Risk Premium Hypothesis in Futures Markets

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I. Introduction

In the last decade the volume of futures trading in commodities has shown a steady increase, and with the introduction of trading in foreign exchanges and debt instruments, financial futures markets were first developed. Concurrently, there have been a growing number of studies on futures markets. A major effort has been directed towards determining the equilibrium futures price in a capital market equilibrium framework. Especially, the downward bias of futures price relative to the expected future spot price, i.e., the risk premium of futures price has been the subject of particular interest.[1] However, most of these studies built market equilibrium models either specifically for a commodity futures market (e.g. Danthine [10], Stoll [22], Anderson and Danthine [1]) or for a financial futures market(e.g. Jacobs and Jones [17]), and none of them investigated the existence (or nonexistence) of risk premium

* Korea Institute of Energy and Resources. This paper is based on the author's Ph. D. dissertation submitted to Washington University in St. Louis. He wishes to thank Robert Parks, William Marshall, and James Little for their helpful comments.

1) This has been a controversial issue of futures maket literature ever since Keynes[19] proposed the "normal backwardation" hypothesis which states that there is a positive risk premium in the futures price as hedgers compensate the speculators for risk bearing. See also Hicks [16], Telser [23, 24], and Cootner [7, 8].

in both markets in a general framework.[2] Also, most of the empirical studies of futures markets were confined primarily to the commodity futures markets (e.g. Dusak [11], Bodie and Rosansky [4], Breeden [5]), and none compared the performance of commodity and financial futures markets on this issue.

This paper notes that if a futures market equilibrium is obtained at the optimal reallocation of risks arising from an uncertain future spot price of a commodity (or a financial asset), then the equilibrium futures prices and the existence of risk premium would fundamentally depend on the net amount of risk in the economy associated with the commodity or the financial asset in consideration. At any future moment of time, there will be a positive amount of commodity in the economy whose value is subject to the uncertain future spot price. Futures trading reallocates this risk among its members; however, it does not eliminate the risk from the perspective of the whole economy. On the other hand, a financial asset is created by a contract between individuals, and its net amount in the economy is identically zero at any moment of time. Based on these observations, this paper proposes the risk premium hypothesis (RPH) whose main tenet is that there is a positive risk premium in the commodity futures market, whereas there is no risk premium in the financial futures market. Empirical evidence of this hypothesis is studied next in the commodity markets of six agricultural products and the foreign exchange markets of five currencies during 1970s, and it is shown that RPH is generally supported empirically.

Rest of the paper is organized as follows. In section II an individual

2) In an economy consisting of producers and speculators, Danthine [10] shows the existence of risk premium. Stoll [22] shows an instance of positive risk premium in an economy of inventory carryover of a commodity stock. Anderson and Danthine [1] posit an economy consisting of producers, processors, and speculators and attribute the risk premium to the imbalance of planned supply and demand for a commodity. Jacobs and Jones [17] derive the equilibrium futures interest rate in a debt instrument futures market.

optimization model of futures transactions is developed, and the market equilibrium solution of futures price is derived. Existence of risk premium is examined in the commodity and financial futures markets, and RPH is proposed. Section III contains the empirical evidence of RPH and section IV concludes the study.

II. Model

To convey the basic idea, the model is developed on a set of highly simplifying assumptions. The natures and implications of some of these assumptions will be discussed later. In this model, an individual in the economy has a one-period planning horizon of utility maximization, the utility being dependent on the mean and the variance of the stochastic terminal wealth. At the beginning of period an individual is endowed with an initial budget and a "future endowment." A future endowment is defined to be a commodity or a financial asset that one is to receive or to deliver at the end of period. It can arise as an output of a given production process or from a contract between individuals which is given at the beginning of period. It is assumed that production of a commodity, if any, is in progress and fixed when the portfolio decision is made at the beginning of period: all inputs are already supplied in the production process, and a non-random positive amount of a commodity is expected to be produced as an outcome of the production process at the end of period.

An individual allocates the budget, first, on purchasing a commodity (or a financial asset) to carry it to the end of period expecting a price advance exceeding the storage cost during the period. This will be called the "carryover stock". In case of a commodity, the carryover stock is nonnegative. For a financial asset, however, there is no such restriction. A positive carryover of a foreign exchange is to lend and a negative carryover is to borrow the foreign currency over the period.[3)]

A positive carryover of a debt instrument is buying and negative carryover is issuing a debt instrument. It is assumed that in a commodity market there is a nonstochastic positive storage cost which is proportional to the amount of carryover stock.[4)] Secondly, the budget is allocated on lending at the prevailing risk free interest rate. If the amount spent on the carryover stock exceeds the budget, one needs to borrow the deficit, which is a negative lending.

Along with the allocation of initial budget, an individual can make a futures contract which is defined to be a commitment to buy or to sell a certain amount of a commodity (or a financial asset) at the end of period at a given futures price. Opening a futures contract is assumed not to incur any cost. Hence the amount of futures positions that one can take is not limited or affected by the size of initial budget.[5)]

Under these conditions, the individual optimization problem is formulated as follows:

$$\underset{N,F,L,}{\text{maximize}}\ U(E(W_1),\ V(W_1)),$$

where $U_1 = \partial U/\partial E(W_1) > 0,\ U_2 = \partial U/\partial V(W_1) < 0,$

subject to $W_1 = (1+r)L + P_1(N+F+G) - P_f F - SN,$

$W_0 = L + P_0 N,$

where U=utility function,

E=expected value operator,

V=variance operator,

3) When there is a positive risk free interest rate, the (positive) carryover of a currency will take the form of lending. Actual holding of a currency at any moment of time is motivated by the purposes other than carrying it to the end of period, and hence should not be considered as a carryover stock.

4) Storage costs in the financial futures markets need different interpretation. It is proposed that there is a negative storage cost, due to interest payment, in carrying a foreign exchange and that there is no storage cost in carrying a debt instrument. Interest payment of a debt instrument is reflected in its discounted price.

5) If a margin deposit as a performance bond of a futures contract is posted by an interest earning asset, there is no opportunity cost of a margin deposit.

N=carryover stock,

F=futures market position,

L=lending,

G=future endowment,

P_0=current spot price,

P_1=future spot price,

P_f=futures price,

S=storage cost,

W_1=terminal wealth,

W_0=initial budget.

All variables other than the future spot price (P_1) are assumed to be nonstochastic, and the uncertainty of terminal wealth originates from the uncertainty of future spot price. At the end of period the future spot price is realized and the uncertainty of terminal wealth is resolved.

In a commodity futures market there is an additional restriction that the carryover stock is nonnegative ($N \geqq 0$), and the first order conditions of the optimization are as follows:

$$\partial U/\partial N = U_1 \cdot (E(P_1) - (1+r)P_0 - S) + 2U_2 V(P_1)\ (N+F+G) \leqq 0, \quad (1)$$

$$N \cdot (\partial U/\partial N) = 0, \quad (2)$$

$$\partial U/\partial F = U_1 \cdot (E(P_1) - P_f) + 2U_2 V(P_1)\ (N+F+G) = 0. \quad (3)$$

From conditions (1) and (3) the futures price is seen to have an upper bound of the sum of current spot price and the implicit and the explicit storage costs ($P_f \leqq (1+r)P_0 + S$). Furthermore, if the futures price is less than this sum, the carryover stock is zero; on the other hand, if the carryover stock is positive, the futures price is equal to this sum ($N \cdot (P_f - (1+r)P_0 - S) = 0$).[6] In a financial futures market, there is no nonnegativity condition of carryover stock, and condition (1) holds

6) This result is well recognized in the early literature. See Blau [3].

an equality while condition (2) is excluded. Conditions (1) and (3) imply that in a financial futures market the futures price is the sum of current spot price and the storage costs.[7]

Sum of carryover stock, futures market position, and future endowment $(N+F+G)$ will be called the "individual net position." It is the net amount of a commodity (or a financial asset) that an individual will be holding at the end of period. From condition (3), the optimum individual net position equation is obtained:

$$N+F+G=\frac{1}{2}\left(-\frac{U_1}{U_2}\right)\frac{1}{V(P_1)}(E(P_1)-P_f). \qquad (4)$$

This equation identifies three factors that determine the individual net position: $(-U_1/U_2)$ measures the degree of risk aversion; $V(P_1)$ measures the degree of uncertainty about the future spot price; and $(E(P_1)-P_f)$ measures the difference between the expected future spot price and the futures price. It shows that the individual net position is smaller in absolute value, if he is more risk averse, if he is more uncertain about the future spot price, or if the smaller is the difference between the expected future spot price and the futures price in absolute value.

As the individual net positions are aggregated over the economy, the following equation is obtained:

$$\sum_j N_j+\sum_j F_j+\sum_j G_j=\frac{1}{2}\sum_j\left\{\left(-\frac{U_1}{U_2}\right)_j\frac{1}{V_j(P_1)}(E_j(P_1)-P_f)\right\}, \qquad (5)$$

where the subscript j denotes the j^{th} individual in the economy. Imposing the market clearing condition of a futures market $(\sum_j F_j=0)$, the market equilibrium furtures price is derived as

$$P_f=\frac{\sum_j w_jE_j(P_1)}{\sum_j w_j}-\frac{(2\sum_j N_j+\sum_j G_j)}{\sum_j w_j}, \qquad (6)$$

7) With suitable interpretation of price variables and storage costs, this spot-futures price relation can be seen to be the parity equation in the foreign exchange market and the term structure equation in the debt instrument market.

where $w_j = (-U_1/U_2)_j / V_j(P_j)$. The first term on the right hand side of (6) is the weighted average of individual expectations of future spot price and will be called the "market expectation of future spot price." The second term on the right hand side shows the discrepancy between the market expectation of future spot price and the equilibrium futures price and represents the "risk premium of futures price." The numerator of the risk premium term $(\sum_j N_j + \sum_j G_j)$ is the sum of individual net positions and will be called the "social net position." It is the net amount of a commodity (or a financial asset) that will be existing in the economy at the end of period.

In the commodity futures market, the sum of carryover stocks$(\sum_j N_j)$ is nonnegative. And the sum of future endowments $(\sum_j G_j)$ is also nonnegative since, while the sum of future endowments arising from production is nonnegative, the sum of future endowments arising from contracts between individuals offset each other and is identically zero. Moreover, it can be argued that the social net position is strictly positive in the commodity market. This is because at any future moment of time there will exist a positive quantity of a commodity in the economy which is either carried over from the past $(\sum_j N_j > 0)$ or produced in the economy at that time $(\sum_j G_j > 0)$. Thus the risk premium should be positive in a commodity futures market. And, other things being equal, the risk premium would be larger when the uncertainty about the future spot price is larger. If it is assumed that the uncertainty increases as the time to maturity of futures contract lengthens, the risk premium would be larger for the futures contract of longer maturity.

In a financial futures market, the carryover stocks are created by contracts between individuals, and hence net carryover of the economy is identically zero. All future endowments are also created by the contracts between individuals, and there is no net addition of them in the economy arising from a production process. Thus the social net position of a financial asset vanishes, and there would be no risk premium in the financial futures market.[8]

Main implications of the model derived so far can be summarized as the following risk premium hypothesis (RPH): (1) there is a positive risk premium in a commodity futures market; (2) the risk premium in a commodity futures market is larger for the contract of longer maturity; and (3) there is no risk premium in a financial futures market.

Now some of the assumptions underlying the development ot RPH need be examined. First, this model treats a futures contract as being identical to a forward contract.[9] The daily settlement feature of futures contract is not captured in a one-period model of arbitrary period length. Nevertheless, under certain assumptions RPH can still be shown to hold in a multiperiod model where the change in futures price in each period is settled at the end of each period (Chung [6]).

Second, the model is developed in an economy of single commodity (or a financial asset). If there are multitude of commodities or financial assets whose future spot prices are correlated to each other, the risk premium of a commodity would depend on the social net positions of

8) Jacobs and Jones [17] develop a similar model of futures market equilibrium in a debt instrument futures market. They conclude that the existence of risk premium depends on the net imbalance of individual lending plans. Their concept of lending plan seems to have a broader scope than the future endowment in this study. While future endowments are amounts committed between individuals, the lending plan includes the additional ex ante demand and supply plans of individuals. This additional demand and supply plans are not included in our model on the ground that there would not be any consistent direction of bias between them, the expected value of net imbalance being zero. Then the existence of risk premium should be determined by the committed amounts of future endowments. Similar issue in a commodity futures market is raised by Anderson and Danthine [1]. This will be examined in detail at the end of this section when the assumption of fixed production scheme is discussed and in the appendix.

9) Differences between these two contracts and implications have been studied by Cox, Ingersoll, and Ross [9], Jarrow and Oldfield [18], French [12], and Richard and Sundaresan [20].

other commodities and financial assets as well. The single commodity model can be rationalized if it is assumed that the variance of the future spot price of a commodity dominates its covariances with the future spot prices of other commodities and financial assets, i.e., if it is not possible to perfectly diversify the risk of holding a commodity.

Third, the model assumes that the production of a commodity is fixed with all inputs already being supplied in the production process at the beginning of period and that there is no further processing of the resulting output as an intermediate good. Various regimes of input-output relations in a commodity market and their implications on the risk premium of futures price are discussed by Anderson and Danthine [1]. They conclude that the bias of futures price depends upon the discrepancy between the planned supply and demand for a commodity and that "it is not possible from a purely theoretical point of view to demonstrate the predominance of backwardation or contango (i.e., positive or negative risk premium)."[10) This result seems basically due to the neglect of the positive correlation of input-output prices of commodities. When there is a planned demand for a commodity as an input of a production process, even though it exceeds its planned supply, the positive output of the production process and the positive covariance of input-output prices would restore the existence of risk premium in a commodity futures market. In appendix, RPH will be discussed in more detail under the various input-output regimes and will be shown to be still valid.

III. Empirical Tests

In this section, the risk premium hypothesis proposed in the last section is empirically examined in the futures markets of six commodities

10) Anderson and Danthine [1, p. 23].

and five foreign exchanges during 1970s. First, using the data of sampling interval shorter than the forecast interval of futures prices, the risk premium term is estimated with the moving average error process (Stockman [21], Hakkio [14]) and, next, using the pooled data of commodities on the one hand and those of foreign exchanges on the other, risk premiums are estimated in the two aggregated markets.

1. Data

The first data set consists of futures and spot prices of six commodities (wheat, corn, oats, soybeans, iced broilers, and plywood) traded in the Chicago Board of Trade during the period of 1972~78. Futures prices (*PF*2, *PF*4, *PF*6, *PF*8, and *PF*10) are the monthly averages of daily closing futures prices during 2, 4, 6, 8, and 10 months prior to each contract month. And the spot price (*PS*) of each contract is the monthly average of daily closing futures prices during the contract month.[11] There are five contract months (March, May, July, September, and December) during a year in wheat, corn, and oats markets; seven contract months (January, March, May, July, August, September, and November) for soybeans; six contract months (January, March, May, July, September, and November) for plywood; and they vary from year to year in the iced broilers market during the data period.

The second data set consists of futures and spot prices of five foreign currencies (British pound, Deutche mark, Swiss franc, Canadian dollar, and Japanese yen) traded in the International Monetary Market division of the Chicago Mercantile Exchange during the period of 1973~81. Futures prices (*PF*2 through *PF*10) are constructed in the same way as those of commodity futures prices. The spot price is the closing futures

11) In a commodity futures market, the delivery can be made on any day of the contract month, given a proper notice, at the seller's option. Hence futures prices in the contract month are effectively the spot prices.

price on the last trading day of each contract.[12] There are four contract months (March, June, September, and December) for each currency.

2. Estimation by the Moving Average Process Fit of the Error Term

Assuming that the market expectation of future spot price is an unbiased estimator of future spot price, the following specification is adopted:

$$\frac{P_{t+k}-P^f_{t,k}}{P^f_{t,k}}=A_k+u_{t,k}\,. \qquad (7)$$

where P_{t+k}=the spot price at time $t+k$,

$P^f_{t,k}$=the futures price at time t for the contract maturing at time $t+k$,

A_k=the risk premium of futures price for the contract maturing in k perods, and

$u_{t,k}$=the random disturbance term with mean zero.

Time to maturity of the contract, k, will be called the "forecast interval" of futures price for the future spot price k periods ahead. The term A_k measures the risk premium relative to the futures price level for the k-month forecast interval. Equation (7) is to be estimated for the forecast intervals ranging from two to ten months in the commodity and the foreign exchange futures markets. In this specification RPH would be confirmed (1) if A_k is significantly positive in the commodity futures markets, (2) if A_k in the commodity market is larger for the futures contract of a longer maturity, i.e., $A_i \geqq A_j$ if $i \geqq j$, and (3) if A_k is not significantly different from zero in the foreign exchange futures markets.

In estimating (7), it is expected that there would be a serial corre-

12) Unlike a commodity futures maket, the delivery date is fixed in the foreign exchange futures market. In the International Monetary Market, the delivery date of foreign exchanges is the third Wednesday of the contract month, and futures trading terminates on the second business day immediately preceding the delivery date. The spot exchange rate on the delivery day and the futures exchange rate on the last trading day is usually negligible.

lation of errors if the forecast interval is longer than the sampling interval of data. This is because a new forecast is made before the error of the previous forecast is realized yet and, thus, before it is added to the information set upon which the new forecast is to be made. Thus, $E(u_{t,k}\ u_{t+h,k})=0$ for $h \geqq k$ and $E(u_{t,k}\ u_{t+h,k}) \neq 0$ for $h<k$ where k is the forecast interval and h is the sampling interval. In this case, for each forecast interval the error term can be specified as following the moving average (MA) process where its order is the number of lags up to which the autocorrelations of errors do not vanish.[13] Thus equation (7) is reformulated with MA process of error term such that

$$\frac{P_{t+k}-P^f_{t,k}}{P^f_{t,k}}=A_k+\varepsilon_t-\theta_1\varepsilon_{t-1}-\theta_2\varepsilon_{t-2}-\cdots-\theta_p\varepsilon_{t-p}, \qquad (8)$$

where p is the order of MA process, ε_t is a normally distributed white noise process and A_k, $\theta_1, \cdots, \theta_p$ are parameters to be estimated. In wheat, corn, and oats markets, the contract months are March, May, July, September, and December. As the length of observation interval is not uniform for a given forecast interval, the number of lags when the autocorrelation of errors starts to vanish varies among different contract months. To limit the complication of estimation, the largest number of lags among different contract months is chosen as the order of MA process. They are 0, 1, 2, 3, and 4 for the forecast intervals of 2, 4, 6, 8, and 10 months respectively. In the soybeans, iced broilers, and plywood markets, the contract months of January, March, May, July, September, and November are chosen for empirical study. The orders of MA processes of errors are then specified as 0, 1, 2, 3, and 4 for the forecast intervals of 2, 4, 6, 8 and 10 months respectively. In the foreign exchange futures markets, the contract months are March, June, September, and December, and the orders of MA processes of errors are

13) Similarly, Stockman [21] and Hakkio [14] specified the fourth order MA process of error term in estimating the risk premium of forward foreign exchange rate using the weekly data of one month forward rates. See also Hansen and Hodrick [15].

specified as 0, 1, 1, 2, and 3 for the forecast intervals of 2, 4, 6, 8, and 10 months respectively.

Table 1 presents the estimation results with given specifications of MA processes of error terms. For some commodities (or foreign exchanges) equations could not be estimated for all maturities, especially for longer ones, due to the intermittent missing values in the data,and as the MA process fit requires the data of non-missing observations. In the case of Japanese yen, absence of data of June 1976 contract made the estimation impossible for all maturities. In addition. Box-Pierce Q-statistics are reported for each estimation. $Q(k)$ tests the null hypothesis that the first k autocorrelations of ϵ_t process are jointly zero. Under the null hypothesis, $Q(k)$ has a chi-square distribution of $(k-p)$ degrees of freedom where p is the number of parameters estimated.

Estimation results show that wheat, corn, and oats have positive but insignificant risk premiums except for the ten-month maturities of wheat and corn. On the otherhand, soybeans, iced broilers,and plywood have significantly positive risk premiums for all of their maturities. And it is notable that the risk premiums increase monotonically as the time to maturity of contracts becomes longer. Except for two cases ($Q(6)$ of ten-month maturities of sopbeans and plywood), all of Box-Pierce Q-statistics of six commodities are not significantly different from zero at the 5% significance level, which indicates that the MA process fit of error term is generally adequate.

In the foreign exchange markets, risk premiums are not significantly different from zero for all maturities of all currencies except for one case (six-month maturity of Canadian dollar). And there is no pronounced relationship between the magnitude of risk premium and the time to maturity of contract; risk premiums of British pound and Deutche mark do not have a uniform sign over different maturities, and Swiss franc and Canadian dollar have positive and negative risk premiums

Table 1. Estimation of Risk Premiums: $\frac{PS-PF}{PF}=A+\varepsilon_t-\theta_1\varepsilon_{t-1}-\theta_2\varepsilon_{t-2}-\cdots-\theta_p\varepsilon_{t-p}$

Commodities			N	P	A	θ_1	θ_2	θ_3	θ_4	$Q(6)$	$Q(12)$	$Q(18)$	$Q(24)$
	wheat	*PF2*	35	0	0.026 (1.15)	—	—	—	—	8.14	12.76	14.46	15.50
		PF4	34	1	0.047 (1.13)	−0.294 (−1.74)	—	—	—	5.73	8.93	10.35	11.85
		PF6	33	2	0.063 (0.73)	−0.907 (5.60)	−0.461 (−2.85)	—	—	2.64	4.67	5.53	6.45
		PF8	32	3	0.113 (0.18)	−0.910 (4.83)	−0.813 (−3.96)	−0.088 (−0.46)	—	2.13	4.77	5.79	6.90
		PF10	31	4	0.187* (1.90)	−1.188 (−5.67)	−1.195 (−3.90)	−0.333 (−1.06)	0.009 (0.04)	3.02	3.37	5.36	6.43
	Corn	*PF2*	35	0	0.014 (1.04)	—	—	—	—	5.86	7.75	9.02	13.76
		PF4	34	1	0.035 (1.07)	−0.500 (−3.26)	—	—	—	1.02	1.44	1.97	5.54
		EP6	33	2	0.050 (0.87)	−1.220 (−11.66)	−0.910 (−8.70)	—	—	3.28	4.89	7.56	8.65
		PF8	32	3	0.063 (1.02)	−1.058 (−5.67)	−1.017 (−5.10)	−0.183 (−0.96)	—	2.21	3.61	5.07	6.73
		PF10	31	4	0.112* (0.97)	−1.539 (−8.88)	−1.211 (−3.88)	−0.904 (−2.84)	−0.601 (−3.39)	2.71	4.40	6.47	8.52
	Oats	*PF2*	35	0	0.013 (0.93)	—	—	—	—	4.80	9.32	11.14	12.80
		PF4	34	1	0.027 (1.09)	−0.242 (−1.41)	—	—	—	3.93	8.51	9.95	13.75
		PF6	33	2	0.045 (1.27)	−0.481 (−2.63)	−0.112 (−0.61)	—	—	5.80	8.79	10.01	13.73
	Soybeans	*PF2*	41	0	0.031* (1.56)	—	—	—	—	7.70	10.43	13.37	17.07

(Table 1 Continued)

			N	P	A	θ_1	θ_2	θ_3	θ_4	Q(6)	Q (12)	Q (18)	Q (24)
	(Soybeans continued)	PF4	40	1	0.083* (1.86)	−0.840 (−7.89)	—	—	—	2.87	5.05	7.63	9.70
		PF6	39	2	0.136* (1.99)	−1.040 (−7.08)	−0.482 (−3.27)	—	—	2.91	5.64	9.04	10.18
		PF8	38	3	0.171* (1.91)	−1.190 (−8.20)	−0.853 (−4.23)	−0.590 (−3.90)	—	0.22	1.64	3.25	7.08
		PF10	37	4	0.215** (2.79)	−1.442 (−9.15)	−1.044 (−4.11)	−0.644 (−2.54)	−0.493 (−3.10)	5.94**	6.60	8.66	11.05
	Iced Broilers	PF2	41	0	0.026** (2.69)	—	—	—	—	10.26	15.99	23.81	25.64
		PF4	40	1	0.051** (2.10)	−0.624 (−4.90)	—	—	—	3.44	6.99	10.65	11.39
		PF6	32	2	0.071** (2.53)	−1.452 (−15.09)	−0.969 (−9.60)	—	—	0.52	3.46	6.09	7.69
	Plywood	PF2	41	0	0.038** (2.18)	—	—	—	—	7.06	12.73	13.24	13.97
		PF4	40	1	0.068** (2.03)	−0.633 (−5.02)	—	—	—	7.86	10.67	11.07	11.54
		PF6	39	2	0.095**	−0.755	−0.991	—	—	7.72	11.64	12.37	13.61
		PF8	38	3	0.115** (2.14)	−0.790 (−6.24)	−0.907 (−7.35)	−0.806 (−6.25)	—	5.33	10.13	11.22	12.15
		PF10	37	4	0.126** (2.81)	−1.149 (−9.43)	−1.076 (−6.23)	−0.924 (−5.25)	0.857 (−6.57)	6.08*	6.97	9.43	10.52
Foreign Exchanges	British Pound	PF2	36	0	−0.00205 (−0.40)	—	—	—	—	3.19	6.07	8.81	10.89
		PF4	36	1	0.00222 (0.20)	−0.43112 (−2.70)	—	—	—	1.60	4.33	7.96	8.84
		PF6	36	1	0.00371 (0.27)	−0.75988 (−6.29)	—	—	—	4.90	7.96	12.53	16.10
		PF8	36	2	0.00067 (0.03)	−1.00688 (−6.42)	−0.43608 (−2.78)	—	—	2.18	3.76	6.92	9.46

(Table 1 Continued)

		N	P	A	θ_1	θ_2	θ_3	θ_4	Q(6)	Q(12)	Q(18)	Q(24)
(British Pound continued)	*PF10*	36	3	−0.00415 (−0.18)	−0.95599 (−6.21)	−1.09307 (−8.05)	−1.52724 (−3.24)	—	2.79	7.01	11.91	14.51
Deutche Mark	*PF2*	36	0	−0.00353	—	—	—	—	2.83	7.13	10.32	12.76
	PF4	36	1	0.00819	−0.64130	—	—	—	1.92	4.99	9.32	11.36
	PF6	36	1	0.00309 (0.19)	−0.84793 (−8.92)	—	—	—	1.28	6.45	9.54	12.48
	PF8	36	2	0.00385 (0.17)	−1.07135 (−6.38)	−0.26541 (−1.54)	—	—	0.09	4.14	5.94	7.71
Swiss Franc	*PF2*	36	0	0.00155 (0.19)	—	—	—	—	3.30	6.49	9.48	13.19
	PF4	36	1	0.01533	−0.38600	—	—	—	1.07	5.08	8.34	10.72
	PF6	36	1	0.01637 (0.86)	−0.81053 (−7.58)	—	—	—	1.50	5.51	6.73	9.79
	PF8	36	2	0.02428 (0.90)	−0.89499 (−5.57)	−0.41090 (−2.54)	—	—	1.18	4.46	5.96	7.42
Canadian Dollar	*PF2*	36	0	−0.00386 (−1.64)	—	—	—	—	7.42	10.12	11.68	13.01
	PF4	36	1	−0.00568 (−1.49)	−0.52874 (−3.56)	—	—	—	3.39	6.58	8.34	11.50
	PF6	36	1	−0.01014** (−2.21)	−0.95272 (−12.40)	—	—	—	3.15	5.32	7.69	9.95

Notes: N is the number of observations. P is the order of MA process. Figures in the parentheses are t-statistics. $Q(k)$ is the Box-Pierce Q-statistic which tests the null hypothesis that the first k autocorrelations are jointly equal to zero. $Q(k)$ is distributed as $\chi^2(k-p)$ where p is is the number of parameters estimated. **(*) of 'A' denotes the significance at 5% (10%) level.
* of Q-statistics denotes the significance at 5% level.

respectively. And none of Box-Pierce Q-statistics are significant. While not clearcut, it may be concluded that RPH is generally supported by this empirical study. All of the estimated risk premium terms are positive in the commodity markets, and they do not have a uniform sign in the foreign exchange markets. Even though eleven out of twenty-seven risk premium terms of the commodity markets are not significant at the 10% significance level, in most of the cases they have higher t-values in absolute value than those of foreign exchange markets. Also, the estimated risk premium terms of the commodity markets are larger in absolute value than those of foreign exchange markets. In addition, for all six commodities the size of the estimated risk premium terms increases as the time to maturity of contract becomes longer confirming the hypothesis that the risk premium increases as the uncertainty about the future spot price increases.

3. Estimation from the Pooled Data

In the last section, to utilize all the contracts that matured during the data period, it was necessary to fit the moving average process of the error term in estimating the risk premium when the forecast interval is longer than the sampling interval. Another approach to increase the data size may be to aggregate the observations across different commodities (or foreign exchanges). Bilson [2] tests the efficiency of forward markets of foreign exchanges from the pooled data of nine foreign currencies. He observes that the large variations of data values across currencies increase the precision of estimation.

In this section the previous data of six commodities are pooled on the one hand, and those of five currencies on the other. Then the risk premiums in these two aggregated markets are estimated and compared to each other. The sampling interval is fixed on six months, and the forecast intervals of two, four, and six months are chosen. As the sampling interval is longer than that of the previous study, the number of observations in each commodity (or foreign exchange) is reduced; however, total number of observations in each of the aggregated markets is larger due to data pooling. And as the forecast intervals are shorter than or equal to the sampling interval, it is expected that there would be no autocorrelations of errors. Specifically, March and September contracts are chosen to form a six-month sampling interval since they are the common contract months of the six-month interval for all commodities and currencies, and equation (7) is estimated for the aggregated commodity and foreign exchange markets.

First, assuming that the variances of disturbance terms are the same for all observations and that they are not correlated across different observations, equation (7) is estimated by the ordinary least squares (OLS). Table 2 reports the OLS estimation results. In the aggreated commodity market, the risk premiums of different maturities are all positive and significant. And they are larger for longer maturities. In the aggregated foreign exchange market, none of the risk premiums are significantly

Table 2. OLS Estimation of Risk Premium from the Pooled Data: $(PS-PF)/PF=A+u$

Commodity Market

	A	$(S.E.)$	N
$PF6$	0.084**	(0.031)	77
$PF4$	0.078**	(0.025)	78
$PF2$	0.030**	(0.015)	84

Foreign Exchange Market

	A	$(S.E.)$	N
$PF6$	0.001	(0.009)	89
$PF4$	0.005	(0.007)	90
$PF2$	−0.002	(0.005)	90

Notes: **denotes the significance 5% level. $S.E.$ in parenthesis is the standard error of 'A'. N is the number of observations.

Table 3. GLS Estimation of Risk Premium From the Pooled Data: $(PS-PF)/PF=A+u$

Commodity Market

	A	$(S.E.)$	N
$PF6$	0.065**	(0.028)	72
$PF4$	0.035	(0.022)	78
$PF2$	0.008	(0.014)	84

Foreign Exchange Matket

	A	$(S.E.)$	N
$PF6$	−0.008	(0.005)	85
$PF4$	−0.004	(0.004)	90
$PF2$	−0.005	(0.004)	90

Notes: Covariance matrix of error terms for GLS estimation is obtained from the sample residuals of OLS estimation. **denotes the significance at 5% level. $S.E.$ in parenthesis is the standard error of 'A'. N is the number of observations.

different from zero, and there is no pronounced trend of risk premium as the maturity lengthens. These results give a strong support for RPH.

The assumptions underlying the OLS estimation are restrictive. The variances of disturbance terms may be different among different commodities (or foreign exchanges), and it is also likely that they are contemporaneously correlated across commodities(or foreign exchanges). To incorporate these considerations, equation (7) is next estimated by the generalized least squares (GLS) using the sample covariance matrix of OLS residuals of equation (7). Table 3 shows the GLS estimation results. While not so favorable as OLS results, they support RPH reasonably well. In the commodity market, risk premiums are positive for all three maturities, and it is significantly different from zero at 5% level for the six-month maturity. And the estimated risk premiums are larger for longer maturities. In the foreign exchange market, risk premiums are all negative but none of them is significantly different from zero, and there is no particular relationship between the size of risk premium and the time to maturity of futures contract.

IV. Concluding Remarks

Based on the notion that the net amount of a commodity in the economy will be positive at any future moment of time, while that of a financial asset will be zero, this paper has proposed the risk premium hypothesis which states that the risk premium is positive in a commodity futures market and zero in a financial futures market. As the purpose of this study is to compare these two markets with regard to the existence or non-existence of risk premium rather than to identify the factors that determine its size, it has ignored some of the fine points developed in the literature but deemed nonessential in the context of this study. If these considerations are incorporated, a more refined model might be obtained. Especially, other than the assumptions examined at the end of Section II, two more points seem to worth further study. One is to consider the market participation of the government. If the government has a utility function not related to the level of its terminal wealth,the (exogenously determined) government net position can be an additional source of futures price bias. The other is to consider that the market participants may evaluate the terminal wealth in real terms. Grauer and Litzenberger [13] address this problem and identify several factors that may cause the bias of futures price from the expected future spot price. The risk premium hypothesis model based on the real wealth maximization perhaps built along the line of Grauer and Litzenberger need be studied.

Results of the empirical study in Section III are seen to generally support the risk premium hypothesis: while some of them are not significant, all the risk premium terms estimated in the commodity futures markets are positive; they are larger for longer maturities;and in foreign exchange futures markets, most of them are not significantly different from zero. Empirical study of financial futures markets is limited to the foreign exchange futures markets in this paper. This is because futures trading in the debt instruments started in 1976, and the size of data available in these markets is not considered large enough for a reliable empirical study. And it is also intended to keep approximately the same data period for both commodities and financial assets in the empirical study. As more data become available in the future, test of the risk premium hypothesis in the debt instrument futures market should be done.

Appendix

As indicated in Section II, the model in this study has assumed that the production process is fixed and already in progress with given inputs and that there is no further processing of the resulting output of the commodity. To examine the validity of RPH when these assumptions are relaxed, the model is recast in three different contexts: (1) when there is a given amount of a planned input supply in the economy which will be used to produce a commodity in question at the end of period; (2) when there is a planned demand for the commodity at the end of period as an input to the second production process; and (3) when there is no fixed plan of producing an input and, for that matter, of processing this input to produce an output, i.e., when the length of the period is a Marshallian long run where every component of the production plan is variable.

Consider a production process involving two commodities: commodity 1 being an input and commodity 2 an output. Assume that the production is accomplished at the end of period, and that the marginal product of commodity 1, which may depend on other factors of production, is a constant $1/k$. Then the plannd market input demand Q_1 and the planned market output supply Q_2 will have the relationship $Q_2=(1/k)Q_1$, and their respective the market expectation of future spot prices will have the relationship $P_2=kP_1$ in a competitive economy. Assume also that there is a fixed, positive amount G_1 of social net position of commodity 1.

If, for notational simplicity, all individuals are assumed to have the same perception of the probability distributions of future spot prices of commodities, a one-period multicommodity mean-variance model will show, as a direct analogue to equation (6), that in the maket equilibrium,

$$\begin{bmatrix} E(P_1)-P_1{}^f \\ E(P_2)-P_2{}^f \end{bmatrix} = 2\left\{\sum_j\left(-\frac{U_1}{U_2}\right)_j\right\}^{-1}\begin{bmatrix} \sigma_{11}\sigma_{12} \\ \sigma_{21}\sigma_{22} \end{bmatrix}\begin{bmatrix} G_1-Q_1 \\ Q_2 \end{bmatrix}, \qquad (A1)$$

where $P_i{}^f$ is a futures price of commodity i and σ_{ij} is the covariance of P_i and P_j. From (A1),

$$\begin{bmatrix} E(P_1)-P_1{}^f \\ E(P_2)-P_2{}^f \end{bmatrix} = 2\left\{\sum_j\left(-\frac{U_1}{U_2}\right)_j\right\}^{-1}\begin{bmatrix} \sigma_{11}G_1+\sigma_{11}(-Q_1+kQ_2) \\ k\sigma_{11}G_1+k\sigma_{11}(-Q_1+kQ_2) \end{bmatrix}, \qquad (A2)$$

since $\sigma_{12}=\sigma_{21}=k\sigma_{11}$ and $\sigma_{22}=k^2\sigma_{11}$. The left hand side of (A2) is the vector of risk premiums of two commodities. As $(-Q_1+kQ_2)=0$, the case of commodity 1 shows that when there is a positive social net position of a commodity ($G_1>0$), the risk premium is positive regardless of the demand for the commodity as an input. For example, the wheat futures price would have a risk premium regardless of the demand for wheat to produce flour. Also, the case of commodity 2 shows that, if there is a positive social net position of an input commodity, the final output commodity would have a risk premium, even if the production process for the final output has not started at the beginning of period. In the previous example, if there is a futures market of flour, it will also have the risk premium, if there is wheat to be produced at the end of period. As the third possibility, if there is no fixed plan of production at all, and every input is variable, assuming that the carryover stock is zero, the social net position of an input commodity is zero ($G_1=0$), and there would be no risk premiums in both input and output commodities. However, given the current trading practice of up to one or two years of maturities in the commodity futures markets, this possibility does not seem relevant in a practical sense.

References

[1] Anderson, Ronald W., and Jean-Pierre Danthine, "Hedger Diversity in Futures Markets: Backwardation and the Coordination of Plans," Research

Working Paper No. 71A, New York, N.Y., Columbia Univ., Graduate School of Business, 1980.

〔2〕 Bilson, J.F.O., "The Speculative Efficiency Hypothesis," *Journal of Business*, Vol. 54 (1981), pp. 435-51.

〔3〕 Blau, G., "Some Aspects of the Theory of Futures Trading," *The Review of Economics and Statistics*, Vol. 12 (1944~45), pp. 1-30.

〔4〕 Bodie, Zvi, and Victor I. Rosansky, "Risk and Return in Commodity Futures," *Financial Analysist's Journal*, May/June 1980, pp. 27-37.

〔5〕 Breeden, Doulas T., "Consumption Risk in Futures Markets," *The Journal of Finance*, Vol. 35 (May 1980), pp. 503-20.

〔6〕 Chung, Dosung., "Individual Optimization and Market Equilibrium in Futures Markets," Unpublised Ph.D. Disseration, St. Louis, Missouri, Washington University, Department of Economics, 1982.

〔7〕 Cootner, P.H., "Returns to Speculators: Telser versus Keynes," *Journal of Political Economy*, Vol. 68 (August 1960), pp. 396-404.

〔8〕 Cootner, P.H., "Rejoinder," *Journal of Political Economy*, Vol. 68 (August 1960), pp. 415-18.

〔9〕 Cox, J.D., Jr. J. E. Ingersoll, and S.A. Ross, "The Relation between Forward Prices and Futures Price," *Journal of Financial Economics*, Vol. 9 (December 1981), pp. 321-46.

〔10〕 Danthine, Jean-Pierre., "Information, Futures Prices, and Stabilizing Speculation," *Journal of Economic Theory*, Vol. 17 (February 1978), pp. 70-98.

〔11〕 Dusak, K., "Futures Trading and Investor Returns: An Investigation of Commodity Market Risk Premiums," *Journal of Political Economy*, Vol. 81 (November/December 1973), pp. 1387-406.

〔12〕 French, K.R., "The Pricing of Futures Contract," Unpublished Manuscript, Rochester, N.Y., University of Rochester, Graduate School of Management, 1981.

〔13〕 Grauer, F.L.A., and R.H. Litzenberger, "The Pricing of Commodity Futures Contracts, Nominal Bonds and Other Risky Assets Under Commodity Price Uncertainy," *The Journal of Finance*, Vol. 34 (March 1979), pp. 769-83.

〔14〕 Hakkio, C.R., "Expectations and the Forward Exchange Rate," Working Paper No. 439, Cambridge, Mass., N.B.E.R., 1980.

〔15〕 Hansen, L.P., and R.J. Hodrick, "Forward Exchange Rates as Optimal Predictors of Future Spot Rates: An Econometric Analysis," *Journal of Political Economy*, Vol. 88 (October 1980), pp. 829-53.

〔16〕 Hicks, J.R., *Value and Capital*, 2nd ed., London, Oxford University Press,

1946.
[17] Jacobs, R.L., and R.A. Jones, "The Treasury-Bill Futures Market," ***Journal of Political Economy***, Vol. 88 (August 1980), pp. 699-721.
[18] Jarrow, R.A., and G.S. Oldfield, "Forward Contracts and Futures Contracts," ***Journal of Financial Economics***, Vol. 9 (December 1981), pp. 373-82.
[19] Keynes, J.M., ***A Treatise on Money Vol. II: The Applied Theory of Money***, London: Macmillan & Co., 1930.
[20] Richard, S.F., and M. Sundaresan, "A Continuous Time Equilibrium Model of Forward Prices and Futures Prices in a Multigood Economy," ***Journal of Financial Economics***, Vol. 9 (December 1981), pp. 347-71.
[21] Stockman, A.C., "Risk, Information, and the Forward Exchange Rates," In ***The Economics of Exchange Rates***, edited by J. Frenkel and H. Johnson, Reading, Mass., Addison-Wesley, 1978.
[22] Stoll, H.R., "Commodity Futures and Spot Price Determination and Hedging in Capital Market Equilibrium," ***Journal of Financial and Quantitative Analysis***, Vol. 14 (November 1971), pp. 873-94.
[23] Telser, L.G., "Futures Trading and the Storage of Cotton and Wheat," ***Journal of Political Economy***, Vol. 66 (June 1958), pp. 233-55.
[24] Telser, L.F., "Reply," ***Journal of Political Economy***, Vol. 68 (August 1960), pp. 404-15.