

# **A Variable Cost Function Approach to the Measurement of Technological Characteristics**

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## **I. Introduction**

The duality theory between production and cost permits us to use cost function model in the empirical study of production technology. Based on this dual approach, recently, a number of empirical studies have utilized total cost function model in order to estimate characteristics of a production technology. For example, Binswanger[1] clarifies its usefulness. Christensen and Greene[5] estimate the scale and substitution characteristics of U.S. electric power generation using the translog total cost function, while Brown, Caves and Christensen[2] utilize the total cost function model to estimate the long run structure of a multi-output technology for U.S. railroads.

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In such models, a key assumption is involved: each firm is assumed to minimize the total cost of producing a given output with respect to all inputs in use at given prices. This assumption implies that there is no fixed or quasi-fixed input; that is, all inputs are instantaneously variable, and thus they are all employed at optimum levels at the point of observations.

In many instances, however, the assumption of this full static equilibrium is suspect, and hence so are the empirical results. The adjustment of certain input to the change in price of input or output level is not instantaneous because of the technical restrictions in substitution or the internal cost of adjustment or regulatory restrictions. Therefore, the firm can rather be assumed to minimize the variable cost with respect to variable inputs at the point of observations. This implies that the firm would be in partial static equilibrium with respect to a subset of inputs (rather than all inputs) conditional on the observed levels of the remaining inputs. The inputs that are employed at optimum levels are referred to as variable inputs and the remaining inputs are designated as fixed or quasi-fixed inputs. Likewise, if the firm does not minimize the total cost with respect to all inputs, the total cost function does not exist. Instead, if the firm minimizes the cost of a subset of inputs (variable inputs) conditional on the levels of the remaining inputs (fixed inputs), then there exists a variable cost function. Therefore, it is wrong to use the total cost function model if the firm actually minimizes the variable cost only for the given amount of fixed inputs.

In the late 1970s and early 1980s, fortunately, this problem has been solved by constructing the variable cost function model which is based on the assumption that the firm minimizes the variable cost with respect to variable inputs conditional on the given amount of fixed inputs. It would seem that this is based on more appropriate and realistic assumptions than the total cost function model is.

In 1980s, some empirical studies began to use this variable cost function model to estimate characteristics of a production technology. For instance, Brown and Christensen[3] apply this model to U.S. agriculture data, while Caves, Christensen, and Swanson[4] utilize this model for Class I U.S. railroads data. Unfortunately, however, an important first derivative property inherent in this model has not been clarified theoretically yet, and thus has been neglected by these authors.

The purpose of this paper is twofold. First, this paper proves the important first derivative property of this new model: the variable cost function is non-increasing in fixed input unless the marginal product of the fixed input is negative. Second, this paper shows that this new model makes it possible to estimate short run production technology as well as long run production technology simultaneously within a single econometric framework.

In the following section, the basic frameworks of both total and variable cost function models are explained briefly. Then section III proves with rigour the first derivative property of the variable cost function with respect to fixed input. Section IV demonstrates that this variable cost function can describe both short run and long run technological characteristics. In section V, our discussions are summarized.

## II. Total Cost Function vs. Variable Cost Function

In the full static equilibrium framework, the firm minimizes total cost with respect to all inputs; that is, the objective function is:

$$\begin{aligned} \text{Min}_{X_i} \text{TC} &= \sum_{i=1}^n w_i X_i \\ \text{s.t. } Q &= f(X_1, \dots, X_n), \end{aligned} \quad (1)$$

where TC is the total cost,  $X_i$  is the input,  $w_i$  is the price of  $X_i$ , and  $Q$  is the output. To solve this minimization problem, we can form a

Lagrangian function:

$$L = \sum_{i=1}^n w_i X_i + \lambda [Q - f(X_1, \dots, X_n)]. \quad (2)$$

The first-order conditions for this problem are:

$$\frac{\partial L}{\partial X_i} = w_i - \lambda f_i = 0, \quad i=1, \dots, n, \quad (3a)$$

$$\frac{\partial L}{\partial \lambda} = Q - f(X_1, \dots, X_n) = 0. \quad (3b)$$

By solving this system of equations for  $X_i$  and  $\lambda$  simultaneously, we can obtain the demand equations for all inputs in terms of output and price of inputs:

$$X_i = X_i(Q, w_1, \dots, w_n). \quad (4)$$

By substituting (4) into (1) for  $X_i$ , we obtain the total cost function:

$$\begin{aligned} TC &= \sum_{i=1}^n w_i X_i(Q, w_1, \dots, w_n) \\ &= TC(Q, w_1, \dots, w_n). \end{aligned} \quad (5)$$

As shown above, the total cost function exists only when all of the inputs are instantaneously variable and they are always employed at optimum levels. However, the fixity of some inputs would not leave the full static assumption untouchable. Capital, for instance, is not instantaneously adjustable to the change in the exogenous circumstances. The sluggish adjustment of fixed input suggests that the instantaneously variable inputs only are employed optimally conditional on the given level of fixed input. If this is true, it should be appropriate to assume that the firm minimizes the variable cost for the given level of output and fixed input (rather than the total cost). Under this assumption, the objective function is:

$$\text{Min}_{X_i} VC = \sum_{i=1}^{n-1} w_i X_i \quad (6)$$

$$s.t. \quad Q = f(X_1, \dots, X_n) \text{ and } X_n \text{ is fixed,}$$

where VC is the variable cost, and  $X_n$  is the fixed input. To solve this minimization problem, we can form another Lagrangian function:

$$L = \sum_{i=1}^{n-1} w_i X_i + \mu [Q - f(X_1, \dots, X_n)]. \quad (7)$$

The first-order conditions for this problem are

$$\frac{\partial L}{\partial X_i} = w_i - \mu f_i = 0, \quad i = 1, \dots, n-1, \quad (8a)$$

$$\frac{\partial L}{\partial \mu} = Q - f(X_1, \dots, X_n) = 0. \quad (8b)$$

By solving this system of equations for  $X_i$  and  $\mu$  simultaneously, we can obtain the short run demand equations for variable inputs in terms of output, fixed input, and price of variable inputs:

$$X_i = X_i(Q, X_n, w_1, \dots, w_{n-1}), \quad i = 1, \dots, n-1. \quad (9)$$

By substituting (9) into (6) for  $X_i$ , we obtain the variable cost function:

$$\begin{aligned} VC &= \sum_{i=1}^{n-1} w_i X_i(Q, X_n, w_1, \dots, w_{n-1}) \\ &= VC(Q, X_n, w_1, \dots, w_{n-1}). \end{aligned} \quad (10)$$

The derivative of VC with respect to  $X_n$ ,  $\partial VC / \partial X_n$ , can be interpreted as the shadow (variable) cost of fixed  $X_n$  since it represents the decrease in the variable cost resulting from the substitution of an additional unit of  $X_n$ , holding  $Q$  and  $w_i$  constant. Based on this interpretation, the hypothetical long run equilibrium point can be derived.

The total cost at the point of observation can be obtained by adding fixed costs to the variable cost:

$$\overline{TC} = VC(Q, X_n, w_1, \dots, w_{n-1}) + w_n X_n. \quad (11)$$

Since  $X_n$  may not be in optimum level, the total cost  $\overline{TC}$  can be minimized with respect to  $X_n$ ; that is,

$$\frac{\partial \overline{TC}}{\partial X_n} = \frac{\partial VC}{\partial X_n} + w_n = 0. \quad (12)$$

By solving (12) for  $X_n$ , the total cost minimizing demand for  $X_n$  can be obtained:

$$X_n^* = X_n^*(Q, w_1, \dots, w_n). \quad (13)$$

Thus we can rewrite (12) as the Envelope Theorem:

$$\left. \frac{\partial VC}{\partial X_n} \right|_{X_n = X_n^*} = -w_n \quad \text{or} \quad \frac{\partial VC^*}{\partial X_n} = -w_n. \quad (14)$$

Furthermore, the minimal total cost at the long run equilibrium point can be written as

$$TC^*(Q, w_1, \dots, w_n) = VC^*(Q, X_n^*, w_1, \dots, w_{n-1}) + w_n X_n^*. \quad (15)$$

Equation (14) and (15) show the tangency of short run and long run cost function at  $X_n = X_n^*$ . That is, the long run marginal cost is equal to the short run marginal cost when  $X_n = X_n^*$ .

$$\begin{aligned} LPMC &= \frac{\partial TC^*}{\partial Q} \\ &= \frac{\partial VC^*}{\partial Q} + \frac{\partial VC^*}{\partial X_n} \cdot \frac{\partial X_n^*}{\partial Q} + w_n \cdot \frac{\partial X_n^*}{\partial Q} \\ &= \frac{\partial VC^*}{\partial Q} \text{ by (14)} \\ &= SRMC. \end{aligned} \quad (16)$$

This result implies that the output gradient of the long run cost function is equal to the output gradient of the short run cost function at a point of long run equilibrium. In terms of the traditional graphical analysis of cost curves, this envelope condition simply reflects the fact that at the tangency between short run and long run cost curves, the slope of the total cost curve must equal the slope of the variable cost curve.

### III. The Variable Cost Function and Fixed Input

It has been shown that the total cost is a function of output and price of inputs. The variable cost function, however, contains an additional argument that is the fixed input. This variable cost function satisfies a set of first derivative properties with respect to its independent variables.

*Property (1):* VC is nondecreasing in  $Q$  and  $w_i$  ( $i=1, \dots, n-1$ ).

*Property (2):* VC is homogeneous of degree 1 in  $w_i$  ( $i=1, \dots, n-1$ ).

*Property (3):* VC is nonincreasing in  $X_n$  unless the marginal product of  $X_n$  is negative.

Among the three properties, the first and second properties are com-

mon for the total cost function when  $i$  is extended to  $n$ , and are proved rigorously by numerous authors. But the third property has not been proved theoretically yet. In what follows, this third property is proved with rigour.

Using (10), property (3) is rewritten mathematically.

*Property (3):*

$$-\frac{\partial VC}{\partial X_n} = \sum_{i=1}^{n-1} w_i \frac{\partial X_i}{\partial X_n} \leq 0, \quad (17)$$

unless the marginal product of  $X_n$  is negative.

*Proof:*

a. Single Output

For a given level of output, the production point will move along the isoquant if the amount of fixed input changes; that is,

$$-\frac{\partial Q}{\partial X_n} = \sum_{i=1}^{n-1} f_i \frac{\partial X_i}{\partial X_n} + f_n = 0, \quad (18)$$

where  $f_i$  and  $f_n$  are partial derivatives of production function  $f$  with respect to  $X_i$  and  $X_n$  respectively. From (18), we obtain

$$-f_n = \sum_{i=1}^{n-1} f_i \frac{\partial X_i}{\partial X_n}. \quad (19)$$

From (8a), the first-order conditions for the variable cost minimization are rewritten

$$f_i = \frac{w_i}{\mu}, \quad i=1, \dots, n-1, \quad (20)$$

where  $\mu$  is the Lagrange multiplier, which is nonnegative. By substituting (20) for  $f_i$  in (19), we obtain

$$-f_n = \sum_{i=1}^{n-1} \frac{w_i}{\mu} \frac{\partial X_i}{\partial X_n}; \text{ i.e., } -\mu f_n = \sum_{i=1}^{n-1} w_i \frac{\partial X_i}{\partial X_n}. \quad (21)$$

From (17) and (21), we can prove the following result unless the marginal productivity of  $X_n$  ( $f_n$ ) is negative

$$\frac{\partial VC}{\partial X_n} = \sum_{i=1}^{n-1} w_i \frac{\partial X_i}{\partial X_n} = -\mu f_n \leq 0. \quad (22)$$

b. Multiple Outputs

The multi-product analysis does not permit us to specify the marginal

product of a specific input unless some conditions are imposed on the vector of multiple outputs. Therefore we use the transformation function for a general structure of production to prove the property. The transformation function can be written as

$$F(Q_1, \dots, Q_n, X_1, \dots, X_n) = 1. \quad (23)$$

Taking the derivative of both sides of (23) with respect to  $X_n$  with all outputs held constant, we obtain

$$\sum_{i=1}^{n-1} F_i \frac{\partial X_i}{\partial X_n} + F_n = 0. \quad (24)$$

Equation (24) can be rewritten

$$-F_n = \sum_{i=1}^{n-1} F_i \frac{\partial X_i}{\partial X_n}. \quad (25)$$

Now the first-order conditions for the variable cost minimization problem are:

$$F_i = \frac{w_i}{\mu}, \quad (26)$$

where  $\mu$  is again the Lagrange multiplier. In (26)  $\mu$  and  $F_i$  must have the same sign so that the variable inputs have positive prices. By substituting (26) for  $F_i$  in (25), we obtain

$$-F_n = \sum_{i=1}^{n-1} \frac{w_i}{\mu} \frac{\partial X_i}{\partial X_n}; \text{ i.e., } -\mu F_n = \sum_{i=1}^{n-1} w_i \frac{\partial X_i}{\partial X_n}. \quad (27)$$

Now it is necessary to show that  $\mu$  and  $F_n$  must also have the same sign unless  $X_n$  has negative marginal product.<sup>1)</sup> In (23), suppose that any single variable input  $X_i$  changes with  $X_n$ , and that only this pair of inputs change for outputs held constant. Then we can obtain

$$F_i dX_i + F_n dX_n = 0; \text{ i.e., } \frac{dX_i}{dX_n} = -\frac{F_n}{F_i}. \quad (28)$$

For outputs held fixed,  $X_i$  and  $X_n$  can not change in the same direction

1) The sign of  $F_i$  and  $F_n$  depends on how to set up the transformation function  $F$ .

For a production function  $Q = \sum_{i=1}^{n-1} X_i + X_n$ , for instance,  $F_i$  and  $F_n$  are positive if the transformation function is formed as  $\sum_{i=1}^{n-1} X_i + X_n - Q = 0$ , and they are negative if it is formed as  $Q - \sum_{i=1}^{n-1} X_i - X_n = 0$ .



unless the marginal product of  $X_n$  is negative. So we can define

$$\frac{dX_i}{dX_n} = -\frac{F_n}{F_i} \leq 0. \quad (29)$$

In (29),  $F_n$  must have the same sign as  $F_i$  and thus as  $\mu$ . Therefore, from (17) and (27) we can also prove the following result.

$$\frac{\partial VC}{\partial X_n} = \sum_{i=1}^{n-1} w_i \frac{\partial X_i}{\partial X_n} = -\mu F_n \leq 0. \quad (30)$$

Caves, Christensen, and Swanson [4] use the generalized translog variable cost function to estimate scale economies and productivity growth for Class I U.S. railroads. But their estimated variable cost function violates property (3) as shown in Table 1. Thus it is questionable to use their function for further analysis because it can no longer be a well-behaved variable cost function. A variable cost function can be a well-behaved variable cost function when and only when it satisfies all properties that economic theory requires.

**Table 1. Estimated Elasticity of Variable Cost with Respect to the Fixed Input in Caves, Christensen, and Swanson Study**

1955	0.208
1963	0.235
1974	0.143

Caves, et al. argue that the elasticity may take a positive sign if the combination of output, prices, and fixed input employment is far from an equilibrium combination. In this case the marginal product of the fixed input becomes negative. Then their analysis faces additional problems for further analysis. First, the basic economic assumption that the firm behaves rationally collapses because it operates outside the ridge lines. Second, the fixed input becomes a complement of any other input not because of its complementary characteristics but because of its negative marginal product.

#### IV. Mathematical Derivation of Technological Characteristics

The total cost function permits us to perform long run analysis only. However, it is possible to perform both short and long run analyses simultaneously when the variable cost function is utilized. It is shown in what follows that we can perform short run analysis with VC, and long run analysis with VC\*. It is obvious that the variable cost function provides more information concerning the technological characteristics by short run characteristics.

##### *Short Run Analysis*

Holding  $X_n$  fixed, the short run characteristics of a production technology can be described.

(i) *Scale Effects (elasticity of scale)*

$$e^s = \frac{SAC}{SMC} = \frac{(VC + w_n X_n)/Q}{\partial(VC + w_n X_n)/\partial Q} = \frac{(VC + w_n X_n)/Q}{\partial VC/\partial Q}. \quad (31)$$

(ii) *Substitution Effects (Allen-Uzawa elasticity of substitution)*

$$\sigma_{ij}^s = \frac{VC}{VC_i} \frac{VC_{ij}}{VC_j}, \quad i \neq j, \quad \begin{matrix} i=1, \dots, n-1 \\ j=1, \dots, n-1. \end{matrix} \quad (32)$$

(iii) *Price Elasticities*

Using Shepherd Lemma, the short run cost minimizing demands for the  $i^{th}$  input can be derived.

$$X_i = \frac{\partial VC}{\partial w_i} = X_i(Q, X_n, w_1, \dots, w_{n-1}). \quad (33)$$

From this derived demand equation, own and cross-price elasticities can be obtained.

$$\eta_{ij}^s = \frac{\partial X_i}{\partial w_j} \cdot \frac{w_j}{X_i} = VC_{ij} \cdot \frac{w_j}{X_i}, \quad \begin{matrix} i=1, \dots, n-1 \\ j=1, \dots, n-1. \end{matrix} \quad (34)$$

(iv) *Demand Elasticities with respect to Fixed Input*

$$\epsilon_{in}^s = \frac{\partial X_i}{\partial X_n} \cdot \frac{X_n}{X_i} = VC_{in} \cdot \frac{X_n}{X_i}, \quad i=1, \dots, n-1. \quad (35)$$

(v) *Elasticity of Variable Cost with respect to Fixed Input*

$$e^s = \frac{\partial VC}{\partial X_n} \cdot \frac{X_n}{VC}. \quad (36)$$

### Long Run Analysis

Letting  $X_n$  be freely adjustable, the long run characteristics of a production technology can be described.

#### (i) Scale Effects

$$e^L = \frac{LAC}{LMC} = \frac{(VC^* + w_n X_n^*)/Q}{\partial (VC^* + w_n X_n^*)/\partial Q} = \frac{(VC^* + w_n X_n^*)/Q^{(2)}}{\partial VC^*/\partial Q}. \quad (37)$$

#### (ii) Substitution Effects

$$\begin{aligned} \sigma_{ij}^L &= \frac{TC^* TC_{ij}^*}{TC_i^* TC_j^*} \\ &= \frac{(VC^* + w_n X_n^*)}{VC_i^* VC_j^*} (VC_{ij}^* - VC_{in}^* \cdot VC_{nj}^* / VC_{nn}^*)^{31}, \quad i=1, \dots, n-1, \quad j=1, \dots, n-1. \end{aligned} \quad (38)$$

$$\begin{aligned} \sigma_{in}^L &= \frac{TC^* TC_{in}^* w_n}{TC_i^* TC_{nn}^*} \\ &= - \frac{(VC^* + w_n X_n^*)}{VC_i^* X_n^*} \cdot \frac{VC_{in}^*{}^{41}}{VC_{nn}^*}, \quad i=1, \dots, n-1. \end{aligned} \quad (39)$$

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$$2) \quad \frac{\partial (VC^* + w_n X_n^*)}{\partial Q} = \frac{\partial VC^*}{\partial Q} + \frac{\partial VC^*}{\partial X_n} \cdot \frac{\partial X_n^*}{\partial Q} + w_n \frac{\partial X_n^*}{\partial Q} = \frac{\partial VC^*}{\partial Q} \text{ by (14)}. \quad (F-1)$$

$$3) \quad TC_i^* = \frac{\partial VC^*}{\partial w_i} + \frac{\partial VC^*}{\partial X_n} \cdot \frac{\partial X_n^*}{\partial w_i} + w_n \frac{\partial X_n^*}{\partial w_i} = VC_i^* \text{ by (14)}. \quad (F-2)$$

$$TC_{ij}^* = VC_{ij}^* + VC_{in}^* \frac{X_n^*}{w_j}. \quad (F-3)$$

From (14), we can obtain

$$VC_{nj}^* + VC_{nn}^* \frac{X_n^*}{w_j} = 0. \text{ Thus,}$$

$$\frac{X_n^*}{w_j} = - \frac{VC_{nj}^*}{VC_{nn}^*}. \quad (F-4)$$

Therefore, by substituting (F-4) in (F-3), we can write

$$TC_{ij}^* = VC_{ij}^* - VC_{in}^* \cdot VC_{nj}^* / VC_{nn}^*. \quad (F-5)$$

By substituting (F-2) and (F-5) in (38) for  $TC_i^*$ ,  $TC_j^*$ , and  $TC_{ij}^*$ , we obtain  $\sigma_{ij}^L$  in terms of derivatives of  $VC^*$ .

$$4) \quad TC_{in}^* = \frac{\partial VC^*}{\partial w_i \partial X_n^*} \cdot \frac{\partial X_n^*}{\partial w_n}. \quad (F-6)$$

From (14), we can obtain

$$\frac{\partial^2 VC^*}{\partial X_n^2} \cdot \frac{\partial X_n^*}{\partial w_n} = -1. \quad (F-7)$$

Thus,

$$\frac{\partial X_n^*}{\partial w_n} = - \frac{1}{\partial^2 VC^* / \partial X_n^2}. \quad (F-8)$$

By substituting (F-8) in (F-6), we obtain

$$TC_{in}^* = -VC_{in}^* / VC_{nn}^* \quad (F-9)$$

And

(iii) *Price Elasticities*

By replacing  $X_n$  by  $X_n^*$  in (33), we obtain the long run cost minimizing demands for  $i^{th}$  input.

$$X_i^* = X_i(Q, X_n^*, w_1, \dots, w_{n-1}). \quad (40)$$

$$\begin{aligned} \eta_{ij}^L &= \left[ \frac{\partial X_i^*}{\partial w_j} + \frac{\partial X_i^*}{\partial X_n} \cdot \frac{\partial X_n^*}{\partial w_j} \right] \frac{w_j}{X_i} \\ &= \left[ VC_{ij}^* - VC_{in}^* \cdot \frac{VC_{nj}^*}{VC_{nn}^*} \right] \frac{W_j^{5)}}{X_i^*}, \quad \begin{matrix} i=1, \dots, n-1 \\ j=1, \dots, n-1. \end{matrix} \end{aligned} \quad (41)$$

$$\begin{aligned} \eta_{in}^L &= \left[ \frac{\partial X_i^*}{\partial w_n} \right] \frac{w_n}{X_i^*} \\ &= \left[ \frac{\partial X_i^*}{\partial X_n} \cdot \frac{\partial X_n^*}{\partial w_n} \right] \frac{w_n}{X_i^*} \\ &= \left[ -\frac{VC_{in}^*}{VC_{nn}^*} \right] \frac{w_n^{6)}}{X_i^*}, \quad i=1, \dots, n-1. \end{aligned} \quad (42)$$

$$\begin{aligned} \eta_{nj}^L &= \left[ \frac{\partial X_n^*}{\partial w_j} \right] \frac{w_j}{X_n^*} \\ &= \left[ -\frac{VC_{nj}^*}{VC_{nn}^*} \right] \frac{W_j^{7)}}{X_n^*}, \quad j=1, \dots, n-1. \end{aligned} \quad (43)$$

*Le Chatelier Principle*

The Le Chatelier principle accounts for the proposition that in the long run, i.e., when all inputs can freely adjust to changes in parameter values, the input demand curves are more elastic than in the short run, when some inputs are held fixed at their previous levels. This principle indicates that an optimizing system will react to a change in a parameter value so as to minimize the impact of that shock.

Our variable cost function model incorporates explicitly the Le Chatelier principle. The formula for long run elasticities discussed above contains an additional term in the bracket. This additional term accounts for the

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$$TC_{w_n}^* = \frac{\partial VC^*}{\partial X_n} \cdot \frac{\partial X_n^*}{\partial w_n} + X_n^* + w_n \frac{\partial X_n^*}{\partial w_n} = X_n^* \text{ by (14).} \quad (F-10)$$

By substituting (F-2), (F-9), and (F-10) in (39), we obtain  $\sigma_{in}^L$  in terms of derivatives of VC.

5) See equation (F-5) in previous footnote for  $\partial X_n^*/\partial w_j$ .

6) See equation (F-8) in previous footnote for  $\partial X_n^*/\partial w_n$ .

7) See equation (F-5) in previous footnote for  $\partial X_n^*/\partial w_j$ .

long run impact of the change in parameter value.

### **V. Concluding Remarks**

It has been the contention of this paper that the variable cost function model is a generalized approach to the measurement of technological characteristics because it is based on the assumption that the variable inputs only may be employed optimally at the point of observations while the remaining inputs may be fixed at nonoptimum levels. More importantly, the variable cost function model makes it possible to perform both short run and long run analyses simultaneously within a single econometric framework. The short run technological characteristics have been described with an input held fixed while the long run technological characteristics have been described by letting the input be freely adjustable.

In order for an estimated variable cost function to be applied to the estimation of production technology, it should satisfy a set of theoretical requirements of which one is an additional property to those for total cost function: variable cost function is nonincreasing in fixed input unless the marginal product of the fixed input is negative. This paper has proved with rigour the additional theoretical requirement for variable cost function.

Furthermore, in order to specify the variable cost function for empirical application, it is necessary to adopt an appropriate functional form. In the 1970s, economists developed flexible functional forms such as translog function, quadratic function, and generalized Leontief function. These functional forms are less restrictive and more convenient and useful than Cobb-Douglas or CES type functions are. Among them, translog functional form has often been used in the variable cost function study.

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