A Note on Friedman's Natural Rate Hypothesis

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In 1968, Friedman formulated his well-known natural rate hypothesis that in the long run a monetary change does not affect a real quantity such as the rate of unemployment, or the level of real income, but affects solely the rate of inflation [Friedman 1968]. He did this by considering only price expectations and overlooking income expectations. Later, realizing the importance of income expectations, Friedman incorporated them into his nominal income theory for the purpose of seeing their roles in determining the short-run and long-run effects of the monetary change on a real quantity and the inflation rate [Friedman 1970 and 1971].[1] He came to a conclusion that the incorporation of income expectations does not change the substance of the original natural rate hypothesis. Unfortunately, this conclusion is questionable because there is a logical slip, which has thus far passed unnoticed, in the manner in which Friedman drew the conclusion. In order to understand the role of income expectations in the natural rate hypothesis, this note will first correct the mistake Friedman made, and then discuss validity of his conclusion, by showing that Friedman's definitions of equilibrium and permanent income, and his differential equations do not support his natural rate hypothesis, but rather they support ironically the trade-off hypothesis. For this purpose, this note will make use of his own key equations and definitions without any modification and see where they will lead us.

We begin with Friedman's equations in his 1970 paper as follows:

(1) $Y=Py$

(2) $Y=vM$

where Y is nominal income, y is real income, P is the price level, M is the

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stock of money, and v is the velocity of circulation. (1) is the definition of nominal income, and (2) is the quantity equation of money. The logarithm of equation (1), differentiated with respect to time, provided that the expected variables also satisfy the corresponding equation, will yield the division of the change in nominal income between prices and real income:

$$(3)\ \frac{d}{dt}\log P = \frac{d}{dt}\log P^{*} + b\left(\frac{d}{dt}\log Y - \frac{d}{dt}\log Y^{*}\right) + c(\log y - \log y^{*})$$

$$(4)\ \frac{d}{dt}\log y = \frac{d}{dt}\log y^{*} + (1-b)\left(\frac{d}{dt}\log Y - \frac{d}{dt}\log Y^{*}\right) - c(\log y - \log y^{*})$$

where the starred symbols stand for the expected values; b and c are positive coefficients. These equations are Friedman's equations (28) and (29) in his paper (1970). Friedman further assumed that the short-run adjustment of nominal income in response to a change in the supply of money is given by

$$(5)\ \frac{d}{dt}\log Y = \frac{d}{dt}\log Y^{*} + v\left(\frac{d}{dt}\log M^{s} - \frac{d}{dt}\log M^{d}\right) + z(\log M^{s} - \log M^{d}).$$

This is Friedman's equation (31) in his paper (1970), which is a dynamic counterpart of equation (2) in the above.

We can also reproduce Friedman's definition of permanent income as follows:

$$(6)\ \log y^{*}(t) = B\int_{T=-\infty}^{t} e^{(B-G)(T-t)}\log y(T)\,dT.$$

This is Friedman's equation (36) in his 1970 paper and also equation (5—17) in his 1957 book (Friedman 1957, p. 144).[2] Permanent income is compounded of two elements: (1) an expected average rate of change to allow for secular growth at a rate of G; (2) the speed of adjustment of income expectations to past experiences at a rate of B. However, a standard assumption of the natural rate hypothesis is that capital is fixed at every point on a long-run equilibrium Phillips curve, so that as far as a long-run equilibrium Phillips curve is concerned, no growth due to capital accumulation is taken into account. This means that G is assumed to be zero for the natural rate hypothesis.[3] To simplify (6), let us differentiate it when G=0. Then we

(7) $\frac{d}{dt}\log y^* = B(\log y - \log y^*), \qquad B > 0.$

Again, the assumption that G=0 is common to all the Phillips curve studies, whether the natural rate hypothesis or the trade-off hypothesis. Equation (7) shows a well-known fact that the definition of permanent income is equivalent to the assumption that the formulation of expectations is adaptive. Let us assume, parallel to the definition of permanent income, that expecations about future prices are also determined by a simple adaptive expectations model. Then the price expectational function is given by

(8) $\frac{d}{dt}\pi^* = A(\pi - \pi^*), \qquad A > 0,$

where $\pi = d\log P/dt$, the actual rate of inflation, and $\pi^* = d\log P^*/dt$, the expected rate of inflation, and A is the speed of adjustment of price expectations to past experiences. This equation corresponds to equation (32) in Friedman (1970).

From equations (3) and (4), one may obtain

(9) $\pi = \pi^* + \frac{c}{1-b}(\log y - \log y^*) + \frac{c}{1-b}\left(\frac{d}{dt}\log y - \frac{d}{dt}\log y^*\right).$

Equation (9) tells us that the actual rate of inflation depends upon three factors: expectations about future prices, the current level of real income, and the expected level of real income. Three features of equation (9) are worthy of note. First, Tobin (1972) identified this equation as the price Phillips curve. He observed that y^* corresponds to the natural rate of unemployment, and that second variable $(\log y - \log y^*)$ is related to the deviation of the actual from the natural unemployment rate, and that the third variable $\left(\frac{d}{dt}\log y - \frac{d}{dt}\log y^*\right)$ is related to its change. If so, when the third variable is zero, the coefficient $c/(1-b)$ is related to the slope of the short-run Phillips curve with given π^* and y^*.

Second, Friedman examined different cases of equation (9) where b and c take different values. He observed among them that setting $b=1$ and $c=0$ is the simple quantity theory assumption that all of the changes in nominal income is in prices and that output is always at its permanent (expected) the economy is always in the long-run equilibrium, the possibility of the existence of the short-run adjustment is ruled out. If there is any adjustment, it takes place so quickly that it could be regarded as instantaneous. Therefore, the assumption, assuring always the equality of the actual inflation rate to the expected inflation rate in equation (9), rejects the existence of the short-run Phillips curve as a temporary adjustment process. This implies that the economy will react to autonomous disturbances not by moving along a short-run Phillips curve before it goes back to a long-run equilibrium position, but moving directly to that equilibrium position along the long-run equilibrium Phillips curve. Under this assumption, there is neither temporary trade-off nor permanent trade-off between inflation and unemployment. Friedman himself regarded this assumption as a special one by saying that "the simple monetary theory of nominal income is of course consistent with these equations [our equations (3) and (4)] in their general form since it does not specify anything about the division of a change in nominal income between prices and output". [Friedman 1971, p. 50]. To this extent, a finite value of $c/(1-b)$ is the requirement for the existence of the short-run Phillips curve as a temporary adjustment process of inflation and unemployment.

Third, Tobin recognized that equation (9) ensures the tenet of the natural rate hypothesis because the coefficient of the expected inflation rate, π^*, is unity. This led him to the conclusion:

> Friedman's particular proposal is simply a Phillips curve trade-off which vanishes in the long run.... The equation [our equation (9)] will be recognized as a standard price Phillips curve.... That the long-run Phillips curve is vertical is ensured by entering expected price change $\dot{P}^*/P^*$ with a coefficient of 1. [Tobin 1972, pp. 858—859].

Equation (9) also led Friedman to a similar conclusion:

> Assume that there is a shift at time $t=t_0$ in the rate of growth of the quantity of money from 3 percent to, say, 8 percent per year.... Let us first ask what the long-run equilibrium solution will be.... If, for the moment, we neglect any effect of this monetary change on real output and the rate of growth of output, this means that prices would be rising at 5 percent.... However, equi-

> effect depends on.... whether it [real output] includes or excludes the nonpecuniary services of money....[and] on the particular growth model.... For simplicity, I shall neglect this possibility and assume that the equilibrium rate of rise in prices is 5 percent per year. [Friedman 1970, pp. 229—231].

This is essentially identical to the conclusion of the original natural rate hypothesis Friedman reached in 1968:

> It [the monetary authority] cannot use its control over nominal quantities to peg a real quantity... the real rate of interest, the rate of unemployment, the level of real national income, or the rate rate of growth of the real quantities of money. [Friedman 1968, p. 11].

That is, Friedman assured the original natural rate hypothesis by excluding the two possibilities of the effect of the monetary change on real income; first, if the nonpecuniary services of money are included in real income, the monetary change will affect equilibrium real income because the former affects productive efficiency through the change in the ratio of money to other inputs in production; second, the monetary change will cause a change in the yield on capital which will affect real income. Friedman's exclusion of the two possibilities was also based on the standard assumption of the natural rate hypothesis that no growth of real income due to capital accumulation is considered.[4]

Friedman and Tobin therefore concluded that equation (9) ensures the natural rate hypothesis. However, this conclusion is questionable. For there is a logical error in the above interpretation of equation (9) because there is a third possibility that the equilibrium real income is affected by the monetary change. This possibility is not related to the two possibilities just mentioned. To emphasize this possibility alone, let us exclude the first two possibilities on the same basis as in the above by assuming that the nonpecuniary services of money are not included in real income, and that the cyclical change in real income is explained not by a particular growth model, but only by the adaptive expectations model of (7). The third possibility is related to the adjustment process of expectations to past experiences in equation (9). This is not to say that Friedman did not discuss the adjust-to go beyond equation (9) to obtain the equation's long-run equilibrium solution because he somehow thought that "it is impossible to carry much farther this verbal statement of the solution of differential equations [our equations (3), (4), and (5)]".[5] It is possible to solve the differential equations, and in what follows the solution will be provided.

Let us substitute equations (7) and (8) into equation (9). The result is

$$(10)\quad \frac{d}{dt}\pi^* = \left(\frac{Ac}{B(1-b)} - \frac{Ab}{(1-b)}\right)\frac{d}{dt}\log y^* + \frac{Ab}{(1-b)}\;\frac{d}{dt}\log y.$$

This equation explains the dynamic adjustment behavior of expectations to experiences at each moment of time during a temporary adjustment process. Let us start with an economy in an initial equilibrium. Following Friedman, we assume that there is a rise at time 1 in the rate of growth of the quantity of money, and that this rise is maintained indefinitely. The economy then begins moving from its initial equilibrium to a new equilibrium. To reach this new state, the economy will adjust continuously to this monetary change. In principle, still it is possible for the adjustment to be explosive rather than damped. If it is explosive, since the economy will not reach the new equilibrium, it is impossible to observe whether or not the monetary change affects equilibrium real income in the long run. To observe this effect, one must assume that the adjustment will be convergent to the new equilibrium. Let us assume that the new equilibrium will be attained at time T. Between time 1 and time T, there must be a cyclical adjustment in the rate of change in expectations and experiences in accordance with equation (10). This cyclical adjustment can be explained by equations (3) to (8). In equation (5), a rise in $d\log M^s/dt$ at time 1 produces a discrepancy between the actual and the expected rate of growth of nominal income. The rate of rise in nominal income will, in turn, be divided into a rise in π and $\log y$ in accordance with equations (3) and (4). As a result, the changed levels of $\log y$ and π start changing $\log y^*$ in equations (7) and (8). Since the demand for money is a function of y^* and π^*, the changes in y^* and π^* start affecting M^d in equation (5). This process will be completed when all actual values are equal will take the time interval T-1 periods from the initial equilibrium to the new equilibrium to make adjustment initiated by the monetary change.

Equation (10) explains an instantaneous change in expectations and experiences at a moment of time during the adjustment. Since the total changes in expectations and experiences over the entire adjustment periods are given by the definite integral of equation (10) from time $t=1$ to time $t=T$, and, according to Friedman's definition of equilibrium, the actual values are equal to the expected values in both equilibria (that is, $y(1)=y^*(1)$, $\pi(1)=\pi^*(1)$, $y(T)=y^*(T)$, and $\pi(T)=\pi^*(T)$), the long-run equilibrium solution of equation (10) is given by

$$(11) \quad \pi(T)-\pi(1)=\frac{Ac}{B(1-b)}(\log y(T)-\log y(1)).$$

Solution (11) gives a number of novel and appealing results. First, it is apparent from (11) that $y(T)>y(1)$ as $\pi(T)>\pi(1)$ if $Ac/B(1-b)$ is a finite value. This implies that when the inflation rate rises from $\pi(1)$ to $\pi(T)$ over the entire adjustment periods in response to a rise in the rate of growth of the quantity of money, the equilibrium level of real income will also rise from $y(1)$ to $y(T)$. Second, it is worth repeating that solution (11) is obtained from Friedman's original equations (1) to (8) without any modification. However, it is a surprise that solution (11) does not support Friedman's natural rate hypothesis, but rather it supports ironically the trade-off hypothesis. Third, notice that the above result is still ensured even though the coefficient of π^* is unity in (9). This may mean that the result does not depend on whether there is money illusion or not. Fourth, since we excluded the two possibilities of the monetary effect on equilibrium real income, the result depends neither upon whether real income includes or excludes the nonpecuniary services of money, nor upon whether a particular growth path affects equilibrium real income or not. Rather, it depends crucially upon whether, among other things, income expectations are included or excluded. If one, like Friedman in his 1968 paper, overlooks income expectations and considers only price expectations, the value of B in equation (7) is zero.

in (11), so that real income is unaffected by the monetary change. The absence of income expectations implies the natural rate hypothesis. If, on the other hand, income expectations are included, $Ac/B(1-b)$ will be a finite value, so that $y(T)>y(1)$ in (11). With income expectations, the monetary change does in the long run affect equilibrium real income. Thus Friedman's own theoretical framework disproves the tenet of his own natural rate hypothesis.

Even though the above lines show that Friedman's conclusion does not follow from his assumptions, it may be worth pointing out that Friedman presented his equations as just one possible linearized version, though admittedly he does not give any other ones. Perhaps one might argue that when an author draws a conclusion that does not follow from his assumptions, he can either drop the conclusion or modify the assumption. If the conclusion is important and the assumption is a minor one made for technical convenience then he should maintain the conclusion. This may or may not be the case here. However, still Friedman's example reveals the importance of income expectations in the natural rate hypothesis on one hand, and a need of reexamination of the natural rate hypothesis with a more general assumption about income and price expectations on the other. Therefore, the remaining task is perhaps to modify Friedman's special assumption about price and income expectations and to see whether his conclusion still holds with these modified assumptions. This is a legitimate open question.

Footnotes

1) Friedman said, "The third approach [the nominal income approach] differs significantly in regard to the elements that are common to the simple quantity theory and simple income-expenditure theory.... It does, unlike the other approaches, give an explicit role to anticipations about economic magnitudes. The differences between anticipated and actual magnitudes are the motive force behind the short-run fluctuations". [Friedman 1971, p. 333].
2) Friedman noted that the same is valid if (5—17) is expressed in logarithmic terms [see footnote 20 in p. 143 of Friedman (1957)].

I found that equation (36) in Friedman (1970) with (5—17) in Friedman (1957) which is the original version:

$$(36)\quad y^*(t)=B\int_{T=-\infty}^{t} e^{(B-G)(t-T)}y(T)dT$$

$$(5\text{—}17)\quad y^*(T)=B\int_{t=-\infty}^{T} e^{(B-G)(t-T)}y(t)dt.$$

Notice that the roles of T and t are reversed in both equations, but not T and t within the parentheses of exponent. This implies that (5—17) defines permanent income as a declining weighted average of past observed values of y, while the weights in (36) increase as we go back in time so that y^* cannot be defined as $T=-\infty$. Furthermore, Friedman made it clear that (5—17) is basically derived from the adaptive expectations function (5—14) as $G=0$ [see Friedman 1957, p. 143]. That is, when we differentiate (5—17) with respect to T as $G=0$, the result is the adaptive expectations function:

$$(5\text{—}14)\quad \frac{dy^*}{dT}=B(y(T)-y^*(T)).$$

On the other hand, when we differentiate (36) with respect to t as $G=0$, we don't obtain the adaptive expectations function. Thus, the correct form of (36) must be:

$$y^*(t)=B\int_{T=-\infty}^{t} e^{(B-G)(T-t)}y(T)dT$$

by reversing T and t within the parenthesis of exponent. The correct form is now identical with (5—17). The logarithmic version of this correct form is equation (6) in this paper. I confirmed this correction through exchanging letters with the Journal of Political Economy.

3) All the Phillips curve studies, whether the natural rate hypothesis or the trade-off hypothesis, in the literature in the past made explicitly or implicitly a common standard assumption regarding a long-run equilibrium situation. One may recognize in the literature that the concept of the long-run equilibrium in the Phillips curve analysis differs from the conventional concept. One conventionally recognizes three types of equilibrium, classified in terms of the relationship between flows and stocks. One is the short-run equilibrium where the influence of the size of flows on stocks may be regarded as negligible. This is the type of equilibrium Keynes had in his mind. The second type is where the flow variables have sufficient time to influence the level of the size of stock but there is not sufficient time for the stocks to attain their equilibrium. This may be called the medium-run equilibrium. The last type is where all the flows and stocks attain their equilibrium. This is the classical long-run equilibrium, and the long-run equilibrium analysis takes into account the growth aspect of economy due to capital accumulation.

On the other hand, in the Phillips curve analysis, these different types of equilibrium are not classified in terms of the flow-stock relationship. A standard on real income is negligible in the short-run as well as in the long-run equilibrium. Only a considerable change in capital is regarded as a factor which shifts a long-run Phillips curve. This means that capital stock is assumed to be fixed at every point on a long-run equilibrium Phillips curve. This does not mean that the Phillips curve analysis assumes that an exogeneous change such as a monetary change does not affect capital formation. Instead, what assumed in the Phillips curve analysis is that its effect is negligible. One may argue that this is the assumption for the short-run or the medium-run equilibrium analysis. It is so in terms of the conventional definition of equilibrium. In fact, the long-run equilibrium in the Phillips curve analysis is a medium-run equilibrium according to the conventional concept. But, it is not necessarily so in the Phillips curve analysis. For the only difference between the long-run and the short-run equilibrium in the Phillips curve analysis is found in the relationship between expectations about the future and experiences in the past. The short-run equilibrium is regarded as determined by an adjustment process in which the rate of adjustment in a variable is a function of the discrepancy between the actual and the expected value of the variable, whereas the long-run equilibrium is defined as determined by an equilibrium process in which the rate of adjustment in a variable is zero, so that the actual value is equal to the expected value of that variable. No growth due to capital accumulation is taken into consideration in the long-run as well as in the short-run equilibrium. This classification of equilibrium in terms of the expectation-experience relationship has been the tradition of the Phillips curve analysis since Friedman (1968).

4) See footnote 3.

5) Friedman (1970), p. 232.

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