

# Computational Costs and Bounded Rationality

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## «Abstract»

In conventional economic theory of choice it is assumed that (optimal) decision making by economic agents is costless. However, any form of problem solving in economic theory involves making use of an effective algorithm, a procedure that requires the use of scarce resources. Thus an optimal decision can only be determined after the cost of reaching the decision has been taken into consideration. A program for bounded rationality is linked to the cost of computation for decision making. In order to distinguish between costs of various decision procedures, a meta-decision problem is established in which it is assumed that a decision about decision methods is costless for an agent.

Key Words: Decision Making, Economic Costs, Bounded Rationality, Computation, Demand Theory.

## 1. Introduction

The dominant method of analysis in the economic theory of choice is to assume that agents behave as if they optimize some criterion function in making their decisions. That they are not actually observed to perform the complex calculations involved in the optimization process is not a basis for the rejection of the assumption according to accepted methodology. Instead, a lack of empirical support for the implications of the assumption of optimizing behavior is the primary basis for doubt (Friedman (1953)). The calculation of the optimal decision by the agent is generally assumed to be costless in this methodological approach. However, certain results in the theory of computation indicate that a useful requirement for a

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function to be 'computable' is that it can be realized by a step-by-step procedure that can be implemented mechanically (Minsky (1967)). Since implementation of each step in the procedure requires the services of some human or mechanical agent, any computation requires the use of scarce resources; and agents may not perform the computation required for the continuous optimization of their criterion functions. Therefore, the optimal decisions of agents in an economy are determined after the cost of reaching the decision is taken into consideration.

The notion that agents in an economy do not continuously maximize profits or utility but instead act according to (optimal) rules of thumb is an old theme in economics. Marshall (1936) suggested that choice is mainly a matter of habit rather than the result of continuous optimal decision-making; however, the particular habit in use by an agent depends upon the environment of the agent.

Baumol and Quandt (1964) simulated situations in which various rules of thumb for price setting by a monopolist were compared to the actual profit maximizing solution. Such rules are used in their scheme because of the costliness of the decision process, and the cost arises because either calculations or data gathering involve the use of resources.

Radner and Rothschild (1975) also propose a model in which decision-makers are assumed to follow simple decision rules because of costly computation, among other reasons. Simple performance criteria such as the survival of the decision-maker or the maximization of the rate of growth of various performance indices are suggested as means of evaluating different simple decision rules.<sup>1)</sup>

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1) The relationship 'rules-of-thumb' and 'optimal decision rules' is not made explicit. But it seems reasonable to assume that they satisfy some specific relation. In this regard, one may adopt C. Futia's view, Futia (1975), that '...one could hope that if the decision maker's environment (=structure of his decision problem) was simple, then so would be his optimal decision rules.' In other words, '...rules of thumb could then be regarded as optimal rules for 'simple' problems; but as I would like to add rules of thumb are less costly than optimal rules for complex problems. For instance, the method of majority decisions is one of the least complex, and hence least costly method of making social decisions, but it may also be optimal for 'simple voting environments' only (Varian (1975)).

An implicit assumption of a theory of bounded rationality is that the cost which drives decision-makers to use a 'non-optimal' decision rule is not explicit in the analysis. The nature of the cost is not discussed except in very general terms, and the cost does not appear as a parameter in the solution to the problem. The decision methods used by these researchers to determine that a particular rule of thumb may be optimal either are too costly for the agent himself to calculate by assumption or are themselves arbitrary rules of thumb.

This essay will explore some implications of assuming that the calculation of decisions is costly and that agents behave as if it is costly. The costliness of decision-making is not assumed to depend principally on the cost of searching for basic data such as prices which may be entered as independent variables into the optimization problem. However, the cost is assumed to lie in the resources used in the implementation of the algorithm which produces the solution of the optimization problem, given the input data.

The assumption of costly decision making leads to a number of inter-related problems which this essay will consider. Among these is the choice of criterion functions which approximate the true criterion functions of the agents in an economy but whose maximization is less costly to compute. An agent's true criterion function may be defined as that which he would use in an environment in which decision making is free. For example, individuals may use approximations to their demand functions for determining a basket of goods to consume instead of using their actual demand functions; and entrepreneurs may use rules of thumb (or special heuristics in the sense of Polya (1957)) in determining optimum vectors of inputs and outputs.<sup>2)</sup>

An application of the choice of approximate criterion functions is the determination of the frequency at which decisions are made over time by agents in an environment in which the data to be used as input into the

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2) In designing a general demand system, Krelle and Pallaschke (1980) have attempted to find an approximative algorithm for solving the general demand system, yet failed to indicate the 'computational budget' of the decision-maker.

optimization problem are stochastic. Since decisions on economic variables such as bundles of goods and asset holdings are costly, they will be made at discrete time intervals instead of being made continuously; and a rule for determining the time of the next decision must be developed.

This essay analyzes a logical dilemma that arises when it is assumed that all decisions are costly. If all decision methods are costly to utilize then so is the decision method by which decision methods are chosen. But then there is no clear method of choosing among the decision methods and no criterion remains with which to decide among the elements of the set. The dilemma is resolved by assuming that a decision about decision methods is costless for an agent. This difficulty was pointed out independently by Winter (1975) in a hypothesis that 'superoptimization', i.e., optimization taking all costs into consideration, is impossible when calculations are costly. The results presented here constitute a proof of this conjecture. The difficulty created by the logical dilemma of costly decision-making is avoided by assuming that the costliness of 'higher level' decisions falls to zero, even though lower level decisions are observed to require scarce resources.

## 2. Approaches for Viewing the Problem of Costly Decisions

In constructing a model for the analysis of observed phenomena, an economist weighs the value of the greater explanatory power that can be derived from increased detail in the model against the cost of the additional complication. The solution to this problem determines not only the complexity of the model itself but also the complexity of the data used in testing the model.

In the sequel, assume that agents in an economy behave as if they make analogous choices in determining the types of problems to solve and the methods to use in their solution. Furthermore, assume that decisions are costly to make because scarce resources are used in the processing of data. For any type of decision that the agent must make, he can choose

from a range of processing techniques which require different quantities of resources and which produce different qualities of decisions.

It will be shown that this assumption can be used to generalize the standard utility approach to demand theory. Furthermore, it can explain the expenditure of resources on techniques which reduce data processing costs.

The theory of demand as derived from utility theory presents the demand function for goods,  $d(p, y)$ , as the solution to the problem of maximizing the utility function for given  $(p, y)$ . Here  $p$  is a vector of  $n$  nominal prices,  $y$  is money income, and  $d(p, y)$  is a vector of quantities of goods in demand. Formally,

$$d(p, y) = x \text{ where } x \text{ solves } \max \{U(x) \text{ subject to } p'x = y\}. \quad (1)$$

In this essay the symbol  $d$  will always refer to the demand functions the consumer would use if decisions were costless or, alternatively, to the demand functions that are usually derived in utility theory.  $U$  is the consumer's utility function.

The processing of the data  $(p, y)$  by (1) implicitly is assumed to be costless; and therefore, the intermediate calculations required to reach the solution are ignored except in those few special cases which allow for explicit solutions. The demand functions  $d$  are black boxes whose inputs and outputs are observed; but a knowledge of their internal mechanisms is unnecessary. Essentially, the free processing of data represents a part of the wealth endowment of the consumer.

The imposition of cost for the use of decision functions creates a different type of problem for the consumer to solve. (The Appendix develops the concept of costly decision making from the point of view of computation theory and presents possible realizations of the concept of 'computation resources'.) If the consumer continues to use  $d$  as defined in (1), he merely has to reduce  $y$  by  $c(d)$ , where  $c$  is a function which indicates the cost of using decision function  $d$ , and to solve  $d(p, y - c(d))$ .

However, if there exists a set  $D$  of possible decision functions whose elements include  $d$ , he may be able to increase his utility by using a

cheaper, less perfect decision function  $d^*$ ,  $d^* \in D$ . Therefore, the consumer's problem is to choose the best decision technique for the choice of bundles of goods rather than to choose the best bundle of goods. This new problem will be referred to as the meta-decision problem of the consumer.

Let us examine two directions that may be taken in constructing a framework in which the meta-decision problem can be solved. The first approach will be rejected because of a problem that arises in assuming that all decision techniques require the use of costly computation resources. This assumption must also apply to the meta-decision technique. The choice among decision functions requires the application of some criterion function to compare the results of each decision function. However, the criterion function of the consumer is his utility function. Since in the most general case the meta-decision function must use the utility function to perform the intermediate steps of evaluating the results of every possible decision function before a choice can be made,  $d$  of (1) will always be chosen since it must be computed anyway. Hence, the meta-decision function will not even be used.

More formally, assume that each algorithm in the set of all possible algorithms is costly for the consumer to use. He must solve a problem of the following type:

$$\max_{d^* \in D} \{U(d^*(p, y - c(d^*)))\}. \quad (2)$$

In (2),  $U$  is the utility function of the consumer and  $c(d^*)$ ,  $c: D \rightarrow R^+$ , is the cost of decision function  $d^*$ .  $D$  is the set of possible decision functions, and it is assumed here to be finite for simplicity.<sup>3)</sup> Members of  $D$  must obey the budget constraint; and therefore, demand functions produced by the solution of (2) for utility-like functions other than  $U$  may also be elements.

Since  $D$  is finite, one algorithm that surely solves this problem is to find  $U(d^*(p, y - c(d^*)))$  for each  $d^* \in D$  and select that  $d^*$  which produces the maximum utility value. Unless more structure is imposed on the

3) In fact, the set of computable functions is countable. Therefore, if an infinite number of decision functions is assumed in  $D$ , then they are also countable.

set  $D$ , this is the only algorithm. By assumption this meta-decision algorithm is costly; in fact, it is more costly to decide which element of  $D$  to select than to use  $d$  directly since the meta-decision algorithm computes  $d(p, y - c(d))$  as part of its operation. Therefore, the consumer should always prefer the use of  $d$  to the use of the meta-decision function;  $d$  may not be the best element of  $D$ , but the consumer has no cost saving method of choosing among the elements of  $D$ . Therefore, the assumption that all decisions are costly implies that it is only possible to decide among the elements of  $D$  at exorbitant cost; but then the meta-decision problem has no solution.

It may be argued that more structure can be imposed on  $D$  so that the meta-decision algorithm can become more efficient. The imposition of structure restricts the domain of a meta-decision function to a sub-set of  $D$ . But then there will be many meta-decision algorithms and one of them must be selected. However, this requires meta-meta-decision algorithms because all decision algorithms are costly. Therefore, the fundamental problem that arises in assuming that all decision functions are costly cannot be removed. All levels of decision functions become elements of  $D$ , and no guide for choosing among these elements remains.

These arguments prove the following impossibility theorem:

Theorem. If

- (i) the consumer must incur a cost to calculate all decision algorithms,
- (ii) for each vector  $(p, y)$ , the consumer chooses the best decision function in  $D$ ,
- (iii)  $D$  is a finite set of decision functions  $d^* : R^{n+1} \rightarrow R^n$  all of which obey the budget constraints,

then it is impossible to solve the meta-decision problem.

This result forces the adoption of a second approach in which it is assumed that the meta-decision function (2) can be run costlessly, i.e., assumption (i) in the theorem is relaxed.<sup>4)</sup> An immediate objection is that

4) The meta-decision function is a Turing machine that simulates all the elements of  $D$  in reaching its result. See the Appendix for a discussion of Turing machines.

$d$  of (1) can be costlessly computed in running (2). Therefore, there is no departure from standard utility theory since the choice of  $d$  and its computation are accomplished in the free processing of (2).

However, this problem can be avoided by assuming that the intermediate calculations of the meta-decision function are not available for the consumer's use. The meta-decision function is then a black-box, as are the demand functions derived in standard demand theory. Its inputs are the price-income data and the set  $D$ , and its output is a number which is an encoding of the decision algorithm to be implemented. The consumer then runs the chosen algorithm, pays the required costs, and purchases the resulting goods vector.

Although at first sight this appears to be a highly restrictive assumption, it seems to be the only means of avoiding the difficulties which have been discussed. Without this assumption, that part of the usual bounded rationality arguments which is based on the assumption of costly calculation is seen to be based on contradictory arguments. Free computation has been removed from the lower level decisions but it remains for higher level decisions. Therefore, that part of the consumer's wealth arising from the standard theory assumption of free decision has not been completely expropriated; the range of free choice has merely been narrowed.

The resulting structure is clearly a generalization of the standard demand theory. If it is assumed that  $c(d^*)=0$  for  $d^* \in D$ , then the demand functions and budget constraint of the consumer will be identical to those of demand theory.

### 3. Example

A simple structure can be constructed to serve as an example illustrating the discussion. The structure will be used specifically to indicate how a consumer might create a single aggregate good for use in decision-making and how he might employ a standard price index.



For this example restrictions will be placed on the elements of  $D$ . If  $U(x)$ ,  $x \in R^{n+}$ , is the criterion function of the consumer, define  $U^i$  as

$$U^i(x) \equiv U(x_0) + \sum_{q=1}^n U_q(x_0)(x^q - x_0^q) + \cdots + \frac{1}{i!} \sum_{q_1, \dots, q_i=1}^n U_{q_1, \dots, q_i}(x_0)(x^{q_1} - x_0^{q_1}) \cdots (x^{q_i} - x_0^{q_i}) \quad (3)$$

the  $i$ th degree Taylor expansion of  $U$  about some  $x_0$ . Then,

$$D \equiv \{d^i | i = 1, 2, 3, \dots\} \quad (4)$$

where

$$\begin{aligned} d^i(p, y) &= x \text{ where } x \text{ solves} \\ \max_x \{U^i(x) \text{ subject to } p'x &= y - c(i)\}. \end{aligned} \quad (5)$$

The sequence of Taylor series approximating  $U$  is considered to be a sequence of utility-like functions of which each element  $U^i$  can determine a function  $d^i$  in the usual manner. Each element of  $D$  is one of these  $d^i$  and can be identified by its index  $i$ , the degree of the approximating polynomial from which it is derived. Therefore,  $D$  is a countable set.

The function  $c(i) \equiv c(d^i)$  produces the money cost of using decision  $d^i$ . The cost of calculation is assumed here to depend on the number of multiplications and additions required to calculate the function  $U^i(x)$ . Hence,  $c(i)$  increases at an increasing rate with  $i$ . As an alternative to requiring all terms of degree  $n$  to be added simultaneously to  $U^{n-1}$  in order to build  $U^n$ , it may be assumed that the degree  $n$  terms are added one by one. Although this expands the number of possible elements of  $D$  and requires a relabeling of them,  $c$  will still be an increasing function. This structure of  $D$  has the technical advantage of allowing the aggregation of subgroups of goods.

Based on this framework, the consumer's meta-decision problem is simply

$$\max_i U(d^i(p, y - c(i))). \quad (6)$$

If  $U$  is a polynomial,  $D$  is finite and the solution of (6) can be found by trying all elements of  $D$ . If  $U$  is not a polynomial, then some structure must be imposed on  $D$  to assure that an algorithm implementing (6) will eventually produce an answer. For instance, if it is assumed that

$$U(\underline{d}^{i+1}(\underline{p}, y - c(i+1))) - U(\underline{d}^i(\underline{p}, y - c(i))) < U(\underline{d}^i(\underline{p}, y - c(i))) - U(\underline{d}^{i-1}(\underline{p}, y - c(i-1))) \quad \forall i = 2, 3, \dots \quad (7)$$

and if for some  $i$

$$U(\underline{d}^{i+1}(\underline{p}, y - c(i+1))) < U(\underline{d}^i(\underline{p}, y - c(i))), \quad (8)$$

then

$$U(\underline{d}^{i+j}(\underline{p}, y - c(i+j))) < U(\underline{d}^i(\underline{p}, y - c(i))) \quad \forall j \geq 1. \quad (9)$$

Proof of Inequality (9)

Given (7), we have

$$U(\underline{d}^{i+2}) = U(\underline{d}^{i+1}) < U(\underline{d}^{i+1}) - U(\underline{d}^i).$$

If we substitute  $U(\underline{d}^i)$  for  $U(\underline{d}^{i+1})$  in the right hand side of the inequality, the inequality is preserved because of (8). Then  $U(\underline{d}^{i+2}) < U(\underline{d}^{i+1})$ . Equation (9) results from continuing this process.

If the  $i$  for which (8) holds is finite, i.e., if the solution exists in  $D$ , then an algorithm implementing (6) will eventually produce a result by running sequentially through the  $\underline{d}^i$  until (8) holds.

The sequence  $U^1, U^2, U^3, \dots$  can be viewed as a sequence of progressive refinements of tastes; each new element in the sequence represents an increased distinction among goods. Essentially, the consumer chooses the degree of differentiation to make among goods according to the cost of that differentiation and to the utility that the differentiation produces; he decides the level of taste to display at any particular time.

Individual tastes can then be defined ordinally by  $i$ , the degree of the polynomial that the consumer uses. A theory of the choice of tastes need not be based only on the quantity of human capital of the consumer except to the extent that such human capital can reduce the cost of the elements of  $D$ . The study of arithmetic or the acquisition of an electronic calculator can be viewed as additions to capital which alter the relative costs of the elements of  $D$ , thereby causing a change in the degree of the approximating polynomial.

This example can be further specialized by assuming that the point  $x_0$  about which the approximation is calculated is taken to be  $x_0 = \underline{d}(\underline{p}, y)$ , the standard theory's vector of goods demanded. Based on this assumption,

one can examine some results that arise in an environment in which the consumer selects the crudest possible tastes, i.e., that in which he selects  $d^1$ . The indifference curves implied by  $U^1$  are linear and they exactly coincide with the budget constraint. Therefore, the consumer does not distinguish at all among goods. His tastes are such that all goods are merely a manifestation of a single aggregate good which he desires to consume, and the amount of any one good that he will demand is indeterminate. Therefore, his decision problem can be reduced from the problem of determining an  $n$ -dimensional vector to one of determining a single number at a saving of computation cost.<sup>5)</sup>

#### 4. Possible Applications

The rationale of aggregation should be to convert  $n$  goods to one composite good at least cost. If an index such as the consumer price index is prepared by the government, the consumer will use it as his price index in order to avoid the computation cost of constructing his own.

There are economies of scale in the central computation of a price index over all  $n$  goods in that every consumer who has chosen the crudest tastes possible can avoid a part of the computation cost of decision-making. Furthermore, the centralized computation of an index provides not only a standardized measurement of the aggregate good but also a uniform price for it.

Aggregation may occur by placing restrictions on  $D$ , the set of decision functions, and  $c$ , the decision cost function. Another tempting way to restrict  $D$  is to require that all  $d^* \in D$  yield the same decisions. The agent would then choose  $d^*$  on the basis of minimum cost alone. This seems to be an implicit assumption in some of the literature on functional separability.

The basis of Leontief's (1947) discussion is that the relationship

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5) For the consumer to be able to choose such a simplified decision technique it must exist in  $D$ .

$$F(x_1, x_2, \dots, x_n) = f_0(f_1(x_1), \dots, f_m(x_m)) \quad (10)$$

holds where  $F, f_0, f_1, \dots, f_m$  are functions and  $(x_1, x_2, \dots, x_n) = (x_1, \dots, x_m)$ . In other words, the symbols  $F$  and the symbols  $f_0(f_1(), \dots, f_m())$  represent algorithms which are realizations of the same function. In the problem that Leontief considered, these algorithms are each computed costlessly; and the purpose of analyzing them is to provide a means by which an analyst can break down complex relations into a sequence of simpler ones. Strotz (1957), however, suggests that this analysis can be applied to actual decision-making by agents in an economy. For instance, decision-making in a firm may be decentralized in separate departments or the budgetary process can be decentralized according to the provision of public goods. Presumably, the reason that decentralization occurs is that such an organization is a less costly structure than centralized decision making, though both organizational forms produce the same decisions (see H. Oniki (1974), Albin and Gottinger (1981)).

However, a difficulty arises in assuming that the decision functions all produce the same result. All of the algorithms in the set  $D$  produce the same decision, yet each of them is costly to use. Therefore, unless the cost of utilizing each of the decision algorithms is the same, they will produce different results because their calculations will be based on different resource constraints.

As an example, consider the consumer's problem. The budget constraint facing the consumer is  $y - c(d^*)$  where  $d^* \in D$ . If  $c(d)$  is different for different  $d$ , then constraining  $D$  such that all  $d \in D$  produce the same result for given  $(p, y)$  can lead to a violation of the budget constraint. Therefore, the results of functional separability cannot consistently be used by agents in an economy as means of economizing on decision-making resources.

## Appendix

### A Model of Costly Computation

Since the basis of the discussion is that decisions require scarce resour-

ces, a motivation for this approach should be presented. This appendix develops the concept of costly decision-making from the point of view of computation theory and presents possible realizations of the concept of 'computation resources.'

The model used here for a computation procedure consists of the operation of a Turing machine, the construction and functioning of which is standard material in automata theory, formal languages and recursive function theory. The description presented here is a brief introduction to the concept of the Turing machine.<sup>6)</sup>

Informally, a Turing machine is a device which manipulates inputs or strings of symbols from a finite set of symbols to produce a new string composed of elements from the same set of symbols. It consists of a control unit, a read-write device, and an infinitely long tape. The control unit contains a finite set of states and a transition function which determines the change from one state to another. The tape is divided into squares of equal size on which can be written any one of a finite set of symbols. The read-write device can read or write a symbol on a given square of the tape and move one square left or right on the tape. The Turing machine operates by reading the symbol on the square scanned by the read-write device; and depending on the symbol read and on the internal state of the machine, it changes state, writes a new symbol on the scanned square, and moves the read-write device one square left or right.

Formally, a Turing machine is a 6-tuple  $T = (K, \Sigma, \Gamma, \delta, q_0, F)$  where

- (i)  $K$  is a finite set of states,
- (ii)  $\Sigma$  is a finite set of input symbols (an alphabet),
- (iii)  $\Gamma$  is a finite set of tape symbols,
- (iv)  $\delta$  is a transition function from  $K \times \Gamma$  into  $K \times \Gamma \times \{L, R\}$  where  $L$  means a move left and  $R$  means a move right,
- (v)  $q_0 \in K$  is the start state,
- (vi)  $F \subset K$  is a set of final or accepting states.

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6) For a more extensive discussion one may refer to M.L. Minsky (1967), Part II.

The Turing machine is started in a configuration in which the internal state is  $q_0$  and in which the read-write device is located on the first square of the tape. The first symbol of an input string is located in the first square of the tape, and the second symbol in the second square and so on. The final symbol of the input string is then followed by an infinite string of blanks. The Turing machine is started and it follows a sequence of moves determined by the transition function and the symbols on the tape; if it ever reaches an internal state  $q \in F$ , the set of final states, it halts. The string of symbols left of the tape when the machine halts is the output of the machine. Thus, a Turing machine is a mapping from the set of strings of elements of the alphabet  $I'$  into itself.

Though the definition of a Turing machine is apparently very restrictive, the class of functions represented by the class of Turing machines is very broad. A basic concept, Church's Thesis, states that any algorithm or procedure, i.e., informal notions for a finite sequence of instructions which can be mechanically implemented, can be realized by a Turing machine. This cannot be deductively proved since it is an empirical statement, but it is used to define a computable function as one that can be realized by a Turing machine. There exists a Turing machine which can duplicate any procedure or algorithm, each instruction of which represents an intermediate step in the production of the solution from the input data. Therefore, any function or decision procedure used by an economic agent can be duplicated by an appropriate Turing machine, including those decision procedures known as demand functions. The resources used by a Turing machine in performing a calculation can then be defined as the number of moves  $N$  that the Turing machine makes and the total  $S$  of the number of spaces occupied on the tape at each move before the machine halts with output. These resources are analogous to the CPU time and the storage area used by a program on a general purpose computer, respectively.<sup>7)</sup>

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7) For practical measures of 'resource requirements within computer processing,' see the survey by Horning and Randall (1973).

More generally, they are analogous to the time of the agent who carries out the step-by-step instructions of the algorithm and to the quantity of storage medium used for intermediate calculations, respectively.

If time and space are priced above zero, then the cost of using the Turing machine  $T_0$  to make a demand decision based on the price-income input vector  $(p, y)$  is  $p_N N(T_0, (p, y)) + p_S S(T_0, (p, y))$ .  $p_N$  and  $p_S$  are the price of time and space, respectively;  $N(T_0, (p, y))$  is the amount of time required by machine  $T_0$  to compute an answer given input  $(p, y)$ ; and  $S(T_0, (p, y))$  is the amount of space required by machine  $T_0$  to compute an answer given input  $(p, y)$ .

In fact, a particular decision function  $H$  can be realized by an infinite number of Turing machines, each of which will use different quantities of resources in computing the same answer. If, for function  $H$ ,  $\phi_H$  is the set of all Turing machines which realize  $H$ , then the cost of decision function  $H$  on input  $(p, y)$  is defined to be

$$f(H, (p, y)) \equiv \min_{T_0 \in \phi_H} \{p_N N(T_0, (p, y)) + p_S S(T_0, (p, y))\} \quad (1)$$

The quantity of each resource used is then

$$g_N(H, (p, y)) \equiv N(T_0^*, (p, y)) \quad (2)$$

and

$$g_S(H, (p, y)) \equiv S(T_0^*, (p, y)) \quad (3)$$

respectively, where  $T_0^*$  is the solution to (1).

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