

OIL SHOCKS AND DEMAND FOR OIL :
AN ANALYSIS OF EX POST FACTOR SUBSTITUTION

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ABSTRACT

This study aims to examine the aggregate oil demand function in the seven leading industrial countries : U. S. A., Japan, Canada, France, West Germany, Italy, and the United Kingdom. A model based on a generalized technological assumption, called 'putty-semiputty hypothesis (PSH)', can explain well the differences among countries in dynamic adjustments of factor demands. For each vintage used in the production, the oil-labor ratio turns out to be a weighted average of those obtained from putty-clay and putty-putty technologies.

Among seven OECD countries included in the sample of pooled regressions, Japan has a higher ex post elasticity of substitution than others. This suggests that oil demand in that country has fallen more rapidly since 1973, and thus, *ceteris paribus*, her economy has suffered less in terms of unemployment, inflation, and labor productivity from the supply shocks. The two polar cases, putty-putty and putty-clay hypotheses, which have been widely used in the literature, are both rejected by the data. This model can be also applied to less developed economies to look at divergent paths after the first oil shocks in 1973-74.

I. INTRODUCTION

This study aims to derive and estimate the aggregate oil demand functions for the major industrialized countries based on a generalized technological assumption. It will be called a "putty-semiputty hypothesis (PSH)". It assumes that, once machines are installed, the production factors are less substitutable than ex ante. Clearly, the PSH is more realistic than putty-putty or putty-clay hypothesis. A putty-clay model will predict a gradual response of oil demand to an oil-price increase since a new lower oil-intensity is assumed to be embodied only in the newest vintage of machines. On the other hand, a neoclassical putty-putty model will imply a rapid change in oil demand because new factor intensities are assumed to be embodied in the entire capital stock. The increased volatility of relative factor prices since the 1970s has added more significance to this issue of ex post factor substitution.

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There have been some efforts to explain the apparent sluggish decline in the oil demand in terms of the putty-clay hypothesis. If the data on the output and inputs which belong to each vintage are given, empirical studies could be carried out more easily. However, those data are not generally available, in particular, for the aggregate economy. With some additional assumptions, Nordhaus(1980) succeeded in deriving and estimating an aggregate oil demand function based on the putty-clay hypothesis. There are at least two issues which still need to be investigated further. First, a more conventional approach such as Hudson and Jorgenson(1978), which is based on the putty-putty hypothesis plus ad hoc adjustment mechanisms, can not explain well changes in oil demand since 1973. An alternative approach using the putty-clay hypothesis also seems to be unsatisfactory, because it usually underestimates the decline of oil demand in most countries. Thus, a third approach based on a more general technological assumption is called for. The second problem is that there have been few studies which have actually tested either the putty-putty or putty-clay hypothesis empirically. For example, Mizon(1974) and Malcomson and Prior(1979) assume, but do not test, the putty-clay hypothesis.

The technological assumption of the putty-semiputty hypothesis has been adopted in several studies.¹⁾ Hu(1970) used the ex post production function in his study of the long-run growth. Fuss(1977) derived a testable specification based on the putty-semiputty hypothesis. In the case of an electricity generating industry, he found that the putty-clay hypothesis could not be rejected. Fuss and McFadden(1980) examined the possible tradeoff between efficiency and ex post flexibility in the choice of technologies. Their studies begin with a generalized Leontief cost function, and use duality results. They do not discuss a vintage model. This study will not use the duality relationship or the cost function. Instead, the ex post production function will be directly derived from the putty-semiputty hypothesis. I will also work with a vintage model, which is suitable for a macro study. It is assumed here that either the efficiency or the ex post flexibility can not be chosen by a firm. But, the firm should choose the size and design of the vintage before it is installed.

The paper is organized as follows. Section 2 will set up the model. Section 3 will derive an aggregate specification of per capita oil demand function. Section 4 will report estimation results from individual country data and from pooled data. Section 5 will discuss statistical tests of alternative technological hypotheses. Section 6 concludes.

II. A PUTTY-SEMIPUTTY PRODUCTION MODEL

The model is based on the following assumptions :

(A1) The economy produces one kind of homogeneous output. Output is perfectly malleable in the sense that it can be used for consumption or to accumulate machines of any type.

1) Park(1966) also allowed ex post factor substitution. He assumed that the ex post elasticity was exactly the same as the ex ante one within some range of capital-labor ratio, but that it was zero outside of the range. It seems to be difficult to identify such ranges from the data.

(A2) There are three factors of production : capital, labor and oil.

(A3) Once the machines are installed, the factors are less substitutable than ex ante (putty-semiputty hypothesis). Hence, depending on the time of installation, the production function of each vintage is different from others.

(A4) The ex ante technology takes a form of Cobb-Douglas production function with constant returns to scale. The technological progress is assumed to be Harrod-neutral.

Then, the ex ante production function of the vintage v can be written as :

$$(1) \quad Y(v, v) = a(v) I(v)^\alpha [e^{\lambda v} L(v, v)]^\beta [e^{\mu v} X(v, v)]^\gamma$$

where

$$\alpha + \beta + \gamma = 1$$

$Y(v, v)$ is the ex ante output of the vintage v ,

$I(v)$ is the capital stock of the vintage v ,

$L(v, v)$ is the ex ante labor requirement,

$X(v, v)$ is the ex ante oil requirement,

$A(v)$ is the efficiency parameter of the vintage v , and

and μ are the rates of embodied factor-augmenting technological progress.

A firm can choose any of the machine type satisfying the technology (1). But, once it installs new machines, the available technology from those machines will no longer be the same as (1). Suppose the firm chose the vintage size, $I(v)$, and the vintage design, $L(v, v)$ and $X(v, v)$. Then, when the vintage v is used, later, the actual production will be affected by these ex ante variables. A general form of the ex post production function may be written as :

$$Y(t, v) = f[I(v), L(v, v), X(v, v), L(t, v), X(t, v), t-v],$$

where (t, v) denotes variables of the vintage v at time $t(t > v)$. The way in which the ex ante variables influence $Y(t, v)$ in this equation will depend on the functional form we assume for the ex post production. In this study, it is assumed that the ex post technology takes a C.E.S. form with the elasticity of substitution less than one. specifically,

$$(2) \quad Y(t, v) = B(v) \{a(v)k(t, v)^\rho + b(v)[e^{\lambda v} L(t, v)]^\rho + c(v)[e^{\mu v} X(t, v)]^\rho\}^{-1/\rho}$$

where $0 < \rho < \infty$, and $a + b + c = 1$. Any technological progress up to time v has already been taken into account through the terms $e^{\lambda v}$ and $e^{\mu v}$.

A new development of production technology after time v will be embodied only in the later vintages, not in the older vintages.

In this ex post production function, the parameters a , b , c , and B depend on the ex ante choice of techniques. First, at the tangent point the ex post level of output will be the same as the ex ante capacity since the input vector used is identical with the ex ante design.

$$(3) \quad Y(t, v) = Y(v, v) \quad \text{if } K(t, v) = I(v), \quad L(t, v) = L(v, v) \\ \text{and } X(t, v) = X(v, v).$$

Secondly, the tangency conditions between ex ante and ex post isoquants at the initial factor ratio can be used. In other words, the ex post marginal rate of technical substitution between factors is equal to the ex ante rate at this ratio. These conditions are :

$$(4) \frac{\partial Y(t, v)/\partial K(t, v)}{\partial Y(t, v)/\partial L(t, v)} = \frac{\partial Y(v, v)/\partial I(v)}{\partial Y(v, v)/\partial L(v, v)}$$

$$(5) \frac{\partial Y(t, v)/\partial K(t, v)}{\partial Y(t, v)/\partial X(t, v)} = \frac{\partial Y(v, v)/\partial I(v)}{\partial Y(v, v)/\partial X(v, v)}$$

$$(6) \frac{\partial Y(t, v)/\partial L(t, v)}{\partial Y(t, v)/\partial X(t, v)} = \frac{\partial Y(v, v)/\partial L(v)}{\partial Y(v, v)/\partial X(v, v)}$$

The equalities hold only when they are evaluated at the tangent point. One of the three equations (4)-(6) is redundant. Since $a+b+c=1$, the four ex post parameters in equation (2) can be obtained by using equations (3), (4) and (5). The results are :

$$(7) a(v) = \frac{\alpha I(v)^{\rho}}{\alpha I(v)^{\rho} + \beta [e^{1\nu} L(v, v)]^{\rho} + \gamma [e^{\mu\nu} X(v, v)]^{\rho}}$$

$$(8) b(v) = \frac{\beta [e^{1\nu} L(v, v)]^{\rho}}{\alpha I(v)^{\rho} + \beta [e^{1\nu} L(v, v)]^{\rho} + \gamma [e^{\mu\nu} X(v, v)]^{\rho}}$$

$$(9) c(v) = \frac{\gamma [e^{\mu\nu} X(v, v)]^{\rho}}{\alpha I(v)^{\rho} + \beta [e^{1\nu} L(v, v)]^{\rho} + \gamma [e^{\mu\nu} X(v, v)]^{\rho}}$$

$$(10) B(v) = \frac{A(v) I(v)^{\rho} [e^{1\nu} L(v, v)]^{\rho} [e^{\mu\nu} X(v, v)]^{\rho}}{\{\alpha I(v)^{\rho} + \beta [e^{1\nu} L(v, v)]^{\rho} + \gamma [e^{\mu\nu} X(v, v)]^{\rho}\}^{1/\rho}}$$

$$= \frac{Y(v, v)}{\{\alpha I(v)^{\rho} + \beta [e^{1\nu} L(v, v)]^{\rho} + \gamma [e^{\mu\nu} X(v, v)]^{\rho}\}^{1/\rho}}$$

Substituting these coefficients into the equation (2), we can get the ex post vintage production function explicitly.

$$(11) Y(t, v)$$

$$= Y(v, v) \{\alpha [K(t, v)/I(v)]^{\rho} + \beta [L(t, v)/L(v, v)]^{\rho} + \gamma [X(t, v)/X(v, v)]^{\rho}\}^{-1/\rho}$$

$$= A(v) I(v)^{\rho} [e^{1\nu} L(v, v)]^{\rho} [e^{\mu\nu} X(v, v)]^{\rho}$$

$$\{\alpha [K(t, v)/I(v)]^{\rho} + \beta [L(t, v)/L(v, v)]^{\rho} + \gamma [X(t, v)/X(v, v)]^{\rho}\}^{-1/\rho}$$

In this equation, the remaining capital stock of the vintage v at time t is $K(t, v) = e^{-\delta(t-v)}$

$I(v)$. The actual production using the vintage v capital will be carried out according to (11) over the lifetime of that vintage. A firm holding the vintage v capital at time t ($t > v$) can still choose $L(t, v)$ and $X(t, v)$ to optimize its objective function. This

is the main difference between putty-clay and putty-semiputty technologies.

Consider the properties of this ex post vintage production function. It has two variables, $L(t, v)$ and $X(t, v)$, which can be chosen ex post. All others are given parameters. There are three groups of parameters: i) the parameters of the ex ante production function ($\alpha, \beta, \gamma, A(v), \rho$) ii) the ex post substitution parameter (ρ) and iii) the variables chosen ex ante ($I(v), L(v, v), X(v, v)$). For the first two groups, a firm has no choice. They are given ex ante as well as ex post. However, for the variables in the last group, a firm can choose them ex ante before machines of the vintage v are installed. They are parameters ex post, but variables ex ante. We may call these the design parameters of the vintage. Equation (11) is different from the ordinary short-run production function in that it depends on these design parameters.²⁾

Consider now a representative firm in the economy. It is assumed that all markets are perfectly competitive. Suppose the firm's objective is to maximize its market value. The market value of the firm at time $t, v(t)$, will be the sum of the market value of the existing vintages of capital stock, which it owns already, and the market value of the future investment projects net of investment costs.

$$(12) \quad V(t) = VO(t) + VN(t)$$

To get the market value of the existing vintages, $VO(t)$, we consider first the market value of the capital of a particular vintage v at time t . It will be the present value of the stream of quasi-rents the machines can earn over their remaining lifetime. Suppose $\ell(t, v)$ is the expected service life of a vintage v at time t . This measures the period from the time of installation to the time when the vintage is expected to retire by economic obsolescence. It need not be the same throughout the life of vintage. As expected factor prices change, $\ell(t, v)$ may become shorter or longer. However, we assume that once a vintage is not used, it is permanently retired. Then, the market value of the existing vintage capital at time $t, VO(t, v)$, will be

$$(13) \quad VO(t, v) = E_t \int_0^{\ell(t, v)} e^{-r(s-t)} \{p(s)Y(s, v) - w(s)L(s, v) - x(s)X(s, v)\} ds,$$

where w is the nominal wage rate, x is the price of oil, and p is the price of output. The firm may have perfect foresight or rational expectations on the future prices. The constant discount rate, r , is used here for simplicity. Suppose the oldest vintage used at time t is (t) . Integrating (13) over all the existing vintages from (t) to t , we can obtain the market value of the old capital stock,³⁾

$$(14) \quad VO(t) = \int_{t-\tau(t)}^t VO(t, v) dv$$

2) The cost function can be also derived from the ex post production function (11). It is a function of i) the prices of variable factors, $w(t), x(t)$, ii) the vintage output level $Y(t, v)$ and iii) the vintage size and design, $I(v), L(v, v), X(v, v)$. One of the main characteristics of the ex post cost function will be that the adjustment costs involved in varying factor intensities are already taken into account.

3) The double integration, i.e., the integration over time and the integration over vintage, is necessary, because we are working with a vintage model.

Next, the market value of machines of a forthcoming vintage v ($v \geq t$), which are new or not installed yet at time t , will be the present value of quasi-rents they are expected to earn from the date of installation throughout their lifetime. Hence, the net worth of a new vintage, $VN(t, v)$, after deducting the investment costs will be

$$(15) \quad VN(t, v) = E_t \int_t^{v+t(t, v)} e^{-r(s-t)} \{p(s)Y(s, v) - w(s)L(s, v) - x(s)X(s, v)\} ds \\ - E_t e^{r(s-t)} q(v) I(v),$$

where $q(v)$ is the price of a new machine. To get the portion of the firm's market value which is contributed by all the investment plans, we should integrate (15) over all the future vintages.

$$(16) \quad VN(t) = \int_t^{\infty} VN(t, v) dv$$

The market value of a firm at time t is the sum of (14) and (16). Now, we are ready to specify the optimization problem of a firm with putty-semiputty technologies. With a help of a dummy variable d^4 , the market value of the firm can be written as

$$(17) \quad v(t) = E_t \int_{t-\tau(t)}^{\infty} \int_v^{v+t(t, v)} d(s) e^{r(s-t)} \{p(s)Y(s, v) - w(s)L(s, v) - x(s)X(s, v)\} ds dv \\ - E_t \int_{t-\tau(t)}^{\infty} d(v) e^{r(v-t)} q(v) I(v) dv.$$

The firm's problem is to maximize its market value (17). This is a dynamic optimization problem. There are no equations of motions for the state variables,⁵⁾ $Y(s, v)$ is given in the form of the ex post vintage production function (11). At time t , the firm should choose the factor intensities of new vintages such as $I(v)$, $L(v, v)$, and $X(v, v)$ for all $v \geq t$. It should also choose the factor intensities of existing vintages such as $L(t, v)$ and $X(t, v)$ for all $v < t$, and $s \geq t$. Finally, it should determine the lifetimes of all the future vintages. The decisions on the current investment, $I(t)$, $L(t, t)$, $X(t, t)$ are irreversible, but a future investment plan made at time t can be modified if the expectations are changed later.

The Hamiltonian of (17) is a function of two sets of control variables. First, it is a function of ex post variables of the existing vintages: $L(t, v)$ and $X(t, v)$ for $v \leq t$. Second, it is also a function of variables which belong to the new vintage t : the ex ante variables, $I(t)$, $L(t, t)$, $X(t, t)$, $\ell(t, t)$, and the ex post variables, $L(s, t)$, $X(s, t)$ of the vintage t , where $s \geq t$.

$$(18) \quad H[I(t), L(t, t), X(t, t), \ell(t, t), L(t, v), X(t, v)] \\ = \int_{t-\tau(t)}^t \{p(t)Y(t, v) - w(t)L(t, v) - x(t)X(t, v)\} dv \\ + E_t \int_t^{t+t(t, t)} e^{r(s-t)} \{p(s)Y(s, t) - w(s)L(s, t) - x(s)X(s, t)\} ds - q(t)I(t) \\ = H_1 + H_2$$

4) $d(v) = 1$ for $v \geq t$, $= 0$ for $v < t$. $d(s) = 1$ for $s \geq t$, $= 0$ for $s < t$.

5) $I(v)$, $L(v, v)$, and $X(v, v)$ may be called state variables in this problem. But, these remain constant over the lifetime of the vintage v . $K(t, v)$ may be regarded as another state variable. It depreciates over time as $K(t, v) = \exp(-\delta(t-v)) I(v)$. However, the change in $K(t, v)$ does not depend on control variables.

The first integral in the Hamiltonian represents the present value of the quasi-rents generated by all the existing vintages at time t . The second integral shows the present value of the quasi-rents which will be obtained by the new machines of the vintage t . All the ex post control variables of the existing vintages appear only in the first integral (H_1) and all the variables related to the new vintage appear in the second integral and in the last term (H_2). According to the maximum principle, this Hamiltonian function should be maximized at each instant. At time t , the firm will maximize H_1 by adjusting variable inputs, $L(t, v)$ and $X(t, v)$. It will also decide the ex ante design and new plant size by choosing $I(t)$, $L(t, t)$, and $X(t, t)$ to maximize H_2 . In (17) or (18), the ex ante and ex post decision problems faced by a firm with putty-semiputty technologies are incorporated together simultaneously.⁶⁾ In other words, a firm faces both short-run and long-run problems at the same time. It will maximize short-run profits by picking the right levels of variable inputs for all the existing different plants.⁷⁾ The first-order conditions for this ex post problem can be derived by differentiating (18) with respect to $L(t, v)$ and $X(t, v)$ for all $t - (t) \leq v \leq t$.

$$(19) \quad \frac{\partial H}{\partial L(t, v)} = \frac{\partial p(t)Y(t, v)}{\partial L(t, v)} - w(t) \\ = b(v) B(v)^{-\rho} e^{-\rho \lambda v} \left[\frac{Y(t, v)}{L(t, v)} \right]^{1+\rho} - w(t)/p(t) = 0.$$

$$(20) \quad \frac{\partial H}{\partial X(t, v)} = \frac{\partial p(t)Y(t, v)}{\partial X(t, v)} - x(t) \\ = c(v) B(v)^{-\rho} e^{-\rho \mu v} \left[\frac{Y(t, v)}{X(t, v)} \right]^{1+\rho} - x(t)/p(t) = 0.$$

The interpretation of these equations is straightforward. In order to maximize short-run profits, the firm should equate the vintage marginal product of each variable factor to its real price. And this should hold for any existing vintage at time t . If, for example, the marginal product of labor for the vintage v is smaller than that for v' , then it is always possible to reduce the short-run total variable cost without affecting the output level. The exact nature of the ex post adjustment of factor proportions can be further examined by taking the ratio of two first-order conditions, equations (19) and (20).

$$\frac{(19)}{(20)} = \left[\frac{X(t, v)}{L(t, v)} \right]^{1+\rho} = e^{(\lambda-\mu)\rho v} \frac{w(t) c(v)}{x(t) b(v)}$$

6) Notice that, in a putty-clay model, there is no ex post decision problem, and the H_1 term drops out from (18). On the other hand, in a putty-putty model, $L(t, t)$ and $X(t, t)$ need not be decided ex ante. The only ex ante problem is to choose an appropriate plant size $I(t)$ in a putty-putty model like Solow (1960).

7) The problem of choosing ex ante variables will not be discussed here. It will be examined in a separate paper.

Next, using equations (8) and (9), the ratio of ex post oil share to labor share becomes

$$\frac{c(v)}{b(v)} = \frac{\gamma (e^{\mu v} X(v,v))^\rho}{\beta (e^{\lambda v} L(v,v))^\rho}$$

Using these two results, and defining $\sigma = 1/(1+\rho)$, we can express the ex post factor intensity between oil and labor for vintage v as follows :

$$(21) \quad \frac{X(t,v)}{L(t,v)} = \left[\frac{\gamma w(t)}{\beta x(t)} \right]^\sigma \left[\frac{X(v,v)}{L(v,v)} \right]^{1-\sigma}$$

The equation (21) says that the ex post oil-labor ratio for the vintage v at time t is a weighted average of two terms on the right-hand side. The first term is an oil-labor ratio when the production function is a usual C.E.S. type where ex ante and ex post production possibility frontiers are identical. The second term is an ex ante choice of the oil-labor ratio for the vintage t . If the ex post elasticity of substitution (σ) is smaller, then the second term will dominate. In this case, the factor intensity will not deviate much from the ex ante value even though relative factor prices fluctuate. In a putty-clay case, the factor intensity will not change at all over the life time of the vintage, since σ is 0. When the ex post elasticity of factor substitution is higher and closer to one, the first term will become more significant. The importance of the choice of ex ante plant design will diminish. The limiting case is a putty-putty technology.

III. AGGREGATE PER-CAPITA OIL DEMAND

In the last section, the ex post oil-labor ratio has been derived for all the existing vintages based on the continuous time model. The discrete time model would also have given us the similar results. In order to get estimable specifications, the time variable will be treated as discrete from now on. If vintage data are available, the ex post substitution parameter σ can be directly estimated from the regression results of the equation (21). At a firm level, such data may be obtained, but for an economy as a whole vintage data do not exist. Therefore, some kind of aggregate measure of per capita oil demand should be derived.

One way is to linearize the vintage oil-labor ratio for each existing vintage (eq. 21), and to add them all up. In order to get a Taylor expansion of the equation (21), consider the vector of factor prices which would support the ex ante choice of input vector, $I(v)$, $L(v, v)$, and $X(v, v)$. Let the implied wage rate and oil price be $w(v)$ and $x(v)$ respectively. Then, they should satisfy the following condition.

$$(22) \quad \frac{w(v)}{x(v)} = \frac{\beta X(v,v)}{\gamma L(v,v)}$$

Suppose that the actual ratio of the wage rate to oil price will vary around $w(v)/x(v)$, and does not deviate too far from it. The linear Taylor approximation of the equation (21) around $w(v)/x(v)$ will be

$$(23) \quad \frac{X(t,v)}{L(t,v)} = \sigma \frac{\gamma w(t)}{\beta x(t)} + (1-\sigma) \frac{X(v,v)}{L(v,v)}, \quad t-\tau(t) \leq v \leq t-1.$$

This relation will hold for all the vintages used in the period t . We assume that the new vintage begins to be used from the next period. There are $\tau(t)$ equations in (23) from the oldest vintage $v=t-\tau(t)$ to the newest vintage $v=t-1$. The oil-labor ratio of each vintage in the period t will be affected by the current movements of the wage rate and the oil price. On the other hand, the labor used in a vintage will become smaller and smaller as the vintage ages. Suppose the weight of the vintage labor can be approximated by the following simple geometric series.

$$(24) \quad L(t,v) = \frac{\delta}{(1+\delta)^{(t-v)}} L(t)$$

Using this weight, each equation in (23) can be rewritten as

$$(25) \quad X(t,v) = \left\{ \sigma \frac{\gamma w(t)}{\beta x(t)} + (1-\sigma) \frac{X(v,v)}{L(v,v)} \right\} \frac{\delta}{(1+\delta)^{(t-v)}} L(t),$$

for $t-\tau(t) \leq v \leq t-1$.

Then, by adding up this (t) equations and dividing by total employment, we can get an expression for the aggregate per-capita oil demand in the period t .

$$(26) \quad \frac{X(t)}{L(t)} = \sigma \frac{\gamma w(t)}{\beta x(t)} + (1-\sigma) \sum_{i=1}^{\tau(t)} \left\{ \frac{\delta}{(1+\delta)^{i-1}} \frac{X(t-i, t-i)}{L(t-i, t-i)} \right\}$$

Notice that the choice of the vintage labor weight, (24), affects only the second term on the right-hand side of (26). The first term, which shows the effect of the current factor prices on the oil demand, does not depend on the choice. Dividing both sides by $L(t)$, we obtain an expression for the aggregate oil-labor ratio in the period t . The second term on the right-hand side represents the effects of the ex ante choices of all the vintages used in time t . As shown in the previous section $X(v, v)/L(v, v)$ is decided before the machines of the vintage v have been installed, and it is a function of the expected future prices at time v . Thus, it will be difficult to get an analytical form for this term, and the equation (26) can not still be estimated directly. When the relative factor prices change rapidly, the vintage which was profitable in $t-1$ may not be so any longer in t . Thus, $\tau(t)$ and $\tau(t-1)$ will be substantially different from each other. Let the set of vintages used in $t-1$, but retired in t , be Δ_1 , and the set of vintages which were not used in $t-1$, but become profitable in t , be Δ_0 . For convenience, the vintage $t-1$ is excluded from Δ_0 , and it is listed separately. Dividing the one-period lagged form of (26) by $(1+\sigma)$, and subtracting this from (26), we get

$$(27) \quad \frac{X(t)}{L(t)} - \frac{1}{1+\delta} \frac{X(t-1)}{L(t-1)} = \sigma \frac{\gamma}{\beta} \frac{w(t)}{x(t)} - \sigma \frac{\gamma}{(1+\delta)\beta} \frac{w(t-1)}{x(t-1)} + (1-\sigma).$$

$$\left\{ \frac{\delta}{1+\delta} \frac{X(t-1, t-1)}{L(t-1, t-1)} + \sum_{i=0}^{\infty} \frac{\delta}{(1+\delta)^{i+1}} \frac{X(t-i, t-i)}{L(t-i, t-i)} - \sum_{i=1}^{\infty} \frac{\delta}{(1+\delta)^{i+1}} \frac{X(t-1-i, t-1-i)}{L(t-1-i, t-1-i)} \right\}$$

Assuming that the last two terms inside the brackets are insignificant, we finally obtain a simplified version of aggregate oil demand function as follows :

$$(28) \quad \frac{X(t)}{L(t)} \cong \frac{1}{1+\delta} \frac{X(t-1)}{L(t-1)} + \sigma \frac{\gamma}{\beta} \frac{w(t)}{x(t)} - \sigma \frac{\gamma}{(1+\delta)\beta} \frac{w(t-1)}{x(t-1)} + (1-\sigma) \delta \frac{X(t-1, t-1)}{L(t-1, t-1)}$$

Let us examine the equation (28). If the putty-clay hypothesis ($\sigma=0$) holds, and the oil-labor ratio of the newest vintage is known, then the factor-price variables, current or lagged, will not be able to explain the movement of the aggregate oil-labor ratio. On the other hand, when the putty-putty hypothesis ($\sigma=1$) is supported, the coefficients of two price terms will become bigger in the absolute values, and will be statistically significant. The oil-labor ratio of the newest vintage will become insignificant, instead.

In equation (28), all terms are observable except the last term on the right-hand side : the ex ante oil-labor ratio of the vintage $t-1$. Therefore, to estimate this equation, we need a proxy for this term. Several alternative proxies can be used, but here results from a rational-expectation proxy will be reported. Let us assume that the factor price ratio, w/x , takes the following stationary autoregressive process :

$$(29) \quad w_t/x_t = \sum_{i=1}^n a_i (w/x)_{t-i} + v_t$$

where v_t is a disturbance term which is independently and identically distributed with mean zero. We can not afford to have a large value of n , the number of lagged terms, since it will make our sample period much shorter. We chose $n=5$. Then, a firm's expectation of future w/x conditional on all the available information at time $t-1$ will be

$$(30) \quad E[w_s/x_s | \Omega_{t-1}] = \sum_{i=1}^n a_i (w/x)_{t-i}, \text{ for all } s > t-1,$$

where Ω_{t-1} is the information set of a firm in period $t-1$. Firms will anticipate that this factor price ratio $E[w_s/x_s | \Omega_{t-1}]$ will prevail over the lifetime of the vintage $t-1$. Hence, the ex ante oil-labor ratio of the vintage $t-1$ will be $(\gamma/\beta) E[w_s/x_s | \Omega_{t-1}]$, and the equation (28) can be written as

$$(31) \quad \frac{X(t)}{L(t)} = \frac{1}{1+\delta} \frac{X(t-1)}{L(t-1)} + \sigma \frac{\gamma}{\beta} \frac{w(t)}{x(t)} - \sigma \frac{\gamma}{(1+\delta)\beta} \frac{w(t-1)}{x(t-1)} \\ + (1-\sigma) \delta \frac{\gamma}{\beta} E \left[\frac{w(s)}{x(s)} \mid \Omega(t-1) \right] + \epsilon_t$$

The equation (29) will be estimated first, and the fitted value will be used as a proxy for the last term in (31)⁸⁾.

IV. ESTIMATION RESULTS

Equation (31) has four explanatory variables, but only three parameters, δ , σ , and r/β . The parameters r and β are not identified separately. There is one overidentifying restriction for the four coefficients. The non-linear least-squares (NLLS) regression will automatically take account of this restriction, since it is minimizing the sum of squared residuals with respect to underlying parameters, not with respect to c_i 's. Hence, the NLLS is used here. Estimation results of the specification (31) are reported in Table I. The sample period is 1955-1984.⁹⁾ To avoid multicollinearity problems, and to get more efficient estimates of σ , we use nonsample information on the depreciation rate. It is restricted at the value of 0.1 a year. Two time variables are added to the right-hand side to account for technological changes: t_1 for the period up to 1973, and t_2 after 1973.

All the estimated parameters have the expected signs. They are significant at the 5% level in all sample countries. The estimates of σ range from 0.14 to 0.30. The difference across countries seems to be substantial. Japan has the highest value of about 0.3. In the United States, the point estimate is 0.203, and the 95% confidence interval is (0.146, 0.261). The estimates and their standard deviations show clearly that the ex post elasticity of substitution is much smaller than the ex ante elasticity in all seven countries. The estimates of r/β are also highly significant with asymptotic t values well over 3. We may compare these values with the sample means of $\alpha X/wL$, the oil-labor factor income ratio, which is observable. Since a Cobb-Douglas form is assumed as the ex ante production function, the sample mean of the factor-income ratio will come closer to the true value of r/β as the sample size increases. They are shown in the last row of the table. In all countries, the value of sample mean is included in the 95% confidence interval of the estimate of r/β . The coefficients of the time variables are less clear-cut. Prior to the 1973-74 oil shock, the labor-saving technological progress seems to have dominated the oil-saving progress. After the shock, that tendency has been weakened or reversed.

The results from individual countries presented above have already shown us much about the dynamic movement of oil demand. But, by combining the experience of the seven countries, the analysis might be sharpened further. A time series-cross sectional model can be specified as

$$(32) \quad (X/L)_{it} = F(\sigma, r/\beta, \delta, t) + \mu_i + \varepsilon_{it},$$

8) Using annual investment data as alternative weights, we can derive another estimable specification for per capita oil demand. The estimates of parameter values were almost the same as from (28).

9) For data sources, see Kim (1987).

where μ_i is the individual country effect, and $i=1, 7$; $t=1955, 1984$. There are two alternative specifications depending upon the treatment of the individual effect. We may treat μ_i as a fixed but unknown constant differing across countries. This is called the fixed effects model. The μ_i 's should be estimated together with other parameters. The alternative specification treats μ_i as a random variable which is independently and identically distributed with zero mean. It is further assumed that μ_i is uncorrelated both with the u_{it} and with the explanatory variables. This is called the random effects or error components model. Since there is no prior justification for using one as opposed to the other, estimation results from both models are presented in Table 2.

The estimates of the fixed effects model are quite satisfactory. All the technological parameters are highly significant. The ratio of ex ante distribution parameters, τ/β , is estimated at 0.096, which is exactly the same as the sample mean of the ratio of oil to labor income share in the seven countries. The estimate of depreciation rate is 0.1, which we have assumed in the individual country regressions. The ex post elasticity of substitution is estimated at 0.26. In the fixed effects model, time variables are also significant. They suggest that the oil-saving technological change has outweighed the labor-saving change since the first oil shock in 1973. The constant term is highly significant in Italy, and somewhat less significant in France and Japan. It is insignificant in the other countries. This result implies, for example, that the observed high per capita oil demand in the United States can be explained mostly by the low oil price or the high wage level. Thus, it may be incorrect to say that the American people use more oil due to non-economic "cultural" factors. On the other hand, the random effects model shows somewhat unsatisfactory results. Two parameters σ and τ/β seem to be underestimated. The estimate of σ is a little higher, but does not change much.

Next, we will allow different values of ex post elasticity of substitution across countries under the fixed effects model, while keeping the assumption that the ex ante production function is the same among countries. As shown in Table 3, the estimates of σ in all countries are highly significant. Japan, France, and Canada seem to have higher values of σ than other countries. Using the results in Table 2 and 3, we can test whether the ex post elasticity is the same across countries. There are six independent linear hypotheses in this test. The value of F statistic is 2.04, which is slightly lower than $F_{0.05}=2.1$, the critical value at the 5% significance level. Hence, we can not reject the null hypothesis that σ is the same in all seven countries with the type I error less than 5%.¹⁰⁾

This result does not necessarily mean that the ex post elasticity of substitution in one country is the same as σ in another country. For example, we can test whether $\sigma(\text{US})=\sigma(\text{JA})$. Under the normality assumption, the test statistic has asymptotically at distribution with T-k degrees of freedom. This test takes account

10) When normality is not assumed, a likelihood test or Wald test is still applicable as Amemiya (1983) has shown. The test results are similar.

of the covariance between two estimates in addition to the variances. The hypothesis $H_0 : \sigma(\text{US}) = \sigma(\text{JA})$ against the alternative hypothesis $H_1 : \sigma(\text{US}) < \sigma(\text{JA})$ can be rejected at the 1% level since the t statistic is -2.60 . As shown in Table 4, the ex post elasticities of substitution in Canada and France are also significantly higher than the estimate of the United States. This implies that the short-run effect of an oil-price change on the per capita oil demand will be smaller in the United States than in these countries. Because of this rigidity in the ex post production function, ceteris paribus, the United States may not get so much benefit from a downward oil price shock as other countries.

Why does one country have more rigidity than others in the ex post production function? One plausible reason might be that the labor is less mobile inside firms or among industries. Consider an unexpected fall of the oil price. It will have asymmetric effects to various industries. Industries operating with oil intensive equipments will be benefited more. Those industries will expand, and factor mobility from other sectors may decide the speed of expansion in the short-run.

V. TESTING TECHNOLOGICAL HYPOTHESES

There are two alternative technological hypotheses which are more extreme forms compared with the putty-semiputty technology used so far. First, consider a putty-clay technology. Since there is no ex post substitution among factors under this hypothesis, the oil demand function (28) will be reduced to the following form under rational expectations :

$$(33) \quad \frac{X(t)}{L(t)} = \frac{1}{1+\delta} \frac{X(t-1)}{L(t-1)} + (1-\sigma)\delta \frac{\gamma}{\beta} E \left\{ \frac{w(s)}{x(s)} \mid \Omega(t-1) \right\} + \epsilon_t$$

This equation shows us that the change in w/x ratio will affect the oil-labor ratio only through one term, the proxy for the oil-labor ratio of the newest vintage. No existing vintages will be affected by the variation in factor prices. To a permanent fall in w/x due to an oil price hike, the aggregate per capita oil demand will go down very gradually as new vintages are introduced year after year.

From earlier estimation results this hypothesis can be easily tested. Under normality assumption, the relevant test is a t test for the coefficient σ . In the pooled regression, the estimate of σ was 0.2606 with the t value 8.3 when a common σ was assumed in Table 2. When different σ 's are allowed across countries in the pooled regression (Table 3), the estimates of σ are still significantly positive with t values well exceeding 2 in all seven countries (Table 5). The results are similar when the equation is estimated country by country as in Table 1. Our data strongly rejects the putty-clay hypothesis in all sample countries. Similarly, we can also reject the putty-putty hypothesis as shown in Table 5.

Many studies of factor demands are implicitly based on the putty-putty hypothesis, but in their empirical applications, they usually assume an ad hoc partial adjustment mechanism. In a factor choice model, a typical adjustment pattern used is, for example,

$$(34) \quad \frac{X(t)}{L(t)} - \frac{X(t-1)}{L(t-1)} = \omega \left[\frac{X(t)^*}{L(t)^*} - \frac{X(t-1)}{L(t-1)} \right] + \varepsilon(t).$$

where ω is the adjustment coefficient, and $[X(t)/L(t)]^*$ is the desired ratio of oil to labor in the period t . With a Cobb-Douglas production function specified in section 2, equation (34) can be rewritten as

$$(35) \quad \frac{X(t)}{L(t)} = (1-\omega) \frac{X(t-1)}{L(t-1)} + \omega \frac{\gamma}{\beta} \frac{u(t)}{x(t)} + \varepsilon(t).$$

Compare this with our putty-semiputty specification, equation (28). The last term $X(t-1, t-1)/L(t-1, t-1)$ which accounts for the newest vintage, is dropped. The ex post substitution parameter σ and the depreciation rate δ in (28) should be equal each other and should be reinterpreted as the partial adjustment parameter. Using the pooled data, we will compare the estimation results of these two specifications.

First, consider Table 6 which presents the regression output of a putty-putty partial-adjustment model. When ω is assumed to be the same across countries, it is estimated at 0.1605. This is smaller than the estimate of the common elasticity of substitution ($\sigma=0.2606$) in Table 2. In fact, it lies between the estimates of σ and δ as the implied restriction requires. When the adjustment coefficient is allowed to vary across countries, it ranges from 0.0986 in the United States to 0.3463 in Japan. The ranking of countries according to the estimates of ω is about the same as that from σ , but the range of ω is estimated much more widely. It can be noticed that the restriction imposed by the putty-putty partial adjustment model is $\sigma=\delta$. When the results of common σ and ω from Tables 2 and 6 are used, the likelihood ratio statistic is 24.88, while Wald statistic is 21.89. These are distributed asymptotically as χ^2 with one degree of freedom. Both of them are much bigger than 1% critical value of χ^2 statistic, 6.63. Thus, the partial adjustment model with the putty-putty technology can not be supported by the data.

VI. CONCLUDING REMARKS

In this paper, we derived an explicit form of the ex post production function, based on the putty-semiputty hypothesis. A representative firm is assumed to be operating with various vintages of machines. These vintages have different designs and sizes. A profit-maximizing problem of a firm is then set up in a framework of optimal control. At any point of time, the firm faces two kinds of decision problems: i) the choice of ex ante variables such as the amount and the types of new machines, and ii) the ex post problems such as determining the levels of variable inputs used in all the existing vintages. From the first-order conditions of the firm's problem, we have obtained several interesting results.

First, the ex post ratio between two variable inputs, such as oil-labor ratio, is a weighted average of the long-run desired ratio and the ratio embodied in the vintage design. The ex post elasticity of factor substitution serves as the weight. The lower is this ex post elasticity, the less the factor ratio will be adjusted toward a long-run

level. A lower ex post elasticity may be interpreted as high adjustment costs. Thus, the two-step structure of a putty-semiputty technology allows for a flexible form of adjustment mechanism. Secondly, a lower ex post elasticity also makes the vintage output smaller relative to capacity. This directly follows from the first result. Suppose that firms in a country are equipped with machines having a low value of ex post elasticity of substitution. The aggregate output of that country will fall more when the price of any input rises. Thirdly, the aggregate per capita demand function of oil has been derived and estimated. The dynamic nature of this function is different from that in a simple partial-adjustment model. The function includes both ex ante and ex post parameters in a production function. The range of estimated ex post elasticity of substitution, σ , is 0.14-0.30, which seems to be reasonable. The United States seems to have a significantly lower ex post elasticity than Japan, Canada, and France.

One of the merits of our specification is that it enables us to test the alternative technological hypotheses which have been most commonly used: the putty-putty and putty-clay hypotheses. Data reject these extreme hypotheses with 1% significance level in all the sample countries. Thus, in the studies of factor demand, any of these technological assumptions should not be taken for granted. A partial-adjustment model is also rejected.

We have worked with a one-good model. Studies using industry or firm data should be possible. Other forms of energy can be also included in the model to take account of inter-fuel substitutions. The oil demand for developing countries can be also studied based on our model. It will be interesting to see whether there is any significant difference in the ex post elasticity of substitution between East Asian and Latin American countries.

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[Table 1] Per Capita Oil Demand

	USA	Canada	France	Germany	Italy	Japan	UK
ex post	0.2027 (0.0304)	0.2746 (0.1216)	0.2848 (0.0832)	0.2530 (0.0855)	0.1414 (0.0525)	0.3032 (0.0550)	0.2441 (0.1387)
r/β	0.1181 (0.0121)	0.1080 (0.0148)	0.1182 (0.0229)	0.0874 (0.0184)	0.1460 (0.0296)	0.1312 (0.0185)	0.0635 (0.0225)
t_1	-0.8272 (0.3994)	0.2141 (0.4265)	0.4342 (0.3166)	0.5321 (0.2950)	0.4817 (0.2984)	0.2540 (0.2027)	0.6778 (0.2715)
t_2	-1.5737 (0.5494)	-0.7091 (0.5947)	-0.6016 (0.5125)	-0.0414 (0.5436)	-0.5981 (0.4854)	-0.8247 (0.4180)	0.2267 (0.4515)
R ²	0.965	0.961	0.987	0.984	0.991	0.992	0.965
SSR	1066	1528	842	994	529	586	617
SER	7.12	8.53	6.33	6.88	5.02	5.28	5.42
D. W.	2.20	1.98	2.23	1.99	2.62	2.60	2.45
Rho	-	-	-	-	-0.461 (0.177)	-	-0.419 (0.182)
sample mean (oil-labor income ratio)	0.92	0.111	0.099	0.078	0.118	0.095	0.081

(asymptotic standard deviations in parentheses)

[Table 2] Pooled Regression : Common Coefficients

	Fixed Effects	Random Effects
σ	0.2606 (0.0313)	0.2989 (0.0475)
r/β	0.0963 (0.0097)	0.0762 (0.0109)
δ	0.1004 (0.0229)	0.0529 (0.0165)
t_1	0.4158 (0.1110)	0.5150 (0.1026)
t_2	-0.4500 (0.2039)	-0.5492 (0.1861)
constant		2.7501 (1.1561)
Canada	2.7175 (2.7816)	
France	2.4835 (1.5517)	
Germany	0.4304 (2.0118)	
Italy	4.6290 (1.5443)	
Japan	2.5120 (1.5123)	
U. K.	-0.9464 (1.6886)	
U. S. A.	-3.4497 (3.3612)	
R ²	0.9940	0.9936
SSR	284.9	303.2
D. W.	1.99	2.02

(asymptotic standard deviations in parentheses)

[Table 3] Pooled Regressions :
Different Ex Post Elasticity Across Countries

	USA	Canada	France	Germany	Italy	Japan	UK
σ	0.1899 (0.0418)	0.3624 (0.0807)	0.3697 (0.0727)	0.2484 (0.0691)	0.2060 (0.0732)	0.3874 (0.0689)	0.2034 (0.0874)
constant	-4.0994 (3.3185)	2.2729 (2.7487)	2.1776 (1.5402)	0.2708 (2.0110)	4.9847 (1.5338)	2.0413 (1.5046)	-0.9766 (1.6778)
r/β	0.0998 (0.0094)			t_1	0.3523 (0.1130)		
δ	0.1045 (0.0227)			t_2	-0.4378 (0.2049)		
	R ² =0.9943,		SSR=267.2,		SER=1.202,		D. W. =2.00

[Table 4] Pooled Regressions :
Testing a Different Ex Post Elasticity

	Estimate	t statistic
$\sigma(\text{US}) - \sigma(\text{CA})$	-0.1725 (0.0863)	-1.999
$\sigma(\text{US}) - \sigma(\text{FR})$	-0.1798 (0.0802)	-2.241
$\sigma(\text{US}) - \sigma(\text{GE})$	-0.0585 (0.0781)	-0.749
$\sigma(\text{US}) - \sigma(\text{IT})$	-0.0161 (0.0829)	-0.195
$\sigma(\text{US}) - \sigma(\text{JA})$	-0.1975 (0.0760)	-2.600
$\sigma(\text{US}) - \sigma(\text{UK})$	-0.0135 (0.0949)	-0.143

[Table 5] Testing Alternative Technological Hypotheses : t statistics

	Putty-Clay	Putty-Putty
United States	4.54	-19.4
Canada	4.49	-7.9
France	5.09	-8.7
Germany	3.59	-10.9
Italy	2.81	-10.8
Japan	5.62	-8.9
United Kingdom	2.33	-9.1

[Table 6] Putty-Putty Technology with Partial Adjustment

	Common Adjustment Factor	Different Adjustment Factor
ω	0.1605 (0.0206)	
Canada		0.1927 (0.0457)
France		0.2918 (0.0443)
Germany		0.1998 (0.0513)
Italy		0.1587 (0.0361)
Japan		0.3463 (0.0560)
U. K.		0.1890 (0.0503)
U. S. A.		0.0986 (0.0244)
r/β	0.0928 (0.0064)	0.1067 (0.0044)
t_1	0.5427 (0.1145)	0.3401 (0.1133)
t_2	-0.1494 (0.2065)	-0.1827 (0.1984)
constant		
Canada	4.7410 (2.9181)	0.1347 (2.4522)
France	2.5199 (1.6454)	0.0043 (1.8545)
Germany	-1.0285 (2.1106)	-4.8875 (2.7349)
Italy	5.5772 (1.6252)	4.8125 (1.5199)
Japan	2.3879 (1.6034)	-0.1470 (1.8634)
U. K.	-2.6332 (1.7543)	-5.0797 (2.2643)
U. S. A.	-4.2551 (3.5601)	-6.7946 (2.3781)
R^2	0.9932	0.9940
SSR	322.0	284.8
D. W.	1.54	1.47