The Relationship Between the Rate and Variability of Inflation

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This paper provides a theoretical justification for the observed relationship between the average inflation rate and the variability of inflation. Using a simple three-equation model with rational expectations, it demonstrates that the average rate and the fluctuation of inflation have a nonlinear, positive relation. The nexus between the two measures of inflationary behavior turns out to be the policy preference parameter measuring the degree of accommodation of monetary policies to inflation.

I. Introduction

During the past decade, there has been considerable interest in the relationship between the rate and the variability of inflation; for example, see Okun (1971), Gordon (1971), Logue and Willett (1976), Foster (1978), Blejer (1979), and Taylor (1981). Their findings indicate that there exists a strong, positive relationship between the average rate of inflation and two alternative measures of inflation fluctuations, the average absolute change and the standard deviation (or variance) of inflation.[1] Most of these studies, however, have concentrated on the empirical nature of the relationship.[2]

Based on a contracting model, Taylor (1981) recently analyzed various relations among the aggregate price level, variance of the aggregate price level, variance of relative price variability, and forecast uncertainty defined as forecast error variance. He obtains, among other interesting results, a possible explanation for the observed relationship between the average rate

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1. Foster (1978) and Blejer (1979) used the former measure of fluctuation.
2. An exception is Taylor (1981).

and the variance of inflation. According to his model, countries with a higher priority on output and employment stability than on inflation could intentionally permit relatively high variability of inflation by letting their monetary policies be more accommodating. Since these countries tend to allow relatively high average inflation rates, the observed relationship could be easily justified. Thus, the trade-off between variabilities of price and output coupled with the choice of optimal policy regime produces the theoretical result behind the empirical observation.

This paper looks into the same problem through a different theoretical framework. Employing the Lucas-type supply behavior with a simple quantity equation for aggregate demand we derive reduced-form equations for the aggregate price level and output, both of which are functions of the accommodation parameter of the monetary policy rule. It is this policy parameter that links the average rate of inflation and its variability. However, unlike the contracting model, there is no trade-off possibility between inflation and output variabilities in this model. Thus the nonlinear, positive relationship between the inflation rate and its variability is confirmed without the assumption of optimizing efforts on the part of policy makers.

The paper is organized as follows. The next section sets up the simple model of an economy and solves for the equilibrium values of output and the aggregate price level under the assumption of rational expectations. Section III discusses the implication of the model for the relationship between the average inflation rate and its variability. Finally, Section IV summarizes the findings and offers some concluding remarks.

II. The Model

The economy is characterized by the Lucas-type aggragate supply and a simple quantity equation for aggregate demand:

$$y_t = y_{nt} + \theta\gamma\,(p_t - \bar{p}_t) + \lambda(y_{t-1} - y_{n,t-1}), \tag{1}$$

$$y_t + p_t = m_t + v_t. \tag{2}$$

Equation (1) describes the suppliers' behavior. It is exactly the same as Lucas' (1973).[3] According to this equation, the deviation of the log of output y_t from its trend $y_{nt} = \bar{\beta} + \beta t$ is affected by the log of the general price level P_t only to the extent that it differs from its expected value $\bar{P}_t$. Strictly speaking, $\bar{P}_t$ is the mathematical expectation of P_t conditional on the infor-

3. See Lucas (1973) for a detailed discussion of the equation.

mation available at time $t-1$. γ is a coefficient giving the individual supplier's response to the unanticipated price change, and $\theta = \tau^2/(\tau^2+\sigma^2)$ where τ^2 and σ^2 are the variances of the market-specific price and the general price level P_t, respectively, about $\bar{P}_t$. The larger the variance σ^2 relative to the variance τ^2, the smaller will be the supply response, Equation (1) also contains the adjustment mechanism whose speed is measured by λ, $|\lambda| < 1$.

Equation (2) is a simple equation where m_t is the log of the money supply and V_t is a velocity shock with mean zero and variance σ_v^2. We assume V_t is serially uncorrelated. The monetary authority is assumed to follow the rule specified by its choice of ρ:[4]

$$m_t = m_{t-1} + \rho(p_t - p_{t-1}) + u_t \tag{3}$$

where u_t is independent of V_t, and serially uncorrelated with mean δ and variance σ_u^2. The accommodation parameter, ρ, $|\rho| < 1$, will receive a key attention in the following discussion. As of now we note that an algebraically larger ρ signifies a more accommodating monetary policy.

Following lucas (1973) and Barro (1976), the solution for the general price level can be obtained using the method of undetermined coefficients. Here we use an efficient solution procedure suggested by McCallum (1983) which also excludes 'bubble' or 'bootstrap' effects. Following this minimal state variables approach we express P_t as

$$\begin{aligned} p_t = {} & \pi_0 + \pi_1 u_t + \pi_2 v_t + \pi_3 y_{t-1} + \pi_4 y_{nt} + \pi_5 y_{n,t-1} \\ & + \pi_6 m_{t-1} + \pi_7 p_{t-1}. \end{aligned} \tag{4}$$

Taking the mathematical expectation of both sides conditional on information available at $t-1$ yields

$$\begin{aligned} \bar{p}_t = {} & \pi_0 + \pi_1 \delta + \pi_3 y_{t-1} + \pi_4 y_{nt} \\ & + \pi_5 y_{n,t-1} + \pi_6 m_{t-1} + \pi_7 p_{t-1}. \end{aligned} \tag{5}$$

Equations (2) and (3) give us

$$y_t = (\rho-1)p_t + m_{t-1} - \rho p_{t-1} + u_t + v_t \tag{6}$$

which, together with (1), results in

$$\begin{aligned} y_{nt} + \theta\gamma(p_t - \bar{p}_t) + \lambda(y_{t-1} - y_{n,t-1}) = {} & (\rho-1)p_t + m_{t-1} \\ & - \rho p_{t-1} + u_t + v_t. \end{aligned} \tag{7}$$

Now, substituting (4) and (5) into (7) and comparing the corresponding coefficients on both sides, we obtain the equations for the undetermined coefficients in (4):

4. Equation (3) is slightly different from the specification used in Taylor (1981).

$$
\begin{aligned}
-\pi_1\delta\theta\gamma &= (\rho-1)\pi_0,\\
\theta\gamma\pi_1 &= (\rho-1)\pi_1 + 1,\\
\theta\gamma\pi_2 &= (\rho-1)\pi_2 + 1,\\
\lambda &= (\rho-1)\pi_3,\\
1 &= (\rho-1)\pi_4,\\
-\lambda &= (\rho-1)\pi_5\\
0 &= (\rho-1)\pi_6 + 1,\\
0 &= (\rho-1)\pi_7 - \rho.
\end{aligned} \tag{8}
$$

In addition, there exists another relationship among parameters which comes directly from (4) and (5)

$$\sigma^2 = \pi_1^2 \; \sigma_u^2 + \pi_2^2 \; \sigma_v^2 \; . \tag{9}$$

Since (8) implies $\pi_1 = \pi_2$, and since $\theta = \tau^2/(\tau^2 + \sigma^2)$ by definition, π_1 should satisfy the following equations:

$$\sigma^2 = \pi_1^2 \; (\sigma_u^2 + \sigma_v^2), \tag{10}$$

$$\pi_1 = \frac{1}{1-\rho+\gamma\dfrac{\tau^2}{\tau^2+\sigma^2}}, \tag{11}$$

which yield a cubic equation for π_1. As we demonstrate below, there is a unique solution for π_1 in (10) and (11). This then implies that we also have a unique solution for σ^2 from (9) and that (8) can be solved for the undetermined coefficients given the value of σ^2. Thus, assuming the unique value of σ^2 is substituted into θ, we obtain the solutions for price and output:

$$
\begin{aligned}
P_t = {} & \frac{\pi_1\delta\theta\gamma}{1-\rho} + \frac{1}{1-\rho+\gamma\theta}(u_t+v_t) - \frac{\lambda}{1-\rho}(y_{t-1}-y_{n,t-1})\\
& - \frac{1}{1-\rho}y_{nt} + \frac{1}{1-\rho}m_{t-1} - \frac{\rho}{1-\rho}P_{t-1}
\end{aligned} \tag{12}
$$

$$y_t = y_{nt} + \frac{\theta\gamma}{1-\rho+\gamma\theta}(u_t + v_t - \delta) + \lambda(y_{t-1} - y_{n,t-1}). \tag{13}$$

The expression for $\Delta P_t = P_t - P_{t-1}$ follows immediately from (12):

$$\Delta P_t = \frac{1}{1-\rho+\gamma\theta}(\Delta u_t + \Delta v_t) - \frac{\lambda}{1-\rho}\left[\frac{\theta\gamma}{1-\rho+\gamma\theta}(\Delta u_{t-1} + \Delta v_{t-1})\right.$$

$$+\gamma\Delta(y_{t-2}-y_{n,t-2})] - \frac{\beta}{1-\rho} + \frac{1}{1-\rho}\Delta m_{t-1} - \frac{\rho}{1-\rho}\Delta P_{t-1}. \tag{14}$$

Interpreting ΔP_t as the inflation rate, the unconditional expectation of ΔP_t is shown to be

$$E[\Delta P_t] = \frac{\delta - \beta}{1-\rho}. \tag{15}$$

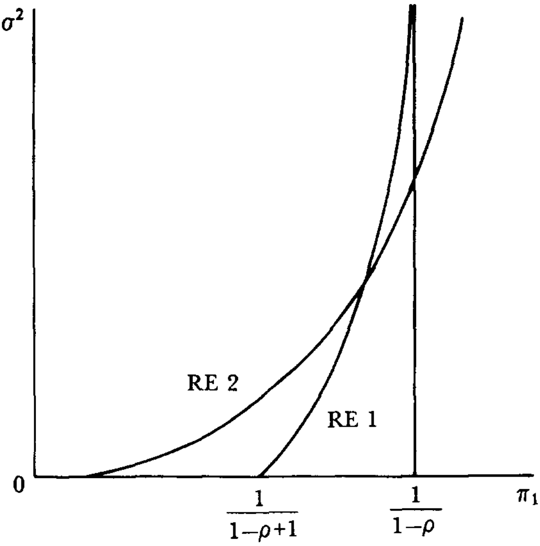
This equation simply says that, in the absence of feedback policy ($\rho = 0$), the average inflation is equal to the growth rate of money, δ, minus the productivity growth rate, β. Since the conditional variance of p_t is time-invariant (see (10)), the variance of ΔP_t, used as the measure of inflation variability, is $2\sigma^2$ ($= \sigma^2_{\Delta P}$). Thus, if we can find a unique solution for π_1, then $\sigma^2_{\Delta P}$ is also uniquely determined.

Equations (10) and (11) give us a cubic equation for π_1, and therefore we have a possibility of multiple solutions. This nonuniqueness problem for a class of rational expectations model was first pointed out by McCafferty and Driskill (1980).[5] Fortunately, our model does possess a unique solution for all reasonable ranges of parameter values. To show this, note that (10) is an equation of parabola in (σ^2, π_1) plane. Given $|\rho| < 1$, its relevant portion is drawn in the first quadrant and labelled RE2 in Figure 1. On the other hand, equation (11) gives us a convex curve cutting π_1 axis at $1/(1-\rho+\gamma)$ and approaching the asymptote $\pi_1 = 1/(1-\rho)$. Assuming $\gamma > 0$, the curve is depicted as RE1 in the figure. Note that RE1 and RE2 meet only at one point and we have a unique solution for π_1 and σ^2.

III. The Relationship Between the Rate and Variability of Inflation

Consider equation (15). Given the values of δ and β, the mean growth rate of money and the exogenously determined slope of the output trend line, it tells us that, the more accommodating the monetary policy ($\rho \to 1$), the larger will be the expected rate of inflation. Thus, this equation establishes the connection between the average rate of inflation and the policy parameter ρ. On the other hand, equation (10) and (11) express the positive relationship between p and σ^2, the variance of the price level. Hence, the positive relationship between the average rate and the variability of inflation is established through the behavioral parameter of the monetary authority.

5. This nonuniqueness is due to a set of nonlinear relationships among model parameters and hence quite different from that arising from an inefficient solution method of undetermined coefficients.

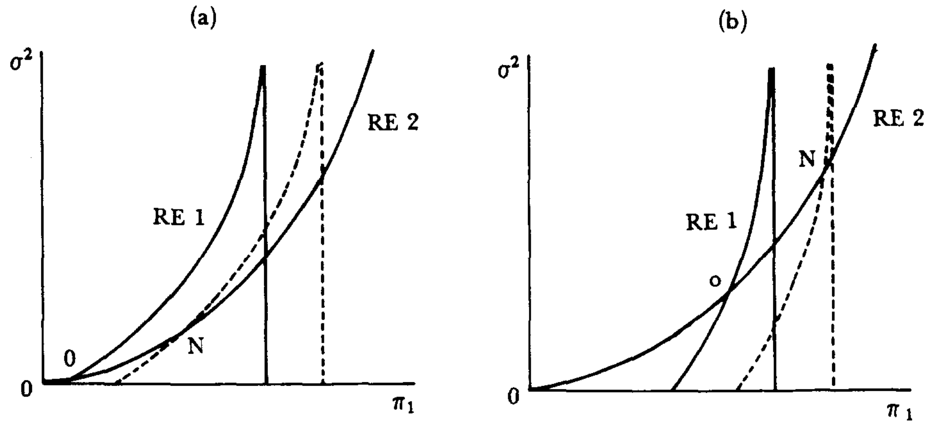


[Figure 1] The Uniqueness of Rational Expectations Solution

Figure 2 illustrates this relationship more clearly. In Panel (a), an initial solution of π_1 and σ^2, given an arbitrary value of ρ, is denoted as 0. Note that $E[\Delta P] = (\delta - \beta)/(1-\rho)$ is proportional to $1/(1-\rho)$, the asymptote on π_1 axis. Now, as ρ increases, both RE1 and its asymptote shift out and the new solution N is obtained (see the dotted curves). Both π_1 and σ^2 at N are larger than those at 0. The higher the average inflation rate (proportional to π_1), the larger will be the variance of inflation ($= 2\sigma^2$).

The nonlinearity of the relationship between $E[\Delta P)$ and $\sigma^2_{\Delta p_t}$ can also be shown in Figure 2. Consider Panel (a). As ρ increases, its effect on π_1 is relatively larger than that on σ^2. Compare the vertical distance to the horizontal distance between 0 and N in the picture. In contrast with Panel (a), when the initial value of σ^2 and π_1 are on the steeper sections of RE1 and RE2 as in Panel (b), the increase in ρ is shown to effect a much larger change in σ^2 than in π_1. Since the value of ρ at the initial point is larger at Panel (b) than at Panel (a), the nonlinearity is also established through the accommodation parameter. At a low level of inflation with a nonaccommodating or a parsimoniously accommodating monetary policy, an increase in accommodation parameter generates a less-than-proportional increase in inflation variability. But as the policy authority tends to be too accommodating, an increase in the average inflation is associated with a greater-than-proportional increase in the fluctuation of inflation.

This nonlinearity stems from two different, but mutually reinforcing factors affecting the general price level. First, a more accommodating



[Figure 2] Different Effects of an Increase in ρ on the Price Variance

monetary policy results in a higher average price level, and at the same time it tends to magnify the effects of velocity and monetary shocks, v_t and u_t on P_t (see the first item on the right hand side of equation (12)). Thus the variance of price level increases with a factor of $(1-\rho+\gamma\theta)^{-2}$, while the mean increases with a factor of $(1-\rho)^{-1}$.

Second, as ρ increases, $\theta = \tau^2/(\tau^2+\sigma^2)$ tends to decrease and this in turn will cause π_1 to increase (equation (11)). This effect will again feed into equation (10) to increase σ^2 further. This channel of the relationship arises because, as the policy maker becomes more accommodating, there tends to be higher variance of the price level and hence the aggregate supply curve becomes steeper implying a reduced trade-off between output and price.

Now consider the trade-off between inflation and output variabilities. From (13) we obtain the unconditional variance of output

$$\sigma_y^2 = \frac{1}{1-\lambda^2}\left(\frac{\theta\gamma}{1-\rho+\theta\gamma}\right)^2(\sigma_u^2+\sigma_v^2), \tag{16}$$

which clearly shows that output variability increases as the degree of accommodation ρ increases. This unambiguous relationship represents two offsetting effects. First, the increase in inflation variability associated with a higher ρ tends to increase the slope of the short-run aggregate supply curve, and this will, ceteris paribus, have a dampening effect on output variability. However, the higher ρ also directly increases the variability of aggregate demand. Equation (16) indicates that the direct positive effect on output variability of the increased ρ through higher aggregate demand variability unambiguously dominates the indirect negative effect that operates through the slope of the short-run aggregate supply curve. Thus, in our model, there exists no trade-off between inflation and output variabilities.

According to Friedman (1977), a higher rate and a larger fluctuation of inflation may retard the growth of the potential output. This is because high variability of inflation induces a loss of efficiency in the price system as the economy characterized by various rigidities due to institutional and political arrangements is forced to undergo the necessary adjustment. Within the context of the present model, this inefficiency can be represented by hypothesizing that $\beta = \beta(\sigma^2)$ with $\partial\beta/\partial\sigma^2 < 0$. In other words, an increase in σ^2 has an adverse effect on the productivity growth rate. Although y_{nt} is now a function of σ^2 through β, the solutions of the model remain the same except that

$$E[\Delta P_t] = \frac{\delta - \beta(\sigma^2)}{1-\rho}. \tag{17}$$

The implication of this new expected inflation is twofold. First, as ρ increases, $E[\Delta P_t]$ in (17) tends to have a higher value relative to that in (15). This is due to the fact that the increase in σ^2 initiated by an increase in ρ will reduce β, thereby reinforcing the initial effect. Second, since the average inflation now varies with the variance of inflation, the previous nonlinearity between them has to be modified. Depending upon the functional form of $\beta(\sigma^2)$ various cases could be considered. For example, if there is a threshold range of inflation over which higher rates of inflation do not induce higher variabilities in inflation, then β could be more or less constant or $\partial\beta/\partial\sigma^2$ may be increasing only slightly over this range.[6] On the other hand, at higher rates of inflation β could change significantly with the increase in σ^2. In this case the original nonlinearity will be enhanced.

IV. Conclusion

In this paper we have demonstrated a theoretical justification for the observed relationship between the average rate of inflation and the variability of inflation. The relationship, shown to be nonlinear and positive as Okun (1971) asserted, can be generated by a particular specification of policy maker behavior. The more accommodating the policy, the larger would be both the expected rate of inflation and its

6. Logue and Willett (1976) suggested this possibility of the threshold level of inflation.

variance.[7] Unlike the contract model, this relationship is established without the possibility of trade-off between price variability and output variability.

Estimating monetary policy behavior is very difficult.[8] Moreover, as equations (10), (11), and (15) indicate, actual establishment of the relationship between the average rate of inflation and its variability involves complete specification of a structural model of the economy which also exhibits the policy invariance property as suggested by Lucas (1976). Extensive empirical research incorporating the theoretical model such as the present study should shed some light on the exact nature of the observed empirical relationship.[9]

REFERENCES

1. Barro, R., "Rational Expectations and the Role of Monetary Policy," *Journal of Monetary Economics,* 2, 1976, pp. 1 – 32.
2. Black, S., "The Use of Monetary Policy for Internal and External Balance in Ten Industrial Countries, "Working Paper 81-W13, Vanderbilt University, 1981.
3. Blejer, M., "Inflation Variability in Latin America: A Note on the Time Series Evidence," *Economic Letters,* 2, 1979, pp. 337-341.
4. Foster, E., "The Variability of Inflation," *Review of Economics and Statistics,* 60, 1978, pp. 346 – 350.
5. Friedman, M., "Inflation and Unemployment," *Journal of Political Economy,* 85, 1977, pp. 451 – 472.
6. Gordon, R., "Steady Anticipated Inflation: Mirage or Oasis?" *Brookings Papers on Economic Activity,* 1, 1971, pp. 499 – 510.
7. Logue, D. and Willett, T., "A Note on the Relation Between the Rate and Variability of Inflation," *Economica,* 43, 1976, pp. 151 – 158.
8. Lucas, R., "Some International Evidence on Output-Inflation Tradeoffs," *American Economic Review,* 63, 1973, pp. 326 – 34.
9. ________, "Econometric Policy Evaluation: A Critique," *Journal of Monetary Eonomics,* 2, 1976, pp. 19 – 46.
10. McCafferty, S. and Driskill, R., "Problems of Existence and Uniqueness in Nonlinear Rational Expectations Models," *Econometrica,* 48, 1980, pp. 1313-1318.
11. McCallum, B, 'On Non-Uniqueness in Rational Expectations Models: An Attempt at Perspective," *Journal of Monetary Economics,* 11, 1980, pp. 139 – 168.

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7. Equation (3) can be modified such that the monetary authority also reacts to the output, that is, $m_t = m_{t-1} + \rho_1(P_t - P_{t-1}) + \rho_2(y_t - y_{t-1}) + U_t$.
Here both parameters, ρ_1 and ρ_2, play the role of relating the average rate of inflation to its variability. Except algebraic complication, all the substantive results remain basically the same as in the simpler reaction function.
8. See, for example, Black (1981) and Taylor (1980).
9. The author is currently conducting the empirical research along this line.

12. Okun, A., "The Mirage of Steady Inflation," *Brookings Papers on Economic Activity*, 1, 1971, pp. 435 – 498.
13. Taylor, J., "Output and Price Stability: An International Comparison," *Journal of Economic Dynamics and Control*, 1, 1980, pp. 109 – 132.
14. ________, "On the Relationship Between the Variability of Inflation and the Average Inflation Rate," in K. Brunner and A. Meltzer eds., *The Costs and Consequences of Inflation*, North-Holland, 1981.