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INCOME TAXATION VS. CONSUMPTION TAXATION IN TERMS OF WELFARE IMPLICATIONS

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I. INTRODUCTION

One of major advantages of substituting a consumption tax for an income tax is based on the old belief that a consumption tax is superior to an income tax in terms of the welfare loss due to taxation. In this paper, we examine this assertion using an overlapping generations model and a differential tax incidence approach. We ask which tax scheme lowers welfare least while yielding a given amount of tax revenues. The feasible set of government fiscal policies contains only income and consumption taxes.

Since J. S. Mill argued for a consumption tax, because an income tax is a double taxation on saving income, there have been consistent beliefs among economists[1] that a consumption tax is superior to an income tax in terms of welfare comparison. There are several ways to compare the two tax schems. First, in the context of tax design an optimal taxation approach can be used and extended to an intertemporal model in which there are three goods-present consumption, future consumption, and leisure. Using the fact that a pure wage income tax is equivalent to a consumption tax[2], one can assume that a wage income tax and an interest income tax are available to the government as a feasible set of tax policies. The taxation of savings can then be investigated, and, in particular, whether or not a tax on saving should be imposed for optimality. King (1980) and Atkinson and Sandmo (1979) addressed this question and concluded that it is difficult to argue for the welfare superiority of either a consumption tax or an income tax on the basis of their results. In their model the tax rate on saving income, t_r, is allowed to be differenr from the tax rate in wage income, t_w. The optimal condition they derived was

$$\text{(TD)} \qquad \frac{t_w}{1-t_w}(\sigma_{LL}-\sigma_{2L}) = \frac{t_r}{P}(\sigma_{L2}-\sigma_{22}),$$

where σ_{LL} and σ_{L2} denote the compensated elasticities of labor supply with respect to wage and the price of furure consumption, P, respectively, and $\sigma_{2L}\sigma_{22}$ denote the

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1) Among others Kaldor (1958), Pigou (1949), and Fisher and Fisher (1942) claimed that a consumption tax is superior to an income tax, because a consumption tax can encourage savings while an income tax discourages savings due to distortions in the capital market.

2) Several conditions are needed for the equivalence result. See Atkinson and Stiglitz (1980) pp. 371-372.

compensated elasticities of future consumption with respect to wage and P respectively. If $\sigma_{LL}=\sigma_{2L}$, then the contion (TD) implies that $t_r=0$. It implies that no tax on saving, i. e., a pure wage income tax is optimal. On the other hand, if $t_r=t_w$ is implied by the condition (TD), then an income tax is optimal. However, the conditions for either case, which are represented by a combination of various elasticities, are hardly ever satisfied in any economy. In general the condition (TD) requires that t_r be different from t_w, meaning that an optimal tax scheme should consist of both an income tax and a consumption tax.

In a practical sense the approach of optimal tax design is not realistic for comparing alternative tax schemes, because, for most cases, there already exist some tax schemes when the government considers a new tax system. This is one reason why we consider a tax reform rather than a tax design in comparing an income tax and a consumption tax. In particular we derive a critical condition, like the condition (TD), for the case of tax reform. From the condition we derive, one can determine whether the tax reform should be toward the consumption tax scheme or the income tax scheme, given the environments of the economy, as represented by various compensated elasticities, tax rates, the interest rate, etc. In any particular circumstances, from this approach one can know which tax reform is better, while the condition (TD) gives only partial answers.

Secondly, using modern computational facilities and Scarf 's (1973) algorithm for general equilibrium analysis, we can estimate the welfare cost of each tax scheme for a specific economic model. In the models of the Applied General Equilibrium Analysis (AGEA), c. f. Shoven and Whalley (1972) ; Ballard, Fullerton, Shoven, and Whalley (1984), the magnitudes of welfare cost for various taxes are computed. Auerbach and Kotlikoff (1983) develop a perfect foresight general equilibrium simulation model of life-cycle savings to compare a capital income taxation with a consumption tax and wage income tax. In their model, however, the assumption of inelastic labor supply combined with the assumption of a positive elasticity of savings means that a consumption tax is preferred to an income tax[3]. Summers (1981) compares steady state utility for a model with fixed labor supply in a multi-period setting. Like Auerbach and Kotlikoff the assumption of fixed labor supply intrinsically implies a superiority of a consumption tax. Auerbach, Kotlikoff and Skinner (1983) improves the model with elastic labor supply by incorporating leisure-consumption choice in the utility function. A superiority of a consumption tax over an income tax is unchanged, though. They notice that a consumption tax has a feature of a lump-sum tax for the elderly at the time of tax reform. They claim that it is this element of lump-sum taxation, and not the exemption from taxation of capital income *per se* that is crucial to the achievement of efficient tax refrom. It is quite true for transitional period. However, once the economy gets to the steady state there is no longer a feature of lump-sum taxation for the elderly because all generations know their life-time pattern of tax schedules. Thus, in the steady state the feature of exemption from captial

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3) This is clearly pointed out by Stiglitz in his comment on Auerbacb and Kotlikoff (1983).

income taxation may play a major role in efficiency gains. In order to know which factor really matters for the steady state comparison of tax reform we need an analytic model.

The approach of AGEA enables us to consider simultaneously all of the interactive effects of large policy changes in a many-sector model. However, the merit of being able to analyze very complicated policy changes results in the so-called, 'black box problem'. Often one cannot explain the results based on sound economic reasoning. We need an analytic model in which we can find general results that, in turn, can be used for better simulation studies. A general theory can predict the direction of the sensitivity analysis and can warn model builders of pitfalls in specifying particular household preferences and firm technologies. Some types of utility function, for example, Cobb-Douglas or a certain C. E. S. utility funtion intrinsically imply the superiority of a consumption tax over an income tax.

In studying various tax reform proposals we need a theoretical analysis, empirical studies, as well as AGEA. All of three are equally crucial. There have been numerous AGEA studies comparing an income tax with a consumption tax in the context of tax reform. However a general theory comparing two tax schemes in the context of tax reform has not been developed, even if it has been done in the context of tax design (King (1980) ; Atkinson and Sandmo (1979) ; Atkinson and Stiglitz (1976)). This paper derives an analytic model which measures the impact of a tax reform in which both the consumption tax and income tax coexist.

We adopt a differential tax incidence approach to compare an income tax with a consumption tax. We assume that initially there exist both a consumption tax and an income tax. If the government increases one tax rate marginally, there will be an equivalent amount of tax rebate through lump-sum transfers, thus keeping the government tax revenues constant. First, we explore the differential incidence of a consumption tax by comparing it with an equal revenue lump-sum tax. Second, we explore the differential incidence of a consumption tax by comparing it with an equal revenue lump-sum tax. In so doing, we directly compare the two tax schemes, since the substitution of a consumption tax for an income tax can be considered as a two--step process, where the first step is to substitute a lump-sum tax for an income tax and the second step is to substitute a consumption tax for the lump-sum tax. Thus we can obtain the exact welfare change from a tax reform that would substitute a consumption tax for an income tax.

In section 2, we describe the model and explain the assumption on the household sector and the government sector. We characterize the steady state equilibrium in this model. In section 3, we discuss the problems of comparing an income tax and a consumption tax. The equal tax revenue constraint is explained in special detail. The standard criterion of welfare comparison is briefly explored. In section 4, we derive the main findings of the paper and investigate the determinants of the relative welfare superiority of one tax as well as the absolute magnitude of the welfare cost from both tax schemes. We also illustrate this with some examples. A final section summarizes the main results of the paper.

Ⅱ. THE OVERLAPPING GENERATIONS MODEL

1. Household

The economy is composed of many identical individuals. The 'identical' assumption is postulated since we are interested in effciency aspects of taxes rather than distributional aspects. Each individual lives for two periods, working in the first period and then retiring. At the end of each period an old generation dies and a new generation is born, so that in each period of time two generations live together. Each individual earns a fixed wage ω. Since the endowment of leisure is 1, wage income is $(1-L)\omega$, where L denotes leisure consummed $(L \leqq 1)$. We assume that the labor supplied by the individual is elastic with respect to the wage rate ω. The number of individuals in each generation is $(1+n)$ times that in the previous one, so there are $N_0(1+n)^i(=N^i)$ workers at time i. Each member of the generation born at time i (referred to as generation i) maximizes life-time utility subject to a life-time budget constraint. The utility function of a representative individual of generation i is

$$U^i = U^i(C_1^i, \ C_2^{i+1}, L^i, \overline{G})$$

where C^i and C_2^{i+1} are consumptions in the first period and the second period respectively, L^i is the hours of leisure that are available per worker in period 1, and $\overline{G}$ is the quantity of public goods per worker. We assume a monotonically increasing strictly concave utility function. In period 1, an individual of generation i consumes C_1^i, pays taxes, and saves the rest of his or her income for the second period. In period 2, he/she exhausts his/her saving by consuming C_2^{i+1} and by paying taxes. To simplify the analysis, we assume that there is no bequest for the future generation. During the lifetime an individual of generation i pays an income tax and a consumption tax, and may recieve a lump-sum transfer, the individual budget constraint is thus

$$(1) \quad (1+t_e)C_1^i + \frac{(1+t_e)C_2^{i+1}}{1+r^{i+1}(1-t_i)} - T_1^i - \frac{T_2^{i+1}}{1+r^{i+1}(1-t_i)} = w^i(1-t_i)(1-L)$$

where t_e=consumption tax rate, t_i=income tax rate on wage income and saving income, T_j^i=lump-sum transfer in period j at time i, and r^i=interest rate at time i, ω^i=wage income at time i, for $i=1, 2, \ldots, j=1, 2$. Wage rates and interest rates are assumed to be constant over time. The individual is rational and has perfect foresight about the future. Even though the individual attains utility from public goods $\overline{G}$ provided by the government, $\overline{G}$ is fixed throughout the analysis.

2. The Government

The government is assumed to balance its budget by collecting taxes to provide a fixed amount of public goods, $\overline{G}$, per worker in each period. At time i the government collects consumption tax $t_e C_1^i$, income tax $\omega(1-t_i)(1-L^i)$, and pays lump-sum transfer T_1^i per head from generation i and collects consumption tax $\frac{t_e C_2^i}{1+n}$, income tax $\frac{t_i r S^i}{1+n}$, and pays lump-sum transfer $\frac{T_2^i}{1+n}$ per head from generation $i-1$, where S denotes saving. So the government budget constraint is

$$t_i w(1-L^i) + \frac{t_i r S^i}{1+n} + t_e(C_1^i + \frac{C_2^i}{1+n}) - T_1^i - \frac{T_2^i}{1+n} = \bar{G}$$

where $i=1, 2, \ldots$ and S^i is the savings of generation $i-1$; that is

$$S^i = w(1-L^{i-1})(1-t_i) - (1+t_e)C_1^{i-1} + T_1^{i-1}$$

Notice that all consumption, leisure, and saving levels for every generation are optimal. Individuals have perfect foresight about government changes in tax policy, so they respond optimally to any new policy. At time i each consumer, regardless of generation, is faced with the same tax rates as well as the same lump-sum transfers, so

$$T_1^i = T_2^i$$

for all i. We assume that the economy is in a long-run equilibrium at the time of tax policy change ; thus

$$T_1^i = T_1^{i-1}$$
$$T_2^i = T_2^{i-1}$$

for all i.

3. Equilibrium in the Steady State

In a steady state, every generation is faced with the same tax rates and has the same decision on C_1, C_2, and leisure. Thus, in particular, for two adjacent generations i and $i+1$, $C_1^i = C_1^{i+1}$, $C_2^i = C_2^{i+1}$, and $L^i = L^{i+1}$, for all i. We assume that the wage rate and the interest rate are constant over time. Samuelson (1958) and Gale (1973) proved that in a two-period overlapping generation model in the context of a pure exchange economy, there are at most two possible equilibria. Either (i) $n=r$ where n is the population growth rate and r is the interest rate or (ii) there is no saving. Since we assume that there is always positive saving from every generation, we would be interested in the $n=r$ steady state equilibrium only, if the Samuelson/Gale result holds for this model as well. We can actually verify that even in the presence of taxes in a steady state equilibrium with $n=r$[4]. Samuelson and

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4) In each period net saving in the economy is zero. Net saving per capita from young generation is $S^y = w(1-t_i)(1-L) - (1+t_e)C_1 + T$, while net saving per capita from old generation is

$$S^o = -\frac{(1+t_e)-T}{1+n} - \frac{t_i r(w(1-t_i)(1-L)-(1+t_e)C_1+T)}{1+n}$$

The sum of both generations' saving gives us the fundamental equilibrium condition $0 = S^y + S^o$ or

(F-1) $S^y + S^o = (w(1-t_i)(1-L) - (1+t_e)C_1 + T)(1 - \frac{t_i r}{1+n}) - \frac{(1+t_e)C_2 - T}{1+n} = 0$

The household budget constraint is

(F-2) $w(1-t_i)(1-L) - (1+t_e)C_1 + T - [\frac{(1+t_e)C_1 - T}{1+r(1-t_i)}] = 0$.

From equations (F-1) and (F-2), it follows that $n=r$.

Gale also showed that the steady state equilibrium is also optimal. This result is also valid in the model with taxes.

Each household maximizes its life-time utility $U(C_1, C_2, L)$ subject to its budget constraint

$$(2) \qquad (1+t_e)C_1 + \frac{1+t_e}{1+r(1-t_i)}C_2 + w(1-t_i)L = w(1-t_i) + T + \frac{T}{1+r(1-t_i)} \, .$$

The first order conditions for household utility maximization are

$$(3) \qquad U_1 - (1+r(1-t_i))\,U_2 = 0$$

$$(4) \qquad w(1-t_i)\,U_1 - (1+t_e)\,U_3 = 0$$

along with the budget constraint, equation (1). The government sets tax rates t_i and t_e such that it can raise a fixed amount of tax revenue $\bar{G}$ per capita, satisfying the government budget constraint

$$(5) \qquad t_e C_1^* + t_e\frac{C_2^*}{1+n} + t_i w(1-L^*) + \frac{t_i r S^*}{1+n} - T - \frac{T}{1+n} = \bar{G}$$

where $S^* = w(1-L^*)-(1+t_e)C^* + T$. Initially $T=0$; in other words, there is no lump-sum transfers to households. When the government increases either t_e or t_i, T adjusts to satisfy the government equal tax revenuė constraint. The equilibrium of the economy in the steady state can be described by equations (2) to (5), which may be used to obtain comparative statics result of changes in C_1^*, C_2^*, and L^* due to tax rate changes $\frac{dC_1^*}{d\tau}$, $\frac{dC_2^*}{d\tau}$, and $\frac{dL^*}{d\tau}$, for $\tau = t_i,\ t_e$.

Ⅲ. WELFARE COMPARSION OF INCOME TAX AND CONSUMPTION TAX

1. Differential Tax Incidence

We have assumed that initially there exist both an income tax and a consumption tax. The government can increase either the income tax or the consumption tax rate, rebating the increased tax revenues through lump-sum transfers to the household. Since an increase in the lump-sum transfer is equivalent to a decrease in the lump-sum tax, we can explore the differential incidence of an income tax by comparing it with an equal revenue lump-sum tax. Similarly we can explore the differential incidence of a consumption tax by comparing it with an equal revenue lump-sum tax. By looking at the respective changes in the economy, we can directly compare an income tax with a consumption tax. The substitution of a consumption tax for an income tax can be considered as a two-step tax substitution, where the first step is to substitute a lump-sum tax for an income tax, and the second step is to substitute a consumption tax for the lump-sum tax.

In our approach, a meaningful comparison of two tax schemes depends on the government's equal tax revenue constraint. For the equal tax revenue constraint to be satisfied, the amounts of the tax rebates from either tax increase should be equivalent. Thus,

$$d(TR_c) = d(TR_i)$$

where $d(TR_c)$ =the change in tax revenues due to an incremental increase in the consumption tax rate, dt_c, and $d(TR_i)$ =the change in tax revenues due to an incremental increase in the income tax rate, dt_i. Assuming differentiability of the demand functions, C_1, C_2, etc.

$$d(TR_c) = \frac{d(TR_c)}{dt_c} dt_c$$

$$d(TR_i) = \frac{d(TR_i)}{dt_i} dt_i$$

The tax revenue change due to a tax rate increase consists of two effects. The primary effect is an increase in tax revenues due to an increase in tax rate given unchanged tax bases. The secondary effect comes from the change in tax bases due to the changes in the relative prices of consumption in both periods and leisure. Define the tax revenue function as follows,

$$TR = t_i w(1-L) + \frac{t_i rS}{1+n} + t_c C_1 + \frac{t_c C_2}{1+n} - T - \frac{T}{1+n}$$

where S is saving from previous generation and $C_1 = C_1(t_i,\ t_c,\ T)$, $L = L(t_i,\ t_c,\ T)$, etc[5]. By taking the total differentials in the tax revenue function, we obtain the change of tax revenues due to an increase in one tax rate. For example, for a consumption tax increase,

$$(6)\quad dTR_c = [\ C\ (1 - \frac{t_i r}{1+n}) + \frac{C_2}{1+n} + (t_c - \frac{t_i r(1+t_c)}{1+n})\frac{\partial C_1}{\partial t_c} + \frac{t_c}{1+n}\frac{\partial C_2}{\partial t_c} - (t_i w + \frac{w(1-t_i)t_i r}{1+n})\frac{\partial L}{\partial t_c}\ ]\, dt_c.$$

While the primary effect is always positive as long as $dt_c > 0$, the secondary effect is undetermined. The consumption tax rate increase lowers the relative price of leisure, providing a greater disincentive to work. On the other hand, a decrease in real income due to the consumption tax rate increase induces less leisure and more labor supply. These two opposing effects can either increase or decrease wage income and its tax base. If a wage income is decreased, then lifetime consumption and its tax base will also be decreased, and vice versa. Thus, the net secondary effect can either mitigate or strengthen the increase in the revenues. If the secondary effect is sufficiently small, then tax revenue is increased by either an increase in the income tax rate or an increase in the consumption tax rate. However if the secondary effect is negative and dominates the primary effect, the overall effect will be negative. Thus it is possible (if unlikely) for tax revenue to be decreased by an increase in either tax rate. Here, we shall assume that when the tax reform occurs the tax revenue function is increasing in its all arguments, thus ruling out any possible inverse relationship between any tax rate and revenue. Either the wage income tax rate increase or the capital income tax rate increase will increase tax revenue. So $\frac{dTR}{dt_c}$,

5) All subscripts are suppressed, since the economy is in the long-run equilibrium at the time of tax reform.

$\frac{dTR}{dt_i} > 0$. Also $\frac{dTR}{dT} < 0$, since T denotes a lump-sum transfer (a negative lump-sum tax). In the case of an income tax rate increase, the change in tax revenue is,

$$(7)\quad dTR_i = [\, w(1-L)(1-\frac{t_i r}{1+n}) + \frac{rS}{1+n} + (t_e - \frac{t_i r(1+t_e)}{1+n})\frac{\partial C_1}{\partial t_i} + \frac{t_e}{1+n}\frac{\partial C_2}{\partial t_i} - (t_i w + \frac{w(1-t_i)t_i r}{1+n})\frac{\partial L}{\partial t_i}\,]\, dt_i .$$

Depending on the magnitude of $\frac{dTR_e}{dt_e}$ and $\frac{dTR_i}{dt_i}$, the required tax rate changes dt_e and dt_i are determined by the equal tax revenue constraint. Since the income tax base is larger than the consumption tax base, the consumption tax rate will have to be greater than the income tax rate if they are to raise the same amount of revenue. This holds for changes as well, that is $\frac{dTR_i}{dt_i} > \frac{dTR_e}{dt_e}$. Thus, for an equal revenue tax change, the increase in the consumption tax rate would have to be larger than the increase in the income tax rate.

2. Welfare Comparison

In comparing two states of the economy in terms of welfare, we have bascially two scales for measuring welfare. The first is a direct one that makes use of the cardinality of the assumed utility function of the model. A second approach would be to use money income as a scale as measured by either the compensating variation or equivalent variation. Assuming the differentiability of the utility function, we adopt the first approach.

Since tax rate increases change the prices of the goods - C_1, C_2, and L, in general it is necessary to compare the welfare changes due to the price changes as well as changes in income. Consider the indirect utility function $V(P_1, P_2, P_3, M)$, where P_1=the price of first period consumption, P_2=the price of second period consumption, P_3=the price of leisure or the wage rate, M=full income. When the prices are changed, the change of utility is

$$dV = V_1 dP_1 + V_2 dP_2 + V_3 dP_3 + V_4 dM$$

where V_i is the partial derivative with respect to ith argument in the indirect utility function, dP_i is the change in the price of the ith good, dM is the change in full income.

From Roy's identity,

$$(8)\qquad dV = \alpha\,(C_1^* dP_1 + C_2^* dP_2 + L^* dP_3 + dM)$$

where α=the marginal utility of income. By taking total differentials from the household budget constraint, $P_1 C_1^* + P_2 C_2^* + P_3 L^* = M$ we may obtain

$$(9)\qquad dP_1 C_1^* + dP_2 C_2^* + dP_3 L^* + P_1 dC_1^* + P_2 dC_2^* + P_3 dL^* = dM$$

Thus the change in utility due to the price changes follows from equation (8) and equation (9),

$$(10)\qquad dV = \alpha\,(dC_1^* P_1 + dC_2^* P_2 + dL^* P_3)$$

So when the government raises one tax rate, changing the prices of goods, the change in utility of the household can be directly measured by measuring the changes of consumptions and leisure.

Ⅳ. RESULTS

1. In this section, we examine the welfare changes due to the tax rate increases. We carefully examine the reasoning behind the alleged claim[6] that a consumption tax is superior to an income tax since the former distorts the labor market only. Recall that $dV_e = \frac{dV_e}{dt_e} dt_e$ and $dV_i = \frac{dV_i}{dt_i} dt_i$, where

$$\frac{dV_\tau}{d\tau} = \alpha \left[\sum_{k=1}^{3} \frac{dX_k^*}{d\tau} P_k \right]$$

where $\tau = t_i$, t_c, $X_1 = C_1$, $X_2 = C_2$, $X_3 = L$, $P_1 = 1 + t_c$, $P_2 = \frac{1+t_e}{1+r(1-t_i)}$, and $P_3 = \omega(1 - t_i)$.
Suppose the consumption tax rate t_c increases. There will be a direct response of C^* and L^* to its increase in t_c through a change in the relative price of C^* and L^*, which we call *the direct effect* of the tax rate increase. There is also an increase in lump-sum transfers which follow the increase in t_c to compensate for the extra tax payments. We call this *the indirect effect*. For example,

$$\frac{dC_1^*}{dt_e} = \frac{\partial C_1^*}{\partial t_e} + \frac{\partial C_1^*}{\partial T} \frac{dT}{dt_e}$$

where the first term in the right hand side is the direct effect and the second term is the indirect effect. This relationship is also valid for C_2^* and L^*. Therefore we have

$$(11) \quad \frac{dX_k^*}{d\tau} = \left(\frac{\partial X_k^*}{\partial \tau} + \frac{\partial X_k^*}{\partial T} \frac{dT}{d\tau} \right)$$

where $\tau = t_c$, t_i, and $k = 1, 2, 3$.

When t_c is increased, both P_1 and P_2 increase. Thus, $\frac{\partial X_k^*}{\partial t_e} = \frac{\partial X_k^*}{\partial P_1} + \frac{\partial X_k^*}{\partial P_2} \frac{1}{\delta}$, where $\delta = 1 + r(1 - t_i)$. Since

$$(+) \quad \sum_{k=1}^{3} \frac{\partial X_k^*}{\partial P_j} P_k = -X_j^*$$

for $j = 1, 2, 3$, welfare loss from the direct effect for a consumption tax is

$$(12) \quad \sum_{k=1}^{3} \frac{\partial X_k^*}{\partial t_e} P_k = -C_1^* - \frac{C_2^*}{\delta}.$$

When t_i is increased, P_3 decreases, while P_2 increases, i. e., $\frac{\partial X_k^*}{\partial t_i} = \left(\frac{\partial X_k^*}{\partial P_3} + \frac{\partial X_k^*}{\partial M} \right)(-w) + \frac{\partial X_k^*}{\partial P_2} \frac{(1+t_e)r}{\delta^2}$. Since $\sum_{k=1}^{3} \frac{\partial X_k^*}{\partial M} = 1$, and (+) holds, the welfare loss from the direct effect for an income tax is

6) Obviously, when it comes to the amount of distortions due to taxation the number of distorted markets does not matter.

(13) $\sum_{k=1}^{3} \frac{\partial X_k^*}{\partial t_i} P_k = -w(1-L^*) - \frac{rS^*}{\delta}$.

The indirect effect (the tax rebate effect) is decomposed into two parts : the primary effect and the secondary effect. It can be also decomposed into the income effect and the tax substitution effect. Following the latter decomposition, we can show that the initial welfare loss from the direct effect is exactly offset by the positive income effect through the tax rebate.

The indirect effect for a consumption tax is (see Appendix A for derivation)

(14) $\sum_{k=1}^{3} \left(\frac{\partial X_k^*}{\partial t_i} \frac{dT}{dt_e}\right) P_k = C_1^* + \frac{C_2^*}{\delta} + \phi_e$,

where ϕ_e represents the substitution effect from a decrease in the relative price of leisure due to the consumption tax rate increase, viz.

(15) $\phi_e = \left[\sum_{j=1}^{3} k_j \left(\frac{\partial X_k^*}{\partial P_3}\Big|_u\right) \frac{w(1-t_i)}{1+t_e}\right] \frac{1}{\theta}$,

where $k_1 = t_e - \frac{t_i r(1+t_e)}{1+n}$, $k_2 = \frac{t_e}{1+n}$, $k_3 = -\left(t_i w + \frac{w(1-t_i)t_i r}{1+n}\right)$, and $\theta = \frac{\partial TR}{\partial T} \frac{\delta}{\delta+1} < 0$

The k_i's for $i=1, 2, 3$ are the tax coefficients of the secondary effect on tax revenues. When the substitution effect of the consumption tax rate increase on tax revenue is negative, i. e., $\phi_e < 0$, since a decrease in tax revenue means less lump-sum transfer. Using the fact that $\sum_{j=1}^{3} \frac{\partial X_k^*}{\partial P_j}\Big|_u P_j = 0$, we can rewrite equation (15) as follows,

(16) $\phi_e = (\rho_1 S_{33} + \rho_2 S_{23}) \frac{w(1-t_i)}{1+t_e} \frac{1}{\theta}$,

where $S_{ij} = \frac{\partial X_i^*}{\partial P_j}\Big|_u$ denotes the Slutsky compensated demand, $\rho_1 = -w\left(\frac{t_i+t_e}{1+t_e}\right)$, and $\rho_2 = \frac{t_e r}{(1+r)\delta}$. Later we will discuss the sign of ϕ_e. Since the direct effect is exactly offset by the positive tax rebate effect, ϕ_e also represents the welfare change due to consumption tax rate increase, $\frac{dV_e}{dt_e} = \phi_e$. The welfare change simply comes from the labor market distortions due to a decrease in relative price of leisure.

The indirect effect for an income tax is

(17) $\sum_{k=1}^{3} \left(\frac{\partial X_k^*}{\partial T} \frac{dT}{dt_i}\right) P_k = -w(1-L^*) + \frac{rS^*}{\delta} + \phi_i$,

where ϕ_i represents the substitution effect due to the income tax rate increase. Sinse an increase in t_i implies a decrease in P_3 and an increase in P_2,

(18) $\phi_i = \left[\sum_{j=1}^{3} k_j \left(\frac{\partial X_j^*}{\partial P_3}\Big|_u\right) w\right] \frac{1}{\theta} - \left[\sum_{j=1}^{3} k_j \left(\frac{\partial X_j^*}{\partial P_2}\Big|_u\right) \frac{r(1+t_e)}{\delta^2}\right] \frac{1}{\theta}$.

Since the direct effect is exactly offset by the positive tax rebate effect, ϕ_i represents the welfare change due to the income tax rate increase, i. e., $\frac{dV_i}{dt_i} = \phi_i$. Notice that the first term in equation (18) can be represented by $\phi_e \left(\frac{1+t_e}{1-t_i}\right)$. Let

(19) $\hat{\phi}_i = [\sum_{j=1}^{3} k_j (\frac{\partial X_j^*}{\partial P_2}|_u) \frac{r(1+t_c)}{\delta^2}] \frac{-1}{\theta}.$

Then

(20) $\phi_i = \phi_c (\frac{1+t_c}{1-t_i}) + \hat{\phi}_i.$

The term $\hat{\phi}_i$ represents the substitution effect from an increase in P_2. It also represents the welfare change due to the capital market distortions. Using the fact (+), we can rewrite equation (19) as

(21) $\hat{\phi}_i = (\rho_1 S_{32} + \rho_2 S_{22}) \frac{r(1+t_c)}{\delta^2} (\frac{-1}{\theta}).$

As expected, both taxes have a common factor in welfare change term, which comes from labor market distortions. On the other hand, only an income tax has $\hat{\phi}_i$ which comes from capital market distrortions.

Now, let us discuss the sign of ϕ_c. Unambiguosly, $S_{33} < 0$, $\rho_1 < 0$, and $\rho_2 > 0$. However S_{23} is not determined by theory. Suppose $S_{23} < 0$ and big enough, then it is quite possible that $\phi_c > 0$. In other words, the consumption tax rate increase which is followed by a tax rebate can improve the welfare of the economy. This is not especially surprising, since when there initially exist distorting taxes, at the time of tax reform a new tax may remedy some of the existing distorions in price mechanism. Similarly $\hat{\phi}_i$ can be also positive when $S_{23} < 0$ and big enough.

In that case, $0 < \frac{dV_c}{dt_c} < \frac{dV_i}{dt_i}$

Many economists[7] have claimed that a consumption tax is welfare-superior to an income tax, since a consumption tax distorts only the labor market, while an income tax distorts the capital market as well as the labor market. Notice that the reasoning behind this claim may not hold in the above case, since $\hat{\phi}_i$ may be positive, i. e., the income tax rate increase can reduce the welfare loss from the capital market.

When does $S_{23} > 0$ hold? When future consumption is substitute for leisure, $S_{23} > 0$. In our three-good economy, we may find a sufficient condition for $S_{23} > 0$, namely Condition A : present consumption is more complementary with leisure than future consumption. Condition A simply means that workers work harder for more saving. When condition A is satisfied, $S_{23} > 0$, thus, $\frac{dV_i}{dt_i} < \frac{dV_c}{dt_c} < 0$, and the alleged claim is valid, given equal tax rate increases.

However, the tax rate for two schemes must be different if they are to yield the equal tax revenues. Under assumption A, one can show that $0 < dt_i < dt_c$, since $\frac{dTR}{dt_c} < \frac{dTR}{dt_i}$ (in Appendix B we prove this claim). Thus, because $dV_c = \frac{dV_c}{dt_c} dt_c$ and $dV_i = \frac{dV_i}{dt_i} dt_i$, either tax can be superior to the other tax in terms of welfare loss.

2. In this section we characterize the factors determining '$dV_c - dV_i$'-the welfare change from the tax reform. In a simulation model we often examine the effect of a pa-

7) See footnote 1.

rameter change on outcomes. For instance, we examine the effect on welfare of the savings elasticity parameter with respect to the interest rate. This section will provide a theoretical basis for such empirical studies and simulation studies. When we say that a consumption tax is more *favorable* than an income tax, we mean the welfare gap between two taxes '$dV_c - dV_i$' is larger. It is, therefore, purely a relative term.

Above we derived

(22) $$dV_c - dV_i = k(\phi_c - dt_c - \phi_i dt_i).$$

The magnitudes of dt_c and dt_i are determined by equal tax revenue constraint (ETRC hereafter) $\frac{dTR}{dt_c}dt_c = \frac{dTR}{dt_i}dt_i$ or $\frac{dT}{dt_c}dt_c = \frac{dT}{dt_i}dt_i$ From the ETRC, we can rewrite equation (22) as

(23) $$dV_c - dV_i = k(\phi_c \frac{dT}{dt_i} - \phi_i \frac{dT}{dt_c}),$$

where $k>0$ is constant. We may derive (see Appendix B) that

(24) $$(\frac{1+\delta}{\delta})\frac{dT}{dt_c} = C_1^* + \frac{C_2^*}{\delta} + \phi_c$$

$$(\frac{1+\delta}{\delta})\frac{dT}{dt_c} = w(1-L^*) + \frac{rS^*}{\delta} + \phi_i.$$

We may obtain from equations (23) and (24)

(25) $$dV_c - dV_i = k[\phi_c(w(1-L^*) + \frac{rS^*}{\delta}) - \hat{\phi}_i(C_1^* + \frac{C_2^*}{\delta})].$$

This equation gives another explanation on the indeterminacy of the superiority of one tax over another one, because $\phi_i < \phi_c < 0$ and $\omega(1 - L^*) + \frac{rS^*}{\delta} > C_1^* + \frac{C_2}{\delta}$. Plugging equation (20) into equation (25), we can derive

(26) $$dV_c - dV_i = k[\phi_c \frac{rS^*}{\delta} - \hat{\phi}_i(C_1^* + \frac{C_2^*}{\delta})].$$

Suppose we compare two economies with the same level of tatal outputs with different rates of time preference. For the economy with lower rate of time preference at the time of tax reform an income tax may be favorable than a consumption tax.

As the rate of time preference is lower for the economy, the optimal saving level S^* becomes larger. Since $\phi_c < 0$, $dV_c - dV_i$ decreases as S^* becomes larger, which can be seen in equation (26). On the other hand, as the rate of time preference is lower $\hat{S}_{22}$ becomes larger, so does $\hat{\phi}_i$. Thus, $dV_c - dV_i$ increases. There are two opposing effects of lower time preference rate on $dV_c - dV_i$. If the former effect dominates the latter effect, we have a counter-intuitive result that an income tax may be favorable than a consumption tax for an economy with the lower rate of time preference.

Now, plugging the expressions for ϕ_c and $\hat{\phi}_i$, from equations (16) and (21), into equation (26), we can obtain

(TR) $$dV_c - dV_i = k'[(t_i + t_c)(\sigma_{11} - \sigma_{21}) + \frac{t_i r(1-t_i)}{1+r}(\sigma_{22} - \sigma_{12})],$$

where $k' = -\frac{kr}{(1+t_c)^2} w(1-L^*)S^* < 0$. σ_{LL} and σ_{L2} denote the compensated elasticities of labor supply with respect to wage and P_2 respectively, and σ_{22} and σ_{2L} denote the compensated elasticities of future consumption with respect to P_2 and wage. From condition (TR) we may notice a couple of crucial propositions for tax reform.

As the own price elasticity of future consumption becomes higher, a consumption tax is more favorable than an income tax. The simple reason is that the higher the elasticity, the larger are the distortions in the capital market. Actually this proposition have been supported by most of simulation studies.

As the compensated elasticity of labor supply with respect to wage becomes larger, an income tax is more favorable than a consumption tax. Both taxes lower the relative price of leisure, inducing less labor supply due to the substitution effect. Given equal tax rate increases, an income tax lowers the price of leisure more than a consumption tax and lowers welfare more. However, the ETRC implies that $dt_c > dt_i$. Thus there are two opposing effects. It turns out that the latter effect simply dominates the former effect so that the proposition holds.

Condition (TR) is critical for tax reform, which is corresponding to condition (TD) for tax design shown on page 2. Unlike condition (TD), condition (TR) clearly tells us which direction the government should head in tax reform, given any economic environments represented by the initial tax rates, and various compensated elasticities, etc. Condition (TR) implies that even if $\sigma_{LL} = \sigma_{2L}$ holds, the consumption tax reform may not be superior to the income tax reform, while condition (TD) implies that if $\sigma_{LL} = \sigma_{2L}$ a consumption tax design is always better than an income tax design. Therefore, even given equivalent economic environments we may have different conclusions on which tax scheme is better depending on which problem we are concerned about betweeen tax reform and tax design.

3. Examples

In this section, we give some examples in which condition (TR) is very usefully used.

Example 1

The Cobb-Douglas utility function: $U = a_1 \log C_1 + a_2 \log C_2 + a_3 \log L$, with $a_1 + a_2 + a_3 = 1$. One may easily show that $\sigma_{LL} = \sigma_{L2}$ and $\sigma_{22} < \sigma_{L2}$ for all values of a_1, a_2, and a_3 such that $a_1 + a_2 + a_3 = 1$. From condition (TR), notice that regardless of parameter values, a_1, a_2, a_3 and t_i, t_c, r, $dV_c - dV_i > 0$. This example illustrates that adopting the Cobb-Douglas utility function as a specification of household preference biases the conclusion in favor of a consumption tax[8]. Once the C-D utility function is chosen, either estimating the parameters of utility function or simulation studies always lead us to the conclusion. In this case, any kind of sensitivity anaylsis is meaningless.

8) Stiglitz also pointed out this in his comment on Auerbach and Kotlikoff (1983).

Similarly, the nested C. E. S. utility function : $[C_1^\rho + C_2^\rho]^{\frac{\alpha}{\rho}} \cdot L^{1-\alpha}$ also biases the conclusion in favor of a consumption tax. For $\sigma_{LL}=\sigma_{L2}$ and $\sigma_{22}-\sigma_{L2} = \frac{(s-1)P_1'}{P_1'+P_2'} < 0$, where $s=\frac{\rho}{\rho-1}<1$

Example 2

We examine a proposal of tax reform namely the substitution of a consumption tax for an income tax in the U. S.. First, we need to describe the current U. S. tax system as a combination of a proportional income tax and consumption tax schemes. The feature of a consumption tax in the U. S. is provided by the general sales taxes of the various states. However, since the sales tax rates differ among the states, we use the representative tax rate of 6%, as computed by the ACIR (Advisory Commision on Intergovernmental Relations). Thus $t_c=.06$ in our model. Although it is very difficult to represent, (i. e. pre-1987 tax reform) the current progressive income tax scheme as a proportional tax, we use a representative tax rate of 20% for an income tax, which is computed by the ACIR. We assume that $r=.02$.

We use the evidence on the relevant elasticities that have been derived from macro data by Boskin and Lau (1978). Their estimates of the following uncompensated elasticities, denoted by n_{ij} are, $n_{LL}=-.08$, $n_{22}=-1.49$, $n_{L2}=-.08$, and $n_{2L}=1.11$. In their model, the wealth elasticites of all goods, C_1, C_2, and leisure are 1. Assuming that the budget share of labor hours is .5 and the saving ratio is 6%, we can compute the compensated elasticities as follows, $\sigma_{LL}=.42$, $\sigma_{22}=-1.46$, $\sigma_{L2}=-.22$, and $\sigma_{2L}=.61$. From these figures, we can conclude that $dV_c-dV_t>0$, i. e., a tax reform toward a consumption tax will improve the walfare of the U. S.. From condition (TR), we may notice that $.26=(t_i+t_c)> \frac{t_r r(1-t_i)}{1+r} =.003$, i. e., the tax coefficient attached to $\sigma_{LL}-\sigma_{2L}$ is almost 90 times as great as that attached to and σ_{22}-σ_{L2}. Thus the estimates of σ_{LL} and σ_{2L} are the key parameters for determining which tax substitution is better ; most economists have focussed attention the estimate of σ_{22}.[9]

On the other hand, evidence from a micro, panel data study (Kim, 1986) estimated $\sigma_{LL}=.59$ and $\sigma_{2L}=-.55$, which implies that the suggested tax reform would result in a welfare loss. Thus, we cannot conclude with certainty on the issue based on the empirical evidence.

Example 3 (Incremental Increases in Consumption Tax Treatment of Saving).

How would a decrease in the tax rate on saving that is replaced by an increase in the consumption tax rate affect the welfare of the current U. S? For this problem, the tax rate on savings differ from that on wage income. We can derive a final equation, corresponding to equation (TR). That is

9) The evidence on the interest rate elasticity of savings has been obtained from time series of aggregate savings on consumption; see Stone (1964), Blinder (1975), Boskin and Lau (1978), and Howery and Hymans (1980).

$$(27) \qquad dV_c - dV_i = \hat{k}\left[(t_i + t_c)(\sigma_{LL} - \sigma_{2L}) + \frac{t_r r(1-t_i)}{1+r}(\sigma_{22} - \sigma_{L2})\right],$$

where t_i denotes the wage income tax rate and t_r denotes the tax rate on savings. For $0 \leqq t_r \leqq t_i$, equation (27) leads us to a similar conclusion to Example 2 i. e., since $t_i + t_c > \frac{t_r r(1-t_i)}{1+r}$, what really matters is the estimate of σ_{LL}-σ_{2L}. Therefore, the evidence from the macro data implies that such a proposed tax reform would improve the walfare in the U. S., while the evidence from the micro data implies the opposite.

V. CONCLUSION

In this paper, in the context of tax refrom we have adopted the differential tax incidence approach for the welfare comparison of an income tax and a consumption tax. We ask which tax scheme lowers welfare least while yielding an equal amount of tax revenue. Given equal tax rate increases, a consumption tax will lower welfare of the economy less than an income tax. However, if both taxes must raise the same amount of tax revenue, the consumption tax rate will have to be greater than the income tax rate increase, since the consumption tax base is smaller than the income tax base. Therefore in terms of total welfare loss, we showed that either tax scheme can be superior. We examined the factors determining the direction of welfare loss due to taxation. We found that as the compensated elasticity of saving with respect to the interest rate becomes higher, a consumption tax is more favorable than an income tax. On the other hand, as the compensated elasticity of labor supply with respect to wage becomes higher, an income tax is more favorable than a consumption tax. Surprisingly, for an economy with a larger saving due to a low time preference rate at the time of tax reform, an income tax may be more favorable than a consumption tax.

We derived a very simple equation for welfare changes due to a tax reform-a substitution of a consumption tax for an income tax. The condition, corresponding to the optimal taxation condition in the context of a tax design, tells whether a tax reform is beneficial, given economic environments. We illustrated several examples of tax proposals. Simulation model builders must be careful about a specification of household preferences, since a certain group of utility function intrinsically favors a consumption tax over an income tax. For the proposed tax reform in U. S., the evidence from the macro data implies that the proposed one will improve the welfare of the U. S., while the evidence from the micro data implies the opposite. Also, our analytic model suggests a solution to current debates over incremental increases in consumption tax treatment of savings.

Appendix A

Given equal tax rate changes for both taxes, $dt_c = dt_i$, the comparison of dV_c and dV_i is equivalent to that of $\frac{dV_c}{dt_c}$ and $\frac{dV_i}{dt_i}$.

$$\frac{dV_c}{dt_c} = \alpha[(1+t_c)\frac{dC_1^*}{dt_c} + \frac{1+t_c}{1+r(1-t_i)}\frac{dC_2^*}{dt_c} + w(1-t_i)\frac{dL^*}{dt_c}]$$

$$= \alpha[(1+t_c)(\frac{\partial C_1^*}{\partial t_c} + \frac{\partial C_1^*}{\partial T}\frac{dT}{dt_c}) + \frac{1+t_c}{1+r(1-t_i)}(\frac{\partial C_2^*}{\partial t_c} + \frac{\partial C_2^*}{\partial T}\frac{dT}{dt_c})]$$

$$+ \alpha[w(1-t_i)(\frac{\partial L^*}{\partial t_c} + \frac{\partial L^*}{\partial T}\frac{dT}{dt_c})]$$

$$= \alpha[(1+t_c)\frac{\partial C_1^*}{\partial t_c} + \frac{1+t_c}{1+r(1-t_i)}\frac{\partial C_2^*}{\partial t_c} + w(1-t_i)\frac{dL^*}{dt_c}]$$

$$+ \alpha[((1+t_c)\frac{\partial C_1^*}{\partial T} + \frac{1+t_c}{1+r(1-t_i)}\frac{\partial C_2^*}{\partial T} + w(1-t_i)\frac{\partial L^*}{\partial T})\frac{dT}{dt_c}]$$

Using the fact that $\frac{\partial X_k}{\partial t_c} = \frac{\partial X_k}{\partial P_1} + \frac{\partial X_k}{\partial P_2}\frac{1}{1+r(1-t_i)}$ and $\sum_{k=1}^{3}\frac{\partial X_k^*}{\partial P_j}P_k = -X_j^*$ for $j, k = 1, 2, 3$, and $X_1 = C_1$, $X_2 = C_2$, and $X_3 = L$, we can derive

$$\frac{dV_c}{dt_c} = \alpha[-C_1^* - \frac{C_2^*}{1+r(1-t_i)} + ((1+t_c)\frac{\partial C_1^*}{\partial T} + \frac{1+t_c}{1+r(1-t_i)}\frac{\partial C_2^*}{\partial T} + w(1-t_i)\frac{\partial L^*}{\partial T})\frac{dT}{dt_c}].$$

Since $\frac{\partial X_k^*}{\partial T} = \frac{\partial X_k^*}{\partial M}\frac{\partial M}{\partial T} = \frac{\partial X_k^*}{\partial M}(1+\frac{1}{1+r(1-t_i)})$ and $\sum_{k=1}^{3}P_k\frac{\partial X_k^*}{\partial M} = 1$ where M = full income = $w(1-t_i)+T+T\frac{1}{1+r(1-t_i)}$

$$\text{(A-1)} \quad \frac{dV_c}{dt_c} = \alpha[-C_1^* - C_2^*\frac{1}{1+r(1-t_i)} + (1+\frac{1}{1+r(1-t_i)})\frac{dT}{dt_c}].$$

Similarly we can derive

$$\text{(A-2)} \quad \frac{dV_i}{dt_i} = \alpha[-w(1-L^*) - \frac{rS^*}{1+r(1-t_i)} + (1+\frac{1}{1+r(1-t_i)})\frac{dT}{dt_i}].$$

Without taking into account the tax rebate effect,

$$-w(1-L^*) - \frac{rS^*}{1+r(1-t_i)} < -C_1^* - \frac{-C_2^*}{1+r(1-t_i)} < 0$$

However, since the tax rebate effect which offsets the initial welfare loss, when it comes to the net welfare loss, it is ambiguous. In order to determine the net welfare loss, we should plug $\frac{dT}{dt_c}$ and $\frac{dT}{dt_i}$ into equation (A-1) and equation (A-2) respectively. Making the tax revenue constraint, $TR(t_i, t_c, T) = \bar{G}$, into implicit functional form, $TR(t_i, t_c, T) - \bar{G} = 0$, we can use the Implicit function Theorem to obtain

$\frac{dT}{dt_c}$ and $\frac{dT}{dt_i}$.

$$\frac{dT}{d\tau}=-\frac{\partial TR/\partial \tau}{\partial TR/\partial T}$$

for $\tau = t_c$ and t_i.

Recall the tax revenue function

$$TR(t_c,t_i,T)=t_i w(1-L^*)+\frac{t_i r S^*}{1+n}+t_c C_1^*+\frac{t_c C_2^*}{1+n}-T-\frac{T}{1+n}$$

where $S^* = w(1-t_i)(1-L^*)-(1+t_c)C_1^*+T$, $C_1^* = C_1^*(t_c, t_i, T)$, $C_2^* = C_2^*(t_c, t_i, T)$, and $L^* = L^*(t_c, t_i, T)$.

For a consumption tax

$$\text{(A-3)}\quad \frac{\partial TR}{\partial t_c} = C_1^*(1-\frac{t_i r}{1+n})+\frac{C_2^*}{1+n}$$

$$+(t_c-\frac{t_i r(1+t_c)}{1+n})\frac{\partial C_1^*}{\partial t_c}+\frac{t_c}{1+n}\frac{\partial C_2^*}{\partial t_c}-(t_i w+\frac{w(1-t_i r)t_i r}{1+n})\frac{\partial L^*}{\partial t_c}.$$

For an income tax

$$\text{(A-4)}\quad \frac{\partial TR}{\partial t_i} = w(1-L^*)(1-\frac{t_i r}{1+n})+\frac{rS^*}{1+n}$$

$$+(t_c-\frac{t_i r(1+t_c)}{1+n})\frac{\partial C_1^*}{\partial t_i}+\frac{t_c}{1+n}\frac{\partial C_2^*}{\partial t_i}-(t_i w+\frac{w(1-t_i)t_i r}{1+n})\frac{\partial L^*}{\partial t_i}.$$

For a lump-sum transfer

$$\frac{\partial TR}{\partial T} = -(1-\frac{t_i r}{1+n})-\frac{1}{1+n}$$

$$\text{(A-5)}\quad +(t_c-\frac{t_i r(1+t_c)}{1+n})\frac{\partial C_1^*}{\partial T}+\frac{t_c}{1+n}\frac{\partial C_2^*}{\partial T}-(t_i w+\frac{w(1-t_i r)t_i r}{1+n})\frac{\partial L^*}{\partial T}.$$

Let the coefficients of the secondary effect terms be denoted as follow, $k_1 = t_c - \frac{t_i r(1+t_c)}{1+n}$, $k_2 = \frac{t_c}{1+n}$, and $k_2 = -(t_i w+\frac{w(1-t_i r)}{1+n})$.

Decompose $\frac{\partial TR}{\partial \tau}$ into the income effect and the substitution effect for $\tau = t_c$, t_i, and T. Let the Slutsky term be denoted by '$|_u$'. Since $P_1 = 1+t_c$, $P_2 = \frac{1+t_c}{\delta}$ where $\sigma = \frac{1}{1+r(1-t_i)}$, for $k=1, 2, 3$

$$\frac{\partial X_k^*}{\partial t_c} = \frac{\partial X_k^*}{\partial P_1}+\frac{\partial X_k^*}{\partial P_2}\frac{1}{\delta}$$

Thus we can rewrite (A-3) as

$$\frac{\partial TR}{\partial t_c} = C_1^*(1-\frac{t_i r}{1+n})+\frac{C_2^*}{1+n}+k_1(\frac{\partial C_1^*}{\partial P_1}+\frac{\partial C_2^*}{\partial P_2}\frac{1}{\delta})+k_2(\frac{\partial C_2^*}{\partial P_1}+\frac{\partial C_2^*}{\partial P_2}\frac{1}{\delta})+k_3(\frac{\partial C^*}{\partial P_1}+\frac{\partial L^*}{\partial P_2}\frac{1}{\delta})$$

$$= C_1^*(\frac{\delta}{1+n})+\frac{C_2^*}{1+n}+\sum_{j=1}^{3}k_j(\frac{\partial X_j^*}{\partial P_1}+\frac{\partial X_j^*}{\partial P_2}\frac{1}{\delta})$$

using the fact that $n=r$ in the equilibrium,

$$= \frac{\delta}{1+n}(C_1^*+\frac{C_2^*}{\delta})+\sum_{j=1}^{3}k_j[(\frac{\partial X_j^*}{\partial P_1}|_u - C_1^*\frac{\partial X_j^*}{\partial M})+(\frac{\partial X_j^*}{\partial P_2}|_u - C_2^*\frac{\partial X_j^*}{\partial M})\frac{1}{\delta}]$$

$$= \frac{\delta}{1+n}(C_1^*+\frac{C_2^*}{\delta})+\sum_{j=1}^{3}k_j[(\frac{\partial X_j^*}{\partial P_2}|_u)(-\frac{w(1-t_i)}{1+t_e})+\sum_{j=1}^{3}k_j\frac{\partial X_j^*}{\partial M}(-C_1^*-\frac{C_2^*}{\delta})$$

because $\sum_{k=1}^{3}(\frac{\partial X_j^*}{\partial P_k}|_u)P_k = 0$ implies $\frac{\partial X_j^*}{\partial P_1}|_u + \frac{\partial X_j^*}{\partial P_2}|_u\frac{1}{\delta} = (\frac{\partial X_j^*}{\partial P_3}|_u)(-\frac{w(1-t_i)}{1+t_e})$.

Thus

$$\text{(A-6)}\quad \frac{\partial TR}{\partial t_e} = \frac{\delta}{1+n}(C_1^*+\frac{C_2^*}{\delta})-\sum_{j=1}^{3}k_j(\frac{\partial X_j^*}{\partial P_3}|_u)\frac{w(1-t_i)}{1+t_e}-\sum_{j=1}^{3}k_j\frac{\partial X_j^*}{\partial M}(C_1^*+\frac{C_2^*}{\delta})$$

$$= (C_1^*+\frac{C_2^*}{\delta})(\frac{\delta}{1+n}-\sum_{j=1}^{3}k_j\frac{\partial X_j^*}{\partial M})-\sum_{j=1}^{3}k_j(\frac{\partial X_j^*}{\partial P_3}|_u)\frac{w(1-t_i)}{1+t_e}$$

where $\frac{\partial TR}{\partial t_e}$ is decomposed into the income effect and the substitution effect.

Similarly we can derive, for an income tax,

$$\text{(A-7)}\quad \frac{\partial TR}{\partial t_i} = \frac{\delta}{1+n}(w(1-L^*)+\frac{rS^*}{\delta})-\sum_{j=1}^{3}k_j[\frac{\partial X_j^*}{\partial M}w(1-L^*)+\frac{\partial X_j^*}{\partial M}\frac{rS^*}{\delta}]$$

$$-\sum_{j=1}^{3}k_j(\frac{\partial X_j^*}{\partial P_3}|_u)w+\sum_{j=1}^{3}k_j(\frac{\partial X_j^*}{\partial P_2}|_u)\frac{r(1+t_e)}{\delta^2}$$

$$= (w(1-L^*)+\frac{rS^*}{\delta})(\frac{\delta}{1+n}-\sum_{j=1}^{3}k_j(\frac{\partial X_j^*}{\partial M}))$$

$$-\sum_{j=1}^{3}k_j(\frac{\partial X_j^*}{\partial P_3}|_u)w+\sum_{j=1}^{3}k_j(\frac{\partial X_j^*}{\partial P_2}|_u)\frac{r(1+t_i)}{\delta^2}$$

Nowe we examine the effect of an increase in lump-sum transfer on tax revenue. Since $M=w(1-t_i)+T+\frac{T}{\delta}$, $\frac{\partial M}{\partial T}=1+\frac{1}{\delta}$. We can derive from the tax revenue function,

$$\text{(A-8)}\quad \frac{\partial TR}{\partial T} = -(1-\frac{t_i r}{1+n}+\frac{1}{1+n}+\sum_{j=1}^{3}k_j\frac{\partial X_j^*}{\partial T}$$

$$= -\frac{\delta+1}{1+n}+(k_1\frac{\partial C_1^*}{\partial M}+k_2\frac{\partial C_2^*}{\partial M}+k_3\frac{\partial L^*}{\partial M})\frac{\delta+1}{\delta}$$

$$= -\frac{\delta+1}{\delta}(\frac{\delta}{1+n}-\sum_{j=1}^{3}k_j\frac{\partial X_j^*}{\partial M})$$

Let $\theta=\frac{\partial TR}{\partial T}\frac{\delta+1}{\delta}$. Then $\theta<0$, since $\frac{\partial TR}{\partial T}<0$. From (A-6) and (A-8) we may obtain

$$\text{(A-9)}\quad \frac{dT}{dt_e} = -\frac{\partial TR/\partial t_e}{\partial TR/\partial T}$$

$$= -\frac{(C_1^*+\frac{C_2^*}{\delta})(\frac{\delta}{1+n}-\sum_{j=1}^{3}k_j\frac{\partial X_j^*}{\partial M})-\sum_{j=1}^{3}k_j(\frac{\partial X_j^*}{\partial P_3}|_u)\frac{w(1-t_i)}{1+t_e}}{-\frac{\delta+1}{\delta}(\frac{\delta}{1+n}-\sum_{j=1}^{3}k_j\frac{\partial X_j^*}{\partial M})}$$

$$= -\left(C_1^* + \frac{C_2^*}{\delta}\right)\frac{\delta}{\delta+1} - \frac{\sum_{j=1}^{3} k_j\left(\frac{\partial X_j^*}{\partial P_3}\Big|_u\right)\frac{w(1-t_i)}{1+t_e}}{\frac{\delta}{1+n} - \sum_{j=1}^{3} k_j \frac{\partial X_j^*}{\partial M}}\ \frac{\delta}{\delta+1}$$

$$= \left(C_1^* + \frac{C_2^*}{\delta}\right)\frac{\delta}{\delta+1} + \phi_e\left(\frac{\delta}{\delta+1}\right)$$

where we define

(A-10) $$\phi_e = \frac{\sum_{j=1}^{3} k_j\left(\frac{\partial X_j^*}{\partial P_3}\Big|_u\right)\frac{w(1-t_i)}{1+t_e}}{-\frac{\delta}{1+n} + \sum_{j=1}^{3} k_j \frac{\partial X_j^*}{\partial M}}$$

Similarly for an income tax, we can derive

(A-11) $$\frac{dT}{dt_i} = -\frac{\partial TR/\partial t_e}{\partial TR/\partial T}$$

$$= -\frac{\left(w(1-L^*) + \frac{rS}{\delta}\right)\left(\frac{\delta}{1+n} - \sum_{j=1}^{3} k_j \frac{\partial X_j^*}{\partial M}\right) - \sum_{j=1}^{3} k_j\left(\frac{\partial X_j^*}{\partial P_3}\Big|_u\right) w + \sum_{j=1}^{3} k_j\left(\frac{\partial X_j^*}{\partial P_2}\Big|_u\right)\frac{r(1+t_e)}{\delta^2}}{-\frac{\delta+1}{\delta}\left(\frac{\delta}{1+n} - \sum_{j=1}^{3} \frac{\partial X_j^*}{\partial M}\right)}$$

$$= \left(w(1-L^*) + \frac{rS}{\delta}\right) + \frac{\sum_{j=1}^{3} k_j\left(\frac{\partial X_j^*}{\partial P_3}\Big|_u\right) w}{\theta}\Bigg)\left(\frac{\delta+1}{\delta}\right) - \frac{\sum_{j=1}^{3} k_j\left(\frac{\partial X_j^*}{\partial P_2}\Big|_u\right)\frac{r(1+t_e)}{\delta^2}}{\theta}\left(\frac{\delta}{\delta+1}\right)$$

$$= \left(w(1-L^*) + \frac{rS^*}{\delta}\right)\left(\frac{\delta}{\delta+1}\right) + \phi_e\left(\frac{1+t_e}{1+t_i}\right) + \hat{\phi}_i\left(\frac{\delta}{\delta+1}\right)$$

where

(A-12) $$\hat{\phi}_i = \frac{-1}{\theta}\sum_{j=1}^{3} k_j\left(\frac{\partial X_j^*}{\partial P_2}\Big|_u\right)\frac{r(1+t_e)}{\delta^2}$$

Plugging $\frac{dT}{dt_e}$ and $\frac{dT}{dt_i}$ into (A-1) and (A-2) respectively, we can find

(A-13) $$\frac{dV_e}{dt_e} = \alpha\phi_e$$

(A-14) $$\frac{dV_i}{dt_i} = \alpha\left[\phi_e\left(\frac{1+t_e}{1-t_i}\right) + \hat{\phi}_i\right]$$

Since $\hat{\phi}_i < 0$ and $\frac{1+t_e}{1-t_i} > 0$, $\frac{dV_i}{dt_i} < \frac{dV_e}{dt_e} < 0$.

Appendix B

$\frac{dTR}{dt_i} - \frac{dTR}{dt_e} > 0$ if and only if $\frac{dT}{dt_i} - \frac{dT}{dt_e} > 0$ since $\frac{dT}{d\tau} = -\frac{\partial TR/\partial\tau}{\partial TR/\partial T}$ and $\frac{\partial TR}{dT} < 0$ for $\tau = t_i$ and t_e.

Form the proof of proposition 1 we show

$$\left(\frac{\delta+1}{\delta}\right)\frac{dT}{dt_e} = \left(C_1^* + \frac{C_2^*}{\delta}\right) + \phi_e$$

$$\left(\frac{\delta+1}{\delta}\right)\frac{dT}{dt_i} = \left(w(1-L^*) + \frac{rS^*}{\delta}\right) + \phi_e\left(\frac{1+t_e}{1-t_i}\right) + \hat{\phi}_i$$

Therefore

$$(*) \qquad \left(\frac{\delta+1}{\delta}\right)\left(\frac{dT}{dt_i} - \frac{dT}{dt_e}\right) = \left(w(1-L^*) + \frac{rS^*}{\delta}\right) - C_1^* - \frac{C_2^*}{\delta} + \left(\frac{1+t_e}{1-t_i} - 1\right)\phi_e + \hat{\phi}_i$$

Using the fact that $C_1^* + \frac{C_2^*}{\delta} = \frac{1-t_e}{1+t_i} w(1-L^*)$ we can rewrite equation (*) as

$$(*) = \left[\left(1 - \frac{1+t_i}{1-t_e}\right) w(1-L^*) + \frac{rS^*}{\delta}\right] + \left(\frac{1+t_e}{1-t_i} - 1\right)\phi_e + \hat{\phi}_i$$

$$= \frac{(t_e+t_i)}{(1+t_e)(1-t_i)}\left[(1-t_i)w(1-L^*) + (1+t_e)\phi_e\right] + \frac{rS^*}{\delta} + \hat{\phi}_i$$

From the assumptions that tax revenue function is increasing in all tax rate increases, $w(1-L^*) + \left(\frac{1+t_e}{1-t_i}\right)\phi_e > 0$ and $\frac{rS^*}{\delta} + \hat{\phi}_i > 0$ Thus $(*) > 0$.

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