

The Factor Substitutability The Place of Working Capital in the Firm's Production Process

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As an exploratory attempt, this study presents a detailed evaluation of the role of working capital as an input factor by estimating the translog cost function model for the aggregate and nine two- and three-digit (SIC code) U.S. manufacturing industries over the period 1958 to 1973. Our major concerned hypothesis is "Is there substitution possibilities between working capital and other productive factors in the firm's production process?" We found the answer is, generally, yes. The computed PES (Partial Elasticities of Substitution) support the existence of substitution possibilities between working capital and other input factors in our cost function model. Furthermore, as an interesting finding, the results indicate that the PES between working capital and labor in some industries are larger than those of capital and labor in our sample industries.

I. Introduction

Considerable efforts have been devoted recently to examine empirically the firm's money holding behavior. Based on both micro- and macro-economic aspects of rationales,¹⁾ the arguments of treating money balances as a productive factor in the firm's production process have been developed widely.²⁾ Unfortunately, however, not many of serious explicit arguments

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1) See for more details, Ji-Sung Yoo, "The Factor Substitutability: The Place of Money in Firm's Production Process," *The Economic Research II* (Hanyang University, 1980).

2) See for details, John R. Moroney, "The Current State of Money and Production Theory," *The American Economic Review*, Proc. 62 (May 1972), pp. 335~43.

on the role of working capital which includes money balances as a part of its definition have been pursued in the literature. That is, it is almost true that economists have been neglected to consider the working capital as a crucial variable in the theory of production.

The money balances are a constituent element of the firm's working capital according to the conventional definition. "The contracting and exchange conditions confronting the firm over a given production interval requires financial and real resources in the form of working capital."³⁾ In other words, working capital is required for increasing the economic efficiency by smoothing the material as well as financial needs in the firm's production process. Thus, working capital could also be treated as a productive factor entering the production function.

The purpose of this study is to present a detailed evaluation of the role of working capital as a productive factor for the aggregate and nine two- and three-digit (SIC code) U.S. manufacturing industries over the period 1958 to 1973.

Section II develops the theoretical arguments of the role of working capital and considers the implications of treating working capital as a productive factor. Section III describes the cost function framework, and then the estimation method is followed in section IV. In section V, we present the empirical results, and the last section summarizes our findings. The data requirements and sources are described in appendix.

II. Theoretical Considerations

In this section, as an exploratory study, we intended to establish a theoretical basis for working capital to be treated as a productive factor entering the production function. Hence, we can measure the substitution possibilities between working capital and other input factors by employing the cost function model and statistical methods shown in the later section.

3) V. Kerry Smith and Raymond J. Kopp, "Econconsidering the Demand for Money by Firms," 1978, p. 2 (Mimeographed).

Working capital can be defined as the current assets of the firm, notably cash, marketable securities, account receivable, and inventories.⁴⁾ These asset forms are typified as short-lived and hence their life span does not in general exceed one year.⁵⁾ Alternatively, "in a broader and perhaps more useful sense, working capital refers to both the current assets and current liabilities, the latter including accounts payable, short term bank loans, and other payables and accruals becoming due within a year."⁶⁾ Therefore, the net working capital is defined generally as the current assets less the current liabilities, and is thus a single dollar value.

It seems worthwhile to pursue the theoretical underpinings of working capital in order to grasp its role clearly in the firm's production process. A necessary step in this pursuit is to understand the difference in interpretation of the concept of working capital by economists and accountants.

A search for the antecedents of asset classification in financial statements leads first of all to a distinction long made by economists in their analysis of capital—the differentiation of capital which is 'fixed' from that which is 'circulating.' "Adam Smith's reformation of Quesney's theory of capital produced the dichotomy between 'fixed' and 'circulating' capital, which persists in the literature of economics and accounting to the present time."⁷⁾ The close relationship between this economic concept and accounting classification standards has long been recognized and emphasized by accountants. Further, among others, Paton and Stevenson observed that the distinction made in economic theory between circulating and fixed capital follows much the same line of demarcation as the

4) Dileep R. Mehta, *Working Capital Management*, (New Jersey: Prentice-Hall, Inc., 1974), p. 1; Keith V. Smith, "An Overview of Working Capital," K.V. Smith, ed., *Management of Working Capital* (Los Angeles: West Publishing Co., 1974), p. 4.

5) Mehta, *Working Capital Management*, p. 1

"In practice, however, some assets that violate this criterion are still classified as working capital."

6) Smith, "An Overview of Working Capital," p. 4.

7) William Huizingh, *Working Capital Classification*, (Ann Arbor, Michigan: Michigan University Press, 1967), p. 22.

distinction between current and fixed assets.⁸⁾ For example, accountants recognize that

the courts (in England) were using the term capital with a meaning quite different from that ascribed to it in business practice, that the public was likely to misinterpret the expressions fixed capital and circulating capital, and that the term asset was better understood and more clearly conveyed the meaning intended.⁹⁾

That is, the permanent or long-lived assets might be acquired through borrowing rather than investment, and hence it is assets and not capital in the businessmen's sense.

Alternatively, Keynes's attempt to develop a trichotomy of capital offers interesting point of consideration for a better understanding of the firm's asset structures. Keynes viewed capital as:

being embodied in (1) goods in use, which are only capable of giving up gradually their full yield of use or enjoyment; (2) goods in progress, i.e., in course of preparation by cultivation or manufacture for use of consumption, or in transport, or with merchants, dealers, and retailers, or awaiting the rotation of the seasons; (3) goods in stock, which are yielding nothing but are capable of being used or consumed at any time.¹⁰⁾

He names these three classifications as the fixed capital, working capital, and liquid capital, respectively. He further acknowledged that there is, of course, no sharp line of division between fixed and liquid goods. On the other hand, he defined the working capital as all goods in the production process, including such minimum stocks as are necessary to assure continuity of operations but excluding surplus stocks (those in excess of the minimum required to avoid interruption of productive activities), which he regarded as liquid capital.

8) William A. Paton and Russel A. Stevenson, *Principles of Accounting*. (New York: Macmillan Company, 1918), p. 37.

9) Huizingh, *Working Capital Classification*, p. 32.

10) J. M. Keynes, *A Treatise on Money*, I (London: Macmillan Company, 1930), pp. 128~29.

Following this line of argument, even though it is slightly deviated, Kieso and Weygandt favor the elimination of inventories from the definition of working capital. They argue that

a satisfactory current assets may be tied up in slow-moving inventories. With inventories, especially raw material and work in progress, there is a question of how long it will take to transform them into the finished product and what ultimately will be realized on the sale of merchandise. Elimination of the inventories, along with any prepaid expenses, from the current assets might provide better information for the short term creditor.¹¹⁾

Nevertheless, the inclusion of inventories in the definition of working capital is widely acceptable and used in the theory of working capital management.

The need of working capital arises from the generation of activity in an ongoing business. For example, as sales increase, accounts receivable increases and funds are required to carry these. Thus, working capital is required to satisfy such needs. Even though borrowed funds may be used for working capital over a fairly long period of time, the amount used fluctuates, depending on the cyclical aspects of business activities. In fact, it is the varying use of greater or lesser amounts of money over a single year that most quickly identifies the funds used on such a fluctuating basis as working capital.¹²⁾

Maintaining the proper amount of liquidity (or working capital) bears cost. That is, in the real world where imperfections exist in the capital markets which result in the borrowing rate exceeding the lending rate, there exists a 'cost' of maintaining liquidity. In a general sense, the cost might be thought to be the differences between interest earned on the investment of funds in liquid assets and the cost of financing.¹³⁾

11) Donald E. Kieso and Jerry J. Weygandt, *Intermediate Accounting*, 2nd ed. (New York: John Wiley and Sons, 1977), pp. 1022~23.

12) Royce Diener, "Analyzing the Financing Potential of a Business," in *Management of Working Capital*, ed Keith V. Smith, p. 261.

13) James C. Van Horne, *Financial Management and Policy*, 4th ed. (New Jersey: Prentice-Hall, Inc., 1977), pp. 327~28.

Nevertheless, firms usually keep large amounts of liquidity to maintain their own liquidity standards and meet their fluctuating cash requirements.¹⁴⁾

The maintaining of working capital in the firm's production process above all depends on the objective of maximizing the overall value of the firm, which, in turn, determines the level of working capital. In other words, the contracting and exchange conditions confronting the firm over a given production interval require the working capital for maximizing the overall value of the firm by smoothing the firm's production process.

Smith and Kopp state that

working capital is productive in a sense very similar to any other factor input....because we observe that working capital serves to maintain continuously the production activities through time. Any deficiency in this capital would lead to interruptions and be perceived with information collected at discrete intervals (i.e., at the quarterly or yearly points) as reducing the output produced in the period. Equally important, it is reasonable to conceive of substituting working capital for other inputs by considering the adoption of these activities...¹⁵⁾

A further reason for working capital being a productive factor is indicated in Keynes' statement that:

...it may be possible to increase the intensity of employment, i.e., the amount of the factors of production applied per unit of time to a unit of product in process, with the result of accelerating the rate of process and shortening the duration of process...It follows that an extra demand for working capital corresponding to an increased volume of employment may be partly offset by a faster rate of process. If the duration of process is halved by doubling the intensity of employment, the demand for working capital is (eventually), other things being equal, halved.¹⁶⁾

14) Paul S. Nadler, "Compensating Balances and the Prime at Twilight," in *Management of Working Capital*, ed Keith V. Smith, p. 261.

15) Smith and Kopp, "Reconsidering Demand for Money," p. 5.

16) Keynes, *A Treatise on Money*, II., p. 120.

Relying on the above rationale, we can extend the nature of our argument further by treating working capital as a productive factor entering the production function as did the money balances in the literature,¹⁷⁾ and we can measure the substitution possibilities between working capital and other input factors.

III. The Cost Function Framework

As an alternative specification for the production technology over the production function, the cost function is frequently applied to estimate the factor-demand. The productions are largely unobservable, and the data points represent a sampling of input and output levels which might take place at different times, as factor or output prices change. The cost function, on the other hand, is a function of factor prices and output levels, all of which are readily observable. Hence, the cost function may be easier to estimate, statistically, than the production function.¹⁸⁾ Although the factor demands are derived for fictitious (or nonexistent) production process, the duality principle¹⁹⁾ assures that there exists a unique underlying production function as long as the cost function satisfies some elementary properties, i.e., the linear homogeneity²⁰⁾ and concavity in factor prices. Furthermore, once estimated, the cost function can be used to derive directly the constant output factor-demand curve using the Shephard Lemma, $X_i^* = \partial C^* / \partial p_i$.²¹⁾

17) Refer to Footnote #2.

18) For further discussion on the advantages of cost function over production function, see Hans P. Binswanger, "A Cost Function Approach to the Measurement of Elasticities of Factor Demand and Elasticities of Substitution," *American Journal of Agricultural Economics* 54 (May 1974). p. 377.

19) V. Kerry Smith, "Duality Principles and Measuring the Production Technology: A Heuristic Introduction," *Socio-Econ. Plan. Sci.* 12 (Jan., 1978). p. 161.

20) Cost functions are homogeneous of degree one in price regardless of the homogeneity properties of the production function, because a doubling of all prices will double the costs but will not affect the factor ratios.

21) Ronald W. Shephard, *Theory of Cost and Production Functions*, (Princeton, N. J.: Princeton University Press, 1970), p. 171.

The Translog Function²²⁾ has proved, in many empirical studies, to be a fruitful way of specifying the production technology in the form of a production function as well as a cost function using the duality theorem. Hence, we try to establish our model within the framework of the translog cost function.

We assume that there exists a minimum cost function for alternative rates of output with given input factors—Labor(L), Capital(K), and Working Capital(W)—in each of our sample industries. We further specify the cost function in the parametric form which permits estimation of partial elasticities of substitution.

A Taylor expansion form of the translog cost function,²³⁾ assumed to be twice differentiable and linearly homogeneous, together with Hicks' neutral technical change taken as a maintained assumption, may be expressed as:

$$\ln C = b_0 + \theta_1 \ln Y + 1/2\theta_2 (\ln Y)^2 + \sum_{i=1}^3 a_i \ln p_i \\ + 1/2 \sum_{i=1}^3 \sum_{j=1}^3 r_{ij} \ln p_i \ln p_j + \sum_{i=1}^3 d_i \ln p_i \ln Y, \quad (1)$$

for $i, j = L, K, \text{ and } W$.

where: C = total cost

Y = output

p_i & p_j = i th and j th factor input's price

The equality of the cross partial derivatives in (1) shows the symmetry constraint,

$$r_{ij} = r_{ji}, \text{ for } i \neq j, \quad (2)$$

and the linear homogeneity in factor prices requires the following restrictions on the parameters of (1):

$$\sum_{i=1}^3 a_i = 1, \quad (3)$$

22) L. R. Christensen, D. W. Jorgenson, and L. J. Lau, "Transcendental Logarithmic Production Frontiers," *The Review of Economics and Statistics*, 55 (February 1973), pp. 28~45.

23) The translog cost function is written as a logarithmic Taylor series expansion to the second term of a twice differentiable analytic cost function around variable of 1 (i.e., $\ln Y = 0$, $\ln p_i = 0$, $i = 1, \dots, n$). See Binswanger, "Cost Function Approach," p. 379.

$$\sum_{i=1}^3 r_{ij} = \sum_{j=1}^3 r_{ij} = \sum_{i=1}^3 \sum_{j=1}^3 r_{ij} = 0, \text{ for } i, j = L, K, \text{ and } W \quad (4)$$

$$\sum_{i=1}^3 d_i = 0 \quad (5)$$

The property that C is a continuously increasing function of factor prices can be expressed by the following three equations:

$$\frac{\partial \ln C}{\partial \ln p_i} = a_i + \sum_{j=1}^3 r_{ij} \ln p_j + d_i \ln Y, \text{ for } i, j = L, K, \text{ and } W \quad (6)$$

Shephard Lemma says that along the minimum cost expansion path the equilibrium employment of the i 'th input is

$$X_i^*(Y, p_i) = \partial C / \partial p_i, \text{ for } i = L, K, \text{ and } W \quad (7)$$

where X_i^* is the optimum level of usage of factor i . And the relative factor shares in the total cost, by definition, is

$$S_i = p_i X_i / C \quad (8)$$

Substituting X_i^* in (7) into (8), the relative shares on the cost minimizing expansion locus become

$$S_i^* = (\partial C / \partial p_i) (p_i / C) = \partial \ln C / \partial \ln p_i, \text{ for } i = L, K, \text{ and } W \quad (9)$$

where S^* is the minimum cost shares. Combining (6) and the observed shares, S_i , in (9), together with classical disturbance terms,²⁴⁾ we specify the three stochastic cost minimizing equations:

$$S_i = a_i + \sum_{j=1}^3 r_{ij} \ln p_j + d_i \ln Y + e_i, \text{ for } i, j = L, K, \text{ and } W \quad (10)$$

and we propose to estimate the parameters of (1) from these three equations in (10).

Although the estimated parameters, r_{ij} , have little economic meaning of their own (i.e., some call them the share elasticities),²⁵⁾ they are related to variable elasticities of substitution (σ_{ij}) and of factor demand

24) Some differences may exist between the minimum-cost shares, S_i^* , and the observed shares, S_i , attributable to entrepreneurial errors for minimizing cost in the production process. Thus, the error term, e_i , is added, which could be justified as $S_i = S_i^* + e_i$.

25) Robert Halvorsen, "Energy Substitution in U. S. Manufacturing," *The Review of Economics and Statistics* 59 (November 1977), p. 385.

(η_{ij}) as follows:²⁶⁾

$$\sigma_{ij} = (1/a_i a_j) r_{ij} + 1, \text{ for all } i, j; i \neq j \quad (11)$$

$$\sigma_{ii} = (1/a_i^2) (r_{ii} + a_i^2 - a_i), \text{ for all } i \quad (12)$$

$$\eta_{ij} = (r_{ij}/a_i) + a_j, \text{ for all } i, j; i \neq j \quad (13)$$

$$\eta_{ii} = (r_{ii}/a_i) + a_i - 1, \text{ for all } i, \quad (14)$$

where $a_i = X_i p_i / \sum X_i p_i$.

IV. Estimation Method

Using the stochastic version of (10), we estimate the parameters of (1). Therefore, the estimating equations for the cost function model are:

$$\begin{aligned} S_L &= a_L + r_{LL} \ln p_L + r_{LK} \ln p_K + r_{LW} \ln p_W + d_L \ln Y + e_L, \\ S_K &= a_K + r_{KL} \ln p_L + r_{KK} \ln p_K + r_{KW} \ln p_W + d_K \ln Y + e_K, \\ S_W &= a_W + r_{WL} \ln p_L + r_{WK} \ln p_K + r_{WW} \ln p_W + d_W \ln Y + e_W. \end{aligned} \quad (15)$$

The first two equations of the three equation model are chosen for estimating parameters by iterative Zellner's Efficient Estimation (IZEF) method.

We assume that the disturbances in each regression equation in (15) have a zero mean and a constant variance; that is, there exists neither a heteroscedasticity nor an autocorrelation problem. The two equations in the model are assumed to be independent. Nevertheless, the disturbance terms e_L and e_K are likely related because the random deviations from cost maximization would affect both the labor and capital markets.²⁷⁾ That is, the regression equations in (15) are seemingly unrelated, or disturbance related.²⁸⁾ One possible approach to estimate the coefficient of the seemingly unrelated regression equations is to apply the ordinary

26) Hirofumi Uzawa, "Production Function with Constant Elasticities of Substitution," *Review of Economic Studies* 29 (October 1962), pp. 291~99.; P. A. Samuelson, *Foundations of Economic Analysis* (Cambridge, Mass.: Harvard University Press, 1947), p. 68.; See, for proof, Binswanger, "Cost Function Approach," p. 380.

27) David B. Humphrey and J. R. Moroney, "Substitution among Capital, Labor, and Natural Resource Products in American Manufacturing," *Journal of Political Economy* 83 (February 1975), p. 65.

28) Henry Theil, *Principles of Econometrics* (New York: John Wiley & Sons, Inc., 1971), p. 298.

least squares (OLS) to each equation separately. We then obtain an unbiased and consistent estimate of the regression coefficients. However, the efficiency of the estimator becomes questionable because we simply ignore the information about the mutual correlation of the disturbances by applying OLS separately.²⁹⁾

The ZEF estimator has proven to provide the best linear unbiased estimates with a minimum variance for the seemingly unrelated regression equations.³⁰⁾ It is more efficient than the single equation least squares. Furthermore, the efficiency gain by this method is greater when contemporaneous disturbance terms in different equations are highly correlated and/or when independent variables in different equations are not correlated (i.e., $X_1 \neq X_2$).³¹⁾

However, the problem with the ZEF approach is that the estimates of the translog parameters are no longer independent of our choice of which two equations to estimate. Since the disturbance covariance matrix in cost function model (15) is singular, the ZEF procedure with any two equations may not be invariant to the equation deleted. Invariance can be attained, however, by iterating the ZEF procedure until the estimated coefficients and residual covariance matrix converge. That is, even though the ZEF parameter estimates depend on the equation deleted, the IZEF estimates do not. The IZEF estimator is asymptotically equivalent to the maximum likelihood estimator, and hence the IZEF coefficient is asymptotically efficient and its asymptotic distribution is

29) Arnold Zellner, "An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias," *Journal of the American Statistical Association* 57 (June 1962), pp. 348~68; also see Kmenta, *Elements of Econometrics*, (New York: Macmillan Publishing Co., 1971), p. 518.

30) Richard W. Parks, "Efficient Estimation of a System of Regression Equations When Disturbances are Both Serially and Contemporaneously Correlated," *Journal of the American Statistical Association* 62 (June 1967), pp. 500~509; A. Zellner and D. S. Huang, "Further Properties of Efficient Estimators for Seemingly Unrelated Regression Equations," *International Economic Review* 3 (September 1962), pp. 300~13.

31) Arnold Zellner, "Estimators for Seemingly Unrelated Regression Equations: Some Exact Finite Sample Results," *Journal of The American Statistical Association* 58 (December 1963), p. 975; Zellner and Huang, "Further Properties of Efficient Estimators," p. 300.

normal.³² For this reason, we employ the IZEF as our estimation method.

The restriction of linear homogeneity in our translog function model, together with the symmetry restriction, ensure that parameter estimates of any two equations in (15) will identify exactly all parameters of the model. For example, consider the estimation of the cost function parameters in (15) by the first two equations. The restriction that $a_L + a_K + a_W = 1$ ensures that $\hat{a}_W = 1 - \hat{a}_L - \hat{a}_K$. Similarly, the symmetry restriction ensures that $\hat{r}_{ij} = \hat{r}_{ji}$ ($i \neq j$). Finally, the restriction of zero sums of the r 's requires that $\hat{r}_{ij} = -(\hat{r}_{ii} + \hat{r}_{kk})$, for $i, j, k = L, K, W$. Therefore, only eight of twelve parameters in (15) are unrestricted.

We cannot obtain a set of unique parameter estimates for any two equations of the system (15) by applying IZEF or least squares to each equation individually because the estimates \hat{r}_{ij} and \hat{r}_{ji} ($i \neq j$) will not generally be equal. That is, we cannot guarantee that $\hat{r}_{ij} = \hat{r}_{ji}$ ($i \neq j$) under the unrestricted estimation for the system. We thus use the method of restricted estimation to cope with this problem by substituting the restrictions $r_{LX} = r_{XL} = (-r_{LL} - r_{KL})$ and $r_{KW} = r_{WK} = (-r_{KK} - r_{KL})$ in the first two equations in (15).

V. Empirical Results

Prior to the estimation of the model, we applied OLS to each single equation in the model and achieved the Durbin-Watson serial correlation statistics, which show the existence of severe serial correlations and are reported under the column of OLS in Table 1. Assuming the first-order autoregressive scheme, we thus employed the Cochrane-Orcutt procedure to the individual equations in order to alleviate the serial correlation problems. By iterating the Cochrane-Orcutt technique, we obtained the estimated serial correlation coefficient (ρ) and reported them in Table 2.

32) E. R. Berndt and L. R. Christensen, "The Translog Function and the Substitution of Equipment, Structures, and Labor in U. S. Manufacturing, 1929~1968," *Journal of Econometrics* 1 (March 1973), p. 89.

Table 1. Durbin-Watson Statistics

Industries	Labor Equation		Capital Equation	
	OLS	C-O AUTO	OLS	C-O AUTO
Aggregate	.9993	1.6617	.8815	2.4899
20	.9757	1.9427	.9771	1.8830
22	1.4161	2.7010	1.0694	2.3368
28	1.9843	1.9956	.8039	1.6115
29	.7494	1.7757	.3008	2.1069
32	1.5630	1.8313	1.4520	2.2384
33	.5580	1.5392	.5189	1.9111
36	1.0024	1.7068	.8606	1.7590
37	1.3884	2.1708	.2941	1.4163
371	1.1586	2.0451	.8686	1.9765

Note: C-O AUTO represents the Cochrane-Orcutt Autocorrelation technique.

The lower and upper limits of Durbin-Watson critical region are 1.35 and 1.48, respectively, at $p \leq .1$

Multiplying by the corresponding values of the serial correlation coefficients (i.e., $X_t - \rho X_{t-1}$), the data were transformed. OLS was then applied to the transformed data and the Durbin-Watson statistic was again obtained. The values of newly obtained Durbin-Watson statistic for the model fell in the vicinity of '2' (i.e., accept the null hypothesis), which implied that serial correlation was no longer a major obstacle in

Table 2. Estimated Correlation Coefficients

Industries	Labor Equation	Capital Equation
Aggregate	.986298	.982153
20	.983982	.979475
22	.982336	.981762
28	.002899	.978904
29	.560081	.939286
32	.983236	.977242
33	.980005	.979744
36	.981497	.984083
37	.288156	.979717
371	.958196	.976695

Table 3. Parameter Estimates

Parameter	Industry									
	Aggregate	20	22	28	29	32	33	36	37	371
a_K	.0051*	.0101*	-.0206*	2.3468*	.5325*	.0055*	-.0079*	-.0011	1.3064*	.0031*
	(.0026)	(.0015)	(.0062)	(.0723)	(.0514)	(.0008)	(.0026)	(.0039)	(.0436)	(.0057)
a_K	.0025	.0042*	.0199*	.0079*	.0171	.0105*	.0214*	.0083*	.015	.0161*
	(.0039)	(.0011)	(.0029)	(.0022)	(.0127)	(.0012)	(.0019)	(.0025)	(.0012)	(.0023)
r_{LL}	.1940*	.1584*	.0030	.0265	.0510	.1860*	.0075*	.0416*	.1708*	-.0018
	(.0147)	(.0248)	(.0062)	(.0186)	(.0311)	(.0112)	(.0044)	(.0111)	(.0204)	(.0048)
r_{LK}	-.1371*	-.0856*	-.0037	-.1315*	-.0330	-.1433*	-.0063*	-.0205*	-.1189*	.0013
	(.0612)	(.0201)	(.0027)	(.0107)	(.0256)	(.0121)	(.0032)	(.0074)	(.0135)	(.0029)
r_{KK}	.1576*	.0965*	.0393*	.1735*	.1133*	.1611*	.0720*	.0661*	.1391*	.0607*
	(.0200)	(.0204)	(.0085)	(.0117)	(.0296)	(.0155)	(.0050)	(.0072)	(.0164)	(.0120)

Note: Standard errors are in parentheses.

*Coefficient is significantly different from zero at $p \leq .5$

the estimation process. The values of the Durbin-Watson statistic are reported under the column of C-O AUTO in Table 1. The following estimation and analysis are based on these transformed data.

Evaluating the parameter estimates by the t -statistic, we find most of them are statistically significant, i.e., 39 out of 50 estimates are significantly different from zero at the 5 percent significant level and reported them in Table 3.

Using the parameter estimates for the cost function model in Table 3, together with the linearity and symmetry restrictions, we estimate Uzawa's partial elasticities of substitution (PES) and demand elasticities for each industry and reported them in Table 4 and 5, respectively. These estimations have been evaluated at the sample mean within the industries.

The results in Table 4 indicate, as we might have suspected, a priori, that working capital is a substitute factor for labor. Furthermore, the working capital turns out to be as crucial as capital in the production process. That is, the cross PES between labor and working capital in many industries (i.e., SIC 371) are larger than those between labor and capital. This somewhat surprising result deserves an explanation. In the firm's production process, the expansion of output most likely requires either the acquisition of more resources or more efficient utilization of the existing factor inputs. Since the characteristic of working capital is easy to vary in rather short period of time for smoothing out the firm's production process, it is very much possible working capital to be an easier substitute for labor than capital. And further our results of the industries' behavior confirm the the Smith and Kopp's finding of the substitute relationship between labor and working capital. That is, their result shows it to be .186, while ours shows .2137 in the aggregate industry.³³⁾

The results also indicate that the working capital is a complementary factor with physical capital in many industries. These are an anticipated

33) Smith and Kopp, "Reconsidering Demand for Money," p. 16.

Table 4. Partial Elasticities of Substitution

Industry	σ_{LK}	σ_{LW}	σ_{KW}	σ_{LL}	σ_{KK}	σ_{WW}
Aggregate	.0139	.2137	.0150	-.0358	-.062	-1.4619
20	.3329	.0612	.4408	-.0923	-1.5754	-7.4681
22	.9649	1.0008	-1.2146	-.3368	-4.1398	-5.0499
28	.1923	.0710	-.0791	-.1098	-.3590	-.1370
29	.7181	.1664	.1227	-3.1656	-.1927	-.8751
32	.0707	.2676	.0469	-.0550	-.2384	-2.3358
33	.9637	.9813	-1.3309	-.5948	-2.4836	-2.3379
36	.7846	.7548	-2.2262	-.2408	-2.7680	-7.3147
37	2.0356	.3452	-.2175	-.0429	-.3115	-2.0618
371	1.0009	1.0063	-.7595	-.5173	-3.4547	-3.0422

Table 5. Demand Elasticities

Industry	η_{LL}	η_{KK}	η_{WW}	η_{LK}	η_{LW}	η_{KW}
Aggregate	-.0250	-.0112	-.1518	.0028	.0222	.0016
20	-.0657	-.2824	-.8141	.0590	.0067	.0481
22	-.2509	-.5793	-.5808	.1350	.1151	-.1397
28	-.0638	-.1007	-.0190	.0539	.0099	-.0110
29	-.5269	-.1356	-.1140	.5053	.0217	.0160
32	-.0382	-.0530	-.1961	.0157	.0225	.0039
33	-.3685	-.6946	-.2357	.2695	.0989	-.1342
36	-.1834	-.3484	-.8278	.0980	.0854	-.2519
37	-.0316	-.0486	-.2219	.3173	.0372	-.0223
371	-.3419	-.7734	-.3506	.2241	.1160	.0875

result. Working capital is employed only when actual production is undertaken. And thus, if output is increased the need for working capital is increased and vice versa. In other words, by increasing the amount of working capital required and thus increase the efficiency of capital, resulting in an increase in total output. Leijonhufvud also concludes³⁴⁾ "Most short-lived physical assets are, in fact, complementary in production to Keynes' fixed capital." And our results of these industries' behavior are consistent with the Smith and Kopp's finding of the complementary

34) A. Leijonhufvud, *On Keynesian Economics and the Economics of Keynes*, (New York: Oxford University Press, 1968), p. 255.

relationship between working capital and physical capital.

However, the substitute relations between working capital and capital, in some industries, seem somewhat surprising and inconsistent result from the above. In attempting to understand the implications of this result, we must recognize that the fundamental factor determines the firm's level of working capital is the amount of risk involved. In general, the risk involved in the firm's working capital management may broadly be classified into three categories: (1) the risk of not maintaining adequate liquidity, (2) the risk of having too much or too little inventory to maintain production and sales, and (3) the risk of not granting adequate credit to support the proper level of sales. Therefore there exist the opportunity to gain if the firm willing to take the risk. That is, the decrease in working capital results in a larger 'gain' than the 'loss' that results from a like increase in working capital.³⁵⁾ Nonetheless, it is difficult to extend our discussion beyond these remarks, since there is little basis in theory or past experience for evaluation of our findings.

In overall aspects, from Table 4, we can conclude that working capital is not only a productive factor, judged by the values of PES, but also as important as labor and/or capital in our sample U.S. manufacturing industries. This conclusion can also be supported by the estimated demand elasticities for each industry reported in Table 5.

VII. Summary and Conclusions

As an exploratory attempt, the objective of the study is to investigate the substitution possibilities between working capital and other productive factors in the firm's production process. Accordingly, cost function which included the price of working capital, measured by the discount rate of the Federal Reserve Bank, were estimated for the aggregate and nine two- and three-digit (SIC code) U.S. manufacturing industries over the

35) E. W. Walker, "Toward a Theory of Working Capital," in *Management of Working Capital*, ed. Keith V. Smith, pp. 329~39.

period 1958 I to 1973 IV.

We argued that working capital served to continuously maintain productive activities, and any deficiency of it causes serious impact on the production process. In turn, it is reasonable to conceive substitution possibilities between working capital and other productive inputs. The estimated PES of our empirical estimation results, judged by its size, support the existence of substitution possibilities between working capital and other productive factors in our sample industries. Another noticeable result indicated that the estimated PES between labor and working capital is larger than that between labor and capital in some industries' behavior. However, this does not imply that working capital rather than capital is the major substitute factor for labor in the overall firm's production process.

The most reasonable summary would suggest that the empirical results here show substantial basis for the role of working capital as a productive factor in the firm's production process. One way to extend this study would be to break down the working capital components (i.e., accounts receivable, government securities, and other assets), and include them as an explicit factor in the model. By doing so, we could identify the role of each component in the production process, and hence the productive characteristic role of working capital could further be clarified.

Appendix

This study uses quarterly basis time series data over the period of 1958 to 1973. The data required are the quantities and prices of the input services. Some of the data is taken directly from the published sources, while others have been constructed as described below.

The average number of employees for each industry is used as the quantity index of labor input, and they are collected from the various issues of U.S. Department of Labor, Bureau of Labor Statistics, *Employment and Earnings*. Since the monthly total number of employees

is available in the sources, we have constructed the quarterly index by adding and averaging the monthly data. The compensation of labor for each industry is available on a quarterly basis from the U.S. Department of Labor, Bureau of Employment Security, *Employment and Wages*. The price index of labor input is computed as the ratio of labor compensation to the quantity index of labor input.

Capital stock can be measured as the net value of structure and equipment which industry holds at a given period of time. The net value of capital stock implies total value of capital stock less depreciation and depletion. A similar definition of capital stock measurement has been widely applied in empirical studies.¹⁾ The net value for capital stock has been collected from the Federal Trade Commission, Securities and Exchange Commission, *Quarterly Financial Report for Manufacturing Corporations*. The capital stock has been deflated by the implicit price deflator for fixed nonresidential investment, and the deflators are collected from the Department of Commerce, Bureau of Economic Analysis, *The National Income and Products of the U.S.*, 1929~1974, *Statistical Tables: A Supplementary to the Survey of Business* (Table 7.1, line 8).

The price of capital, or the firm's internal cost of capital, is measured by the interest rate on long-term corporate bonds. Miller and Modigliani, in their study on the cost of capital in industry, conclude that the estimates of cost of capital are nearly equal to the long-term rate of interest on bonds.²⁾ The data on long-term rates are collected from the Board of Governors, Federal Reserve System, *Federal Reserve Bulletin*.

The quantity of working capital is measured as the net working capital. Working capital is sometimes defined as the current assets of the firm—namely cash, marketable securities, accounts receivable, and inventories. Sometimes, in a broader and perhaps more useful sense, working capital

1) Grayham E. Mizon, "Inferential Procedures in Nonlinear Model: An Application in a UK Industrial Cross Section Study of Factor Substitution and Returns to Scale," *Econometrica* 45 (July 1977), p. 1232.

2) M. H. Miller and F. Modigliani, "Some Estimates of the Cost of Capital to the Electric Utility, 1954~1957," *The American Economic Review* 56 (June 1966), pp. 333~91.

refers to both the current assets and current liabilities, the latter including accounts payable, short-term bank loans, and other payables and accruals becoming due within a year. The net working capital is therefore defined as the current assets less the current liabilities and thus is a single dollar value. The data on net working capital are collected from *Quarterly Financial Report on Manufacturing Corporations*. Since the data are described in dollar terms and inventories are the major component in the definition of current assets, the net working capital is deflated by the implicit price deflator for fixed nonresidential investment.

The price of working capital is measured by the discount rate of the Federal Reserve Bank. Two general characteristics of the current assets as a major component of net working capital are illustrated in the literature:

One is they are short-lived; typically life span does not exceed one year. Secondly, each component is swiftly transformed into other asset forms. Thus cash utilized to replenish inventories; inventories are diminished when credit sales occur that augment accounts receivable; collection of accounts receivable increases the cash balances.³⁾

In financial analysis liquidity must be raised through financing. Because of its short-term characteristic, the short-term prime loan rate is an appropriate measure of the cost of working capital or cost of financing. Furthermore, the prime rate is closely related (or almost equivalent) to the discount rate by the Federal Reserve Banks. The data on the discount rates are collected from various issues of *Business Statistics*.

The output data are defined as the total level of sales (net of returns, allowance, and discounts) in each industry, and are collected from the *Quarterly Financial Reports for Manufacturing Corporations*.

Based on the above data collections, we have constructed the relative cost share for each input factor. The total cost is the sum of the

3) Dileep R. Mehta, *Working Capital Management* (New Jersey: Prentice-Hall, Inc., 1974), pp. 1~2.

expenditures on each input factor (i.e., quantity times price). The relative cost share of each input factor is then the ratio of the respective input factor expenditures to the total cost.

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