# An Integer Goal Programming Model for the Location-Allocation Problem

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## Introduction

The location-allocation problem consists of multiple shipping destinations, utilizing known demands for a given product and known area shipping costs. The problem is to determine the number of facilities and their locations in order to best service the shipping destinations. Typically, the problem concerns itself with the tradeoff between the cost of building and operating facilities to meet product demand and the cost of transportation. The opening of a greater number of facilities would decrease transportation costs, while at the same time fixed costs for the establishment and operation of facilities would escalate. Conversely, the centralization of facilities will decrease the total fixed costs of the facilities, but the transportation costs would be at a high level.

While cost tradeoff remains to be an essential consideration, the trend of the 1970's toward more socially conscious business activity reflects the increasing importance of noneconomic considerations of location along with cost and profit. A more comprehensive approach is required to analyze the ramifications of various multiple and often conflicting objectives. The need for models of this type can be seen in the results of a longitudinal survey of industrial development executives in the public utility industry. The 1967 survey revealed that the primary factors of site selection, in order of their importance, were labor availability, availability of site and proximity to the market. The primary location considerations as reported in the same survey administered in 1972, however, were as follows: environmental considerations labor quality and supply, and availability of utilities [16]. From these results it is clearly evident that pure cost minimization models are no longer as applicable in today's energy and environment conscious era we live in.

Revelle *et al.* [19] point out that governmental regulations which legislate pollution control, equal opportunity, and preservation of resources and aesthe-

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tics can also dictate a non-optimal site choice in terms of economic criteria. They also suggest that the stochastic nature of the supply and demand in regard to seasonal or periodic fluctuations, as well as changes in economic conditions and population patterns, should be considered in the facility location problem. Student [23] emphasizes the need to consider human values in plant location. These findings have been supported by studies of Simon [20] and Shubik [27] that the traditional economic motives are not the only consideration of real world managers.

Location analysis models are classified into two structural categories, neither of which have been developed to consider multiple conflicting objectives in site choice. The classifications are 1) location on a plane and 2) locationon a network [19]. The first type models, location on a plane, are clearly based on a single objective. They assume total cost to be proportional to distance traveled, and thus attempt to determine some point or points which minimize the sum of the possible distances from the source to the destinations. Locati on literature includes many examples and modifications of both rectilinear-distance and Euclidean-distance solutions to this problem[5], [18], [2], [19].

The location of facilities as a point on a plane technique clearly does not treat many of today's realities. The location indicated may be in conflict with many corporate or legislated objectives. For example, industrial sites may be available, the community may be opposed to that particular commercial activity, or the area may be environmentally stressed such that it would be unwise to locate there.

The second type of models based on the network location enumerate previously determined alternative facility sites of demand as nodes on a network. Screening procedures can be used, as suggested by Nutt [17], to develop a list of alternative locations which will be acceptable choices. Although these initial checklist-type methods do consider noneconomic criteria and provide a list of satisficing alternatives, mechanisms for choosing between those alternatives revert to maximizing profit or minimizing costs associated with that location.

Locational costs associated with the network models include total transportation costs associated with that site as well as the amortized facility cost or fixed cost associated with that facility. The problem then becomes a search for the optimal solution to the tradeoff between fixed costs and transportation costs. Many authors have considered solutions to the fixed charge problem in facility location [5], [9], [4]. The primary focus of the research has been to develop more exact solution methods [22], [3] and models which can efficiently handle larger sized problems [25], [8]. However, no consideration of multiple objective fixed charge facility location problems is apparent in the literature.

This paper will present an approach to the facility location problem which will allow the analysis of multiple conflicting goals as an extension of previous solution approaches. Specifically, the paper a applies a branch and bound nteger goal programming approach to the location-allocation problem.

## The Goal Programming Approach

The concept of goal programming (GP) is best described as a relatively recent extension of linear programming which attempts to resolve the problem of conflicting multiple objectives. The concept was originally presented by Charnes and Cooper [1] and further refined by the work of Ijiri [7], Lee [10], and others. Lee developed the modified simplex method as a solution technique and numerous studies [11], [12], [13], [14], [15] demonstrated a variety of applications of the technique. Additionally, Lee and Morris [see 15] developed and tested integer goal programming algorithms based on the cutting plane method, implicit enumeration, and the branch and bound method.

## The GP Location-Allocation Model

In order to demonstrate the applicability of goal programming to facility location problems, a simple illustrative example is given. A west coast manufacturer wishes to construct a manufacturing facility or facilities which would subsequently service four major eastern distribution centers. For simplicity's sake, it is assumed that a single product is produced. Preliminary screening of sites in the distribution sector has identified five potential communities for plant location which meet general production requirements such as available utilities, labor, access to transportation and raw materials. The first two sites are in the same state and the next two sites are in a second state while the last site in a third state. Because of differing construction costs and varying levels of community development incentives such as free or partially free construction sites, the fixed costs vary from community to community. A list of five plant sites and associated annual fixed costs is given below:

| Potential Sites | Fixed Cost ($1,000) |
|---|---|
| Location 1 (state A) | 825 |
| Location 2 (state A) | 750 |

| | |
|---|---|
| Location 3 (state B) | 600 |
| Location 4 (state B) | 600 |
| Location 5 (state C) | 650 |

**TABLE 1**

| | | DIST CTR 1 | DIST CTR 2 | DIST CTR 3 | DIST CTR 4 |
|---|---|---|---|---|---|
| Transportation Cost | Location 1 | 200 | 110 | 40 | 90 |
| | Location 2 | 180 | 90 | 40 | 80 |
| | Location 3 | 50 | 200 | 225 | 25 |
| | Location 4 | 35 | 160 | 250 | 35 |
| | Location 5 | 210 | 35 | 125 | 50 |
| | DEMAND | 400 | 300 (in cases) | 200 | 100 |

The forecasted annual product demand for the four distribution centers and costs for shipping each ease of goods are given in Table 1.

**Table 1 Goes Here**

The firm wants to ensure that the manufacturing process would not violate the air quality standards set by individual communities and by the states. It wishes to use the same equipment design as is used in the west coast plants, without modification if possible. The manufacturing process discharges 2. 88 lb per hour particle emissions which is an acceptable amount as long as the poundage of materials processed does not exceed the norms given in the state air quality standards for that rate [24]. These standards vary among the states with State A having the most stringent standards. Given the two-shift day the plant intends to operate, maximum yearly case of the product that the firm will be allowed to produce by a single facility in any given year are as follows for each state:

| | |
|---|---|
| State A | 600 cases per plant |
| State B | 480 cases per plant |
| State C | 800 cases per plant |

Distribution center 3 is located in State A and is a particularly favored customer. In order to provide a high level of service to this account, at least one facility needs to be located in State A which must be able to provide at least 50 cases to the center on short notice. Additionally, due to a conservative outlook and relatively weak cash position, the firm wishes to limt fixed costs to 1. 3 million dollars a year.

**Goals and Priorities**

Given above inforomation, as well as the company's desire to meet product demand and to achieve the more traditional goals of keeping a balance between fixed costs of opening new plants and transportation costs, the following goals are listed by the management of the company. They are listed in the order priority with $P_1$ indicating the most important goal.

$P_1$ : Satisfy the product demands of the four distribution centers.

$P_2$ : Limit fixed costs to \$1,300,000 per year.

$P_3$ : Keep facility production within acceptable air pollution emission standards for the given state.

$P_4$ : Ensure that distribution center 3 retains its favored customer status.

$P_5$ : Minimize total costs (fixed costst plus transportation).

$P_6$ : Minimize transportation costs.

**Variables**

In order to formulate the goal programming model of the facilty location problem, the following variable will be used:

$X_{ij}$=the amount of the product (in cases) to be assigned from facility i to distribution center j.

$Y_i$=zero-one decision variables where $Y_i=1$ if facility i is opened, but $Y_i=O$ otherwise.

$C_{ij}$=cost per case to transport goods from facility i to distribution center j.

$d_k^-$=underachievement of goals or constraints in the kth equation.

$d_k^+$=overachiement of goals or constraints in the kth equation.

**Model Constraints**

The constraints for the facility location model can be formulated as follows:

1. Product Demand Goal

This set of constraints assures that each distribution center's demand is met.

$$\sum_{i=1}^{5} X_{i1}+d_1^-=400 \qquad (1)$$

$$\sum_{i=1}^{5} X_{i2}+d_2^-=300 \qquad (2)$$

$$\sum_{i=1}^{5} X_{i3}+d_3^-=200 \qquad (3)$$

$$\sum_{i=1}^{5} X_{i4} + d_4^- = 100 \tag{4}$$

2. Fixed Cost Limitation Goal (In $1,000s)

$$825Y_1 + 750Y_2 + 600Y_3 + 600Y_4 + 650Y_5 + d_5^- - d_5^+ = 1300 \tag{5}$$

3. Air Standards Goal

In order to meet the air standards goal, production must be limited to the following case amounts per new facility.

$$\sum_{j=1}^{4} X_{1j} + d_6^- - d_6^+ = 600 \tag{6}$$

$$\sum_{j=1}^{4} X_{2j} + d_7^- - d_7^+ = 600 \tag{7}$$

$$\sum_{j=1}^{4} X_{3j} + d_8^- - d_8^+ = 480 \tag{8}$$

$$\sum_{j=1}^{4} X_{4j} + d_9^- - d_9^+ = 480 \tag{9}$$

$$\sum_{j=1}^{4} X_{5j} + d_{10}^- - d_{10}^+ = 800 \tag{10}$$

4. Favored Customer Service Goal

It is desired that distribution center 3 will receive good service by having a facility in the same general area (state).

$$\sum_{i=1}^{2} X_{i3} + d_{11}^- = 50 \tag{11}$$

5. Total Cost Minimization Goal

It is desired to minize total fixed and transportation costs.

$$825Y_1 + 750Y_2 + 600Y_3 + 600Y_4 + 650Y_5 + \sum_{i=1}^{5} \sum_{j=1}^{4} C_{ij} X_{ij} + d_{12}^- - d_{12}^+ = 0 \tag{12}$$

6. Transportation Cost Minimization Goal

$$\sum_{i=1}^{5} \sum_{j=1}^{4} C_{ij} X_{ij} + d_{13}^- - d_{13}^+ = 0 \tag{13}$$

7. Zero-One Constraints

Additionally, the following constaints must be included for forcing $Y_i$ to have a value of 1, thus incurring fixed costs, if any units are produced at plant i.

$$\sum_{j=1}^{4} X_{1j} - 3000Y_1 \leq 0$$

$$\sum_{j=1}^{4} X_{2j} - 3000Y_2 \leq 0$$

$$\sum_{j=1}^{4} X_{3j} - 3000Y_3 \leq 0$$

$$\sum_{j=1}^{4} X_{4j} - 3000Y_4 \leq 0$$

$$\sum_{j=1}^{4} X_{5j} - 3000Y_5 \leq 0$$

where the $Y_i$ coefficient of 3000 is chosen as a sufficiently large value to always force $Y_i$ to be 1 if $\sum X_{ij} \gneq 0$ for that facility. The following non-negativity constraints must also hold: $X_{ij} \geq 0$ and $Y_i \leq 1$.

**The Objective Function**

Given the above constaints and considering the priorities assigned to the firm's achievement of goals, the objective function can be formulated as

$$\min Z = P_1 \sum_{i=1}^{4} d_i^- + P_2 d_5^+ + P_3 \sum_{i=6}^{10} d_i^+ + P_4 d_{11}^- + P_5 d_{12}^+ + P_6 d_{13}^+$$

## RESULTS AND DISCUSSION

The preceding facility-allocation problem was solved using a computer program employing a branch and bound algoithm of integer goal programming. The solution is as follows:

Real Variables

$X_{41}=400$ cases shipped from plant 4 to distribution center 1

$X_{52}=300$ cases shipped from plant 5 to distribution center 2

$X_{53}=200$ cases shipped from plant 5 to distribution center 3

$X_{54}=100$ cases shipped from plant 5 to distridution center 4

Zero-one Variables

$Y_4=1$ A plant is opened at location 4

$Y_5=1$ A plant is opened at location 5

Deviational Variables

$d_5^-=50,000$—underutilization of the allowed fixed costs

$d_6^-=600$
$d_7^-=600$
$d_8^-=480$
$d_9^-=80$
$d_{10}^-=200$
} underachievement of production allowed under air quality standards for each facility location

$d_{11}^-=50$—underachievement of minimal shipment to favored customers

$d_{12}^+=1,302,000$—total costt

$d_{13}^+=52,000$—total transportation cost

All other real, zero-one and deviational variables=0

Analysis of the goal achievement or nonachievement shows that the goals regarding satisfaction of demand, fixed costs limitations, and anti-pollution requirements were completely satisfied by this solution. The fourth goal, the delivering of preferred service to distribution center 3, could not be met. Additionally, the goals of minimizing total costs and transportation costs, goals 5 and 6, were not achieved. However, this to be expected since the formutation attempts to minimize the deviation of costs to zero, which could never realistically be achieved.

Interpretation of the deviationa variables associated with these goals show that this solution has a total yearly cost of $1,302,000 and a ransportation cost of $52,000. Additionally, the firm can increase production tat facility 4 by 80 cases and at facility 5 by 200 cases without exceeding pollution emission limitations. This solution leaves the company the company $50,000 for investment in other projects which incur fixed costs.

## CONCLUSION

Traditional models developed for facility location analysis have focused on the minimization of either transportation costs or a combination or a combination of fixed costs and tranportation costs. The facility location problem, however, is complex, generally consisting of multiple conflicting goals. Few models have been able to consider a solution taking these factors into account.

This paper suggests the integer goal programming approach to facility location and demonstrates the application of the model to such a problem. Although the example presented is made simple purposely the formulation and solution of more complex problems can be easily seen. The model strives to achieve as many of the management goals, in order of their priority, as is possible given the constraints of the situation. Further sensitivity analysis can be performed by restructuring priorities or constraints and by interpreting both the overachievement and underachievement of goals. An interactive mode based on the CMS (conversational monitor system) is available for this purpose.

The goal proramming approach to the facility location problem offers a meningful management science tool, which hopefully can provide more useful and practical information for decision making to management.

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