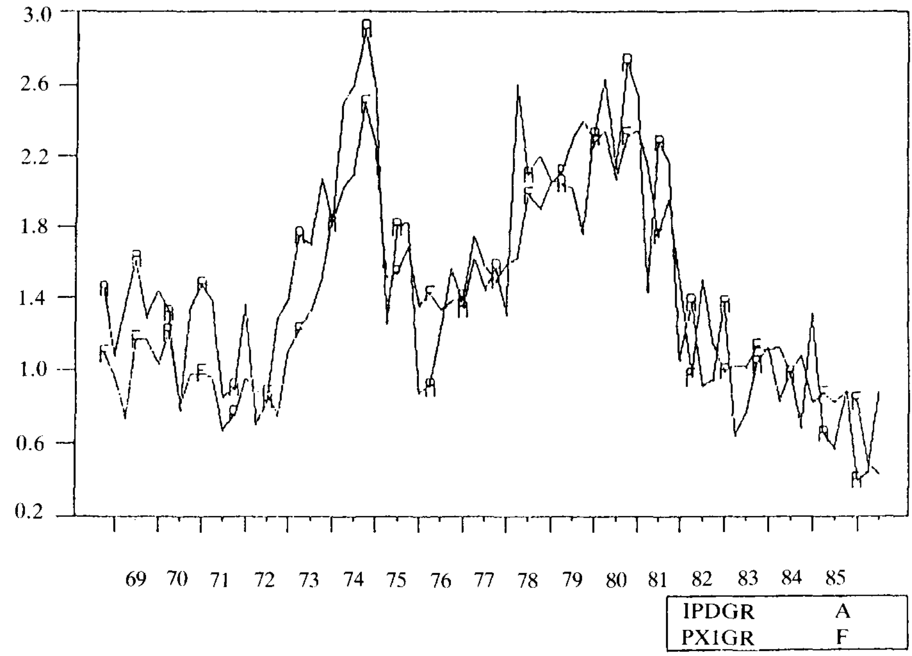
Measures of Feedback

F(X,Y)	F(Y to X)	F(X to Y)
0.019 (1.9%)	0.048 (4.7%)	0.284 (24.8%)

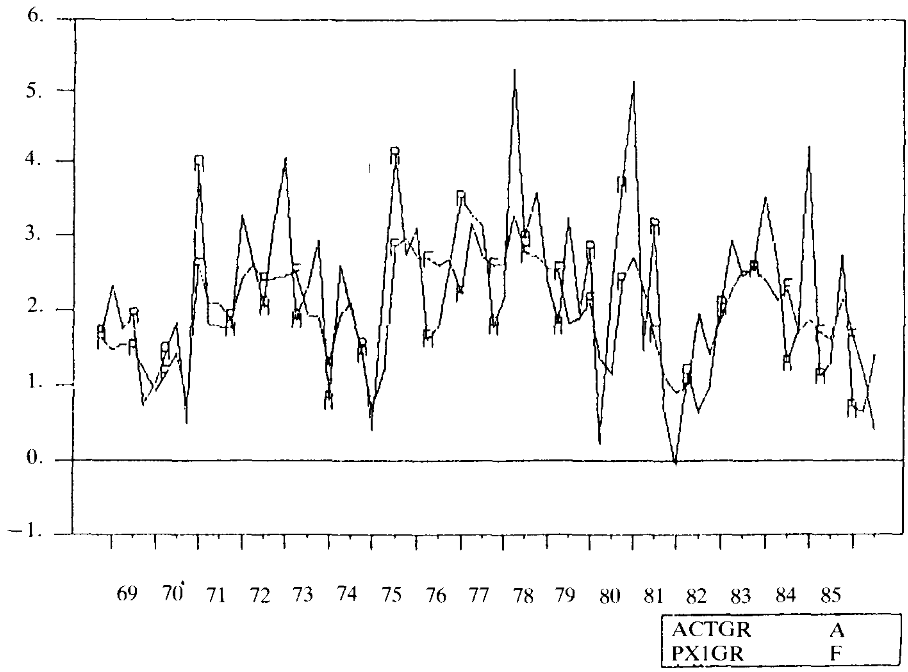
Frequency	Period	F(Y to X)	F(X to Y)
0.000PI		0.172 (15.8%)	0.761 (53.3%)
0.050PI	40.000	0.148 (13.8%)	0.737 (52.2%)
0.100PI	20.000	0.066 (6.4%)	0.665 (48.6%)
0.150PI	13.333	0.006 (0.6%)	0.565 (43.2%)
0.200PI	10.000	0.017 (1.7%)	0.469 (37.4%)
0.250PI	8.000	0.038 (3.7%)	0.392 (32.4%)
0.300PI	6.667	0.051 (5.0%)	0.336 (28.5%)
0.350PI	5.714	0.057 (5.6%)	0.295 (25.6%)
0.400PI	5.000	0.058 (5.7%)	0.262 (23.1%)
0.450PI	4.444	0.053 (5.2%)	0.228 (20.3%)
0.500PI	4.000	0.038 (3.7%)	0.185 (16.9%)
0.550PI	3.636	0.013 (1.3%)	0.140 (13.1%)
0.600PI	3.333	0.015 (1.5%)	0.105 (9.9%)
0.650PI	3.077	0.036 (3.5%)	0.086 (8.2%)
0.700PI	2.857	0.048 (4.7%)	0.083 (8.0%)
0.750PI	2.667	0.053 (5.1%)	0.094 (9.0%)
0.800PI	2.500	0.053 (5.1%)	0.115 (10.9%)
0.850PI	2.353	0.049 (4.8%)	0.139 (13.0%)
0.900PI	2.222	0.042 (4.1%)	0.158 (14.6%)
0.950PI	2.105	0.031 (3.0%)	0.168 (15.5%)
1.000PI	2.000	0.024 (2.4%)	0.171 (15.7%)

	Estimate	Mean	25.00%	75.00%	Adjusted
F(Y to X)	0.05	0.12	0.08	0.16	0.02
F(X to Y)	0.28	0.32	0.27	0.40	0.25
F(X,Y)	0.02	0.04	0.01	0.04	0.01

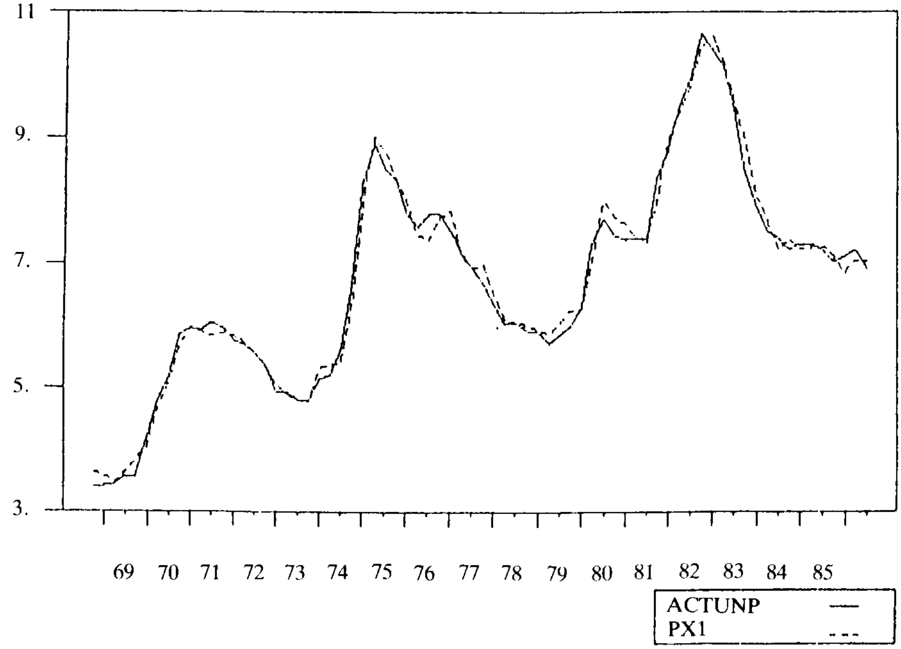
APPENDIX D Actual and Expected Inflation



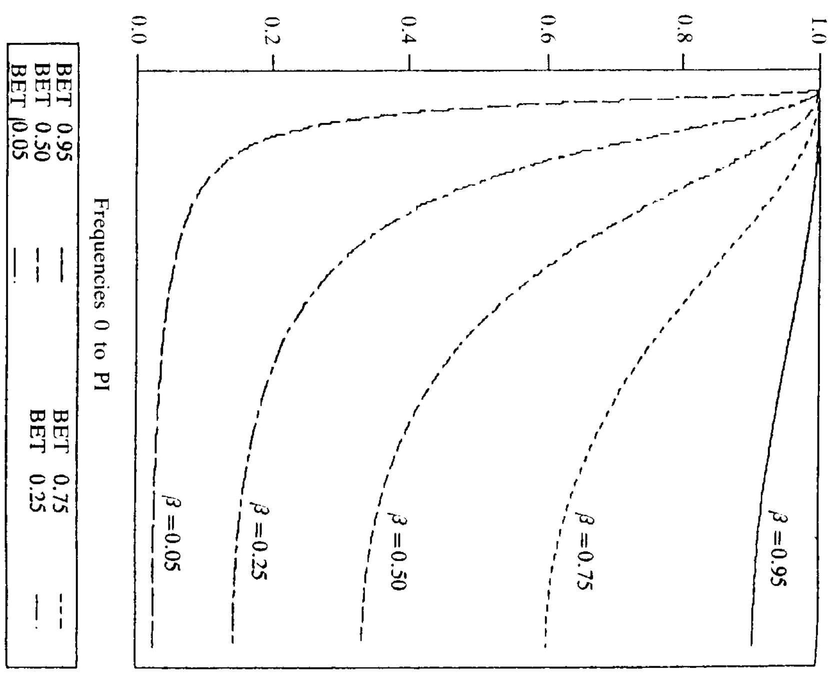
APPENDIX E Actual and Expected GNP Growth Rate



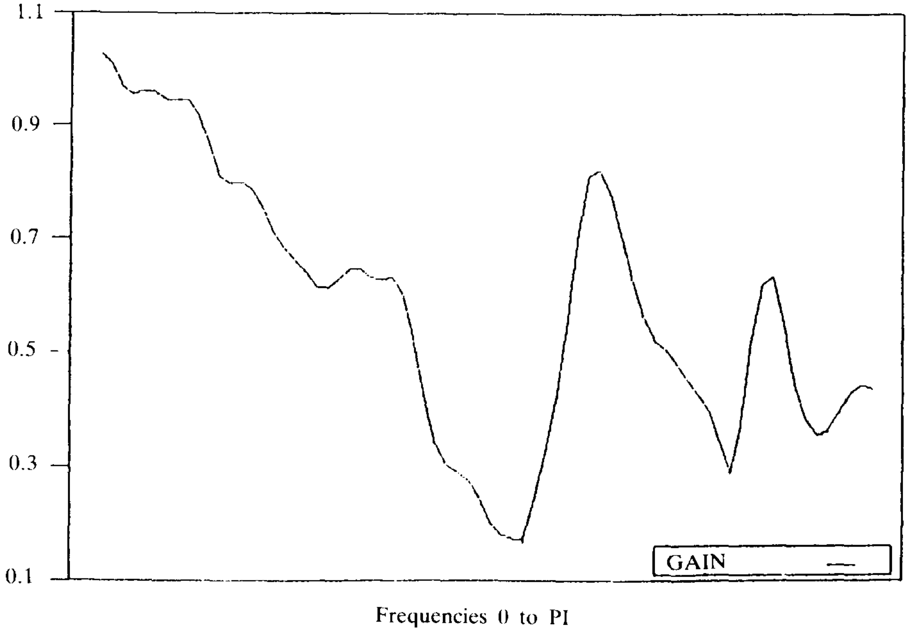
APPENDIX F Actual and Expected Unemployment



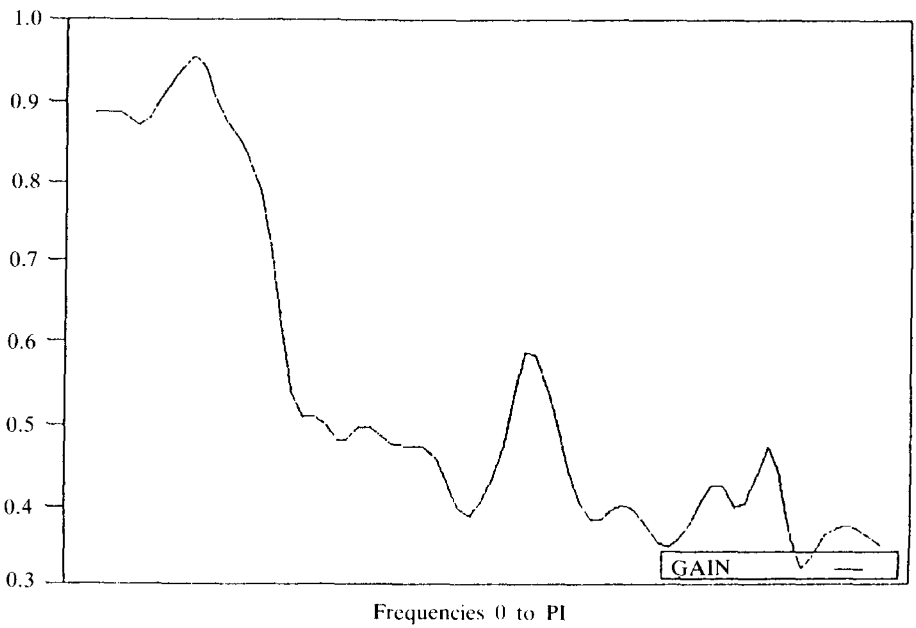
APPENDIX G Theoretical Shape of Gain Function for Varying β



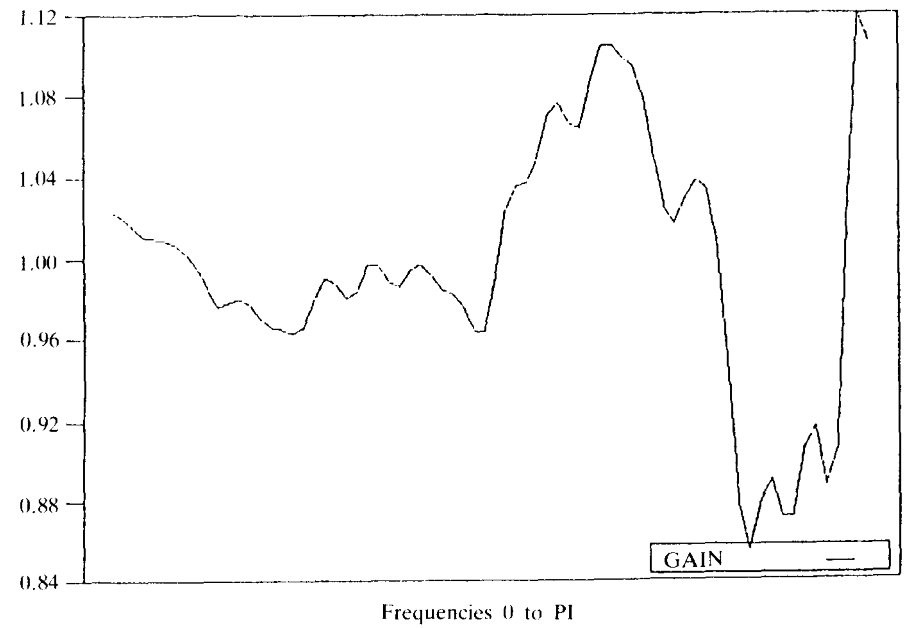
APPENDIX H Gain of EXP–IPD on ACT–IPD



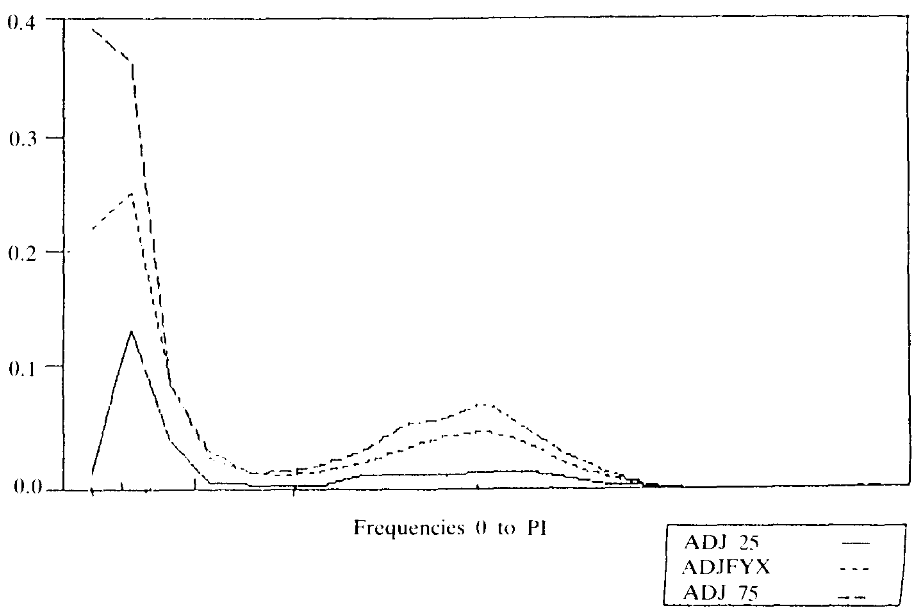
APPENDIX I Gain of EXP–GNP on ACT–GNP



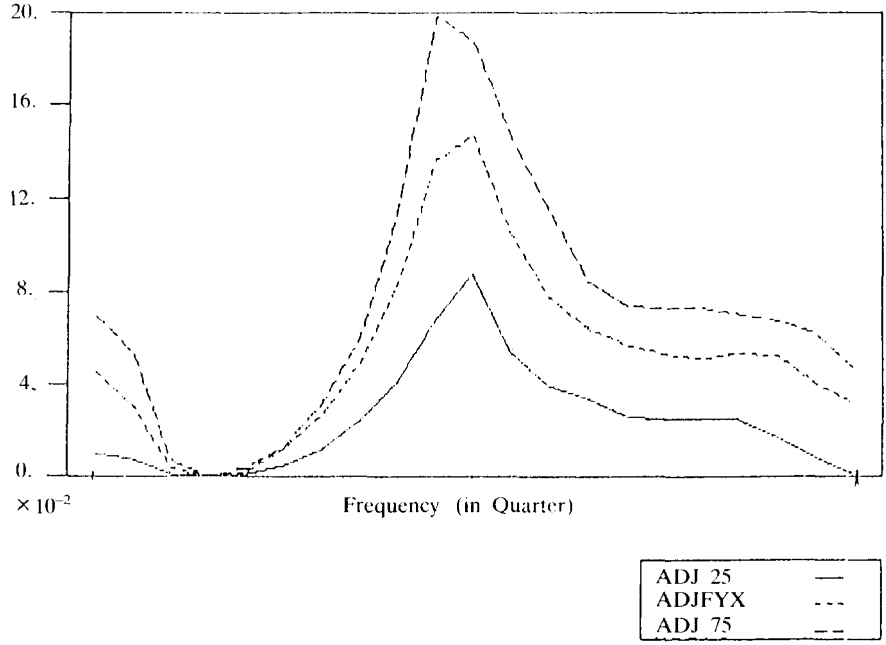
APPENDIX J Gain of EXP–UNP on ACT–UNP



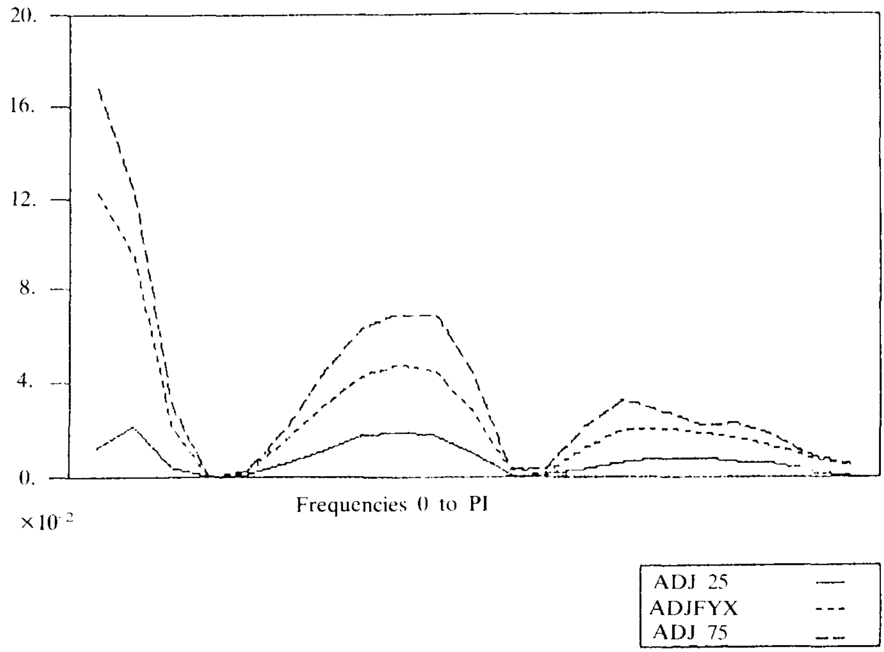
APPENDIX K Causality from FOR–IPD to Error



APPENDIX L Causality from FOR–GNP to Error



APPENDIX M Causality from FOR–UNP to Error



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