A Competitive Interregional Model

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A Partial Static Study in Mainland Australia

ABSTRACT

The main aim of this paper is to present an alternative technique for solving the spatial equilibrium model in the static competitive economic system, in conjunction with the assumption of linearity of the functions. Mathematical programming is not used at all, however, the trade pattern is effected by the simulated motivation of the profit-seeking entrepreneurs. After showing how this algorithm demonstrates the crude but plausible results, further adaptation and improvement is explored and could be developed with comparative ease.

I. INTRODUCTION

Many recent studies on the spatial equilibrium model have employed mathematical programming such as the transportation model, the simplex method in linear programming and quadratic programming. Most of these studies have discussed various methods with economic implication. Some of the more realistic suggestions introduce iterative methods or parametric programming techniques.[1] For instance, the quadratic programming technique is the first method that has been developed to determine a competitive pattern of equilibrium in the interregional model by using linear demand curves and linear supply curves for the commodity; the method of solution is an iterative procedure of the basic algorithm.

The unrealistic assumptions of linearity of supply and demand curves and of competitive pattern of the economy are relatively innocuous by comparison. Therefore, the development of quadratic programming in regional studies has

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1. P.A. Samuelson, "Spatial Price Equilibrium and Linear Programming", *American Economic Review*, June 1952, pp. 283—303, and K.A. Fox and R.C. Taeuber, "Spatial Equilibrium Models of the Livestock-Feed Economy," *American Economic Review*, September 1955, pp. 584—608.

been regarded as more promising.

It has been argued that the extension of quadratic programming to multiple products, multiple time periods and to non-competitive market structure would be possible and that empirical work in these area would be undertaken in the near future.[2]

Despite all these prospects, there are some practical disadvantages to quadratic programming technique:

(1) The computer programming uses much core space. The basic tableau for both the *Wolfe* and the *Van de panne* algorithms requires n^2+2n rows in the matrix, which is more than most computers can accommodate.

(2) A high level of mathematical knowledge and technique is needed to use the programming confidently. A program like the quadratic method is not likely to facilitate extensions from the initial step of feeding the data to the computer, and probably the further step of refinement in internal computation in the computer system is required for the analysis.

The purpose of this paper is to present an alternative technique for solving the spatial equilibrium model in the static competitive economic system, incorporating the assumption of linearity of the demand and supply functions. Mathematical programming is not used at all; the trade pattern is generated by the simulated motivation of the profit-seeking entrepeneur. This algorithm demonstrates that crude but plausible results can be obtained. Further adaptation and improvement could be achieved with comparative ease. The main advantage of the technique is that it uses much less computer space with many regions and to tape augmentation of core space for the possible extension in complex models.

II. THE GRADIENT METHOD

The method of finding a spatial equilibrium pattern of price and trade in this study is called the gradient method because it is derived from a loose analogy to the physical laws of heat transfer, where temperature gradient figures prominently.[3]

Trade will occur between two regions if the initial price differential exceeds

2. As the method of reactive programming the quadratic programming is reported in the application aspect to interregional competition analysis. In fact, the reactive programming can deal with non-linear supply and demand curves as well. See Thomas E. Tramel, "Reactive programming-an Algorithm for Solving Spatial Equilibrium Problems," *Agricultural Economics, Technical Bulletin No.* 9, Mississippi Agricultural Experiment Station, and T. Takayama and G.G. Judge, "Non-linear Formulation of Spatial Equilibrium Models and Methods of Obtaining Solutions", *A.E.R.R.* 66, Illinois Agricultural Experiment Station. Tramel argues that economic realism assurss convergence to a global maximum, not just a local maximum.

3. G. Hadley, *Nonlinear and Dynamic Programming*, Addison Wesley Publishing Co., 1964.

the cost of transportation between the two regions, just as heat transfer will occur if the temperature gradient is high enough. Since heating and cooling are continual, permanent temperature gradients arise, but heat transfer occurs most rapidly across the steepest gradients of temperature. The pattern of heat transfer, after sufficient time has elapsed to establish a stable state, will equate temperature gradients in all channels between regions in which heat transfer actually occurs.[4]

The Trade Gradient is defined as

$$\text{GRADIENT}(I,J)=\frac{P(J)-P(I)}{T(I,J)}$$

where $P(I)$ = price of potential exporting region I,
$P(J)$ = price of potential importing region J, and
$T(I,J)$ = transportation cost from region I to region J.

In much the same way as with heat, trade is postulated to take place across gradients and will occur first across the highest price gradient. This trading activity is postulated to continue until the price difference between trading regions (like I and J regions) have all been reduced to the cost of transportation between the trading regions, at which time all price gradients, Gradient (I,J), between regions where trading actually occurs will equal one.[5]

As is the usual practice, consumption and production are assumed to be concentrated at one point in each region and trade occurs, if it occurs at all, between these points. Also note that $T(I,J)$ need not equal $T(J,I)$. All possible gradients are computed, and they are then arranged in descending order, to determine the highest and next highest and so on. The highest gradient is reduced either to the level of the next highest, or to an appropriate fraction of its previous value (currently, about 80 per cent of the excess over unity), whichever is lower. A gradient, GRADIENT (I,J), is reduced by introducing trade from region I to region J, which involves movements along the demand and supply schedules and also could involve shifts in the supply curves in the

4. Let Q=quantity of heat, t=time,
A=cross sectional area
T_1T_2=temperatures at two points,
L=distance between T_1 and T_2, and
K=constant of conductivity

$$\frac{Q}{t}=-KA\frac{T_2-T_1}{L}$$

See D. Halliday and R. Resnick, *Physics for Students of Science and Engineering*, Wiley 1962. Also see S. Enke, "Equilibrium Among Spatially Separated Markets: Solution by Electric Analogue", *Econometrica*, January, 1951.

5. For the explicit functional presentation, see H.K. Sohn, "Theoretical Presentation of Interregional Competition Model of the Beef Industry", *The Korean Economic Review*, December, 1971, pp. 164—173, and H.K. Sohn and A.B. Larson, "The Spatial Study of the Market for Mainland Beef Cattle of the U.S.", *Research Bulletin* 168, Hawaii Agricultural Experiment Station, University of Hawaii (forthcoming 1974).

two regions. However this dynamic aspect of shifts in supply schedules is not explored here. As a result, price level is increased in region I and decreased in region J. This process is repeated until all gradients between active trading partners are equal to one, within a predetermined threshold of acceptable precision.

When trade occurs between regions I and J, all gradients involving region I or region J are affected. It is possible that the gradient between regions I and K, for instance, may be reduced below unity even though trade had previously occurred between these two regions. Indeed, this will usually occur during the later stages of the process of convergence to a final solution. If this happens, trade must be redirected to the opposite direction, from region K to region I, until the gradient is restored to greater than or equal to unity. The initial trade might be completely erased during this process.

Although it need not be true in reality, it is convenient to have a region be either an importer, or an exporter, or neither, but not both. Initially, the highest gradient must be between two adjoining regions, but the importing region for trades made early in the adjustment process may subsequently become an exporter. To eliminate re-export trading of this kind, imports which come into the trade of a previous exporter are transferred to a region that previously imported from that exporter; similar adjustments are made for exports from a previous importer. Therefore all trades shown in the final solution are direct trading between ultimate importer and exporter.

Because of the sequential nature of the method spurious signals are given at initial and intermediate stages of the convergence process, and various complex and unacceptable trade patterns can emerge which must be eliminated. For example, two regions might both export to the same two other regions (i.e. there will be four trade routes). Total transportation cost can always be reduced by transferring trades among these partners, and eliminating at least one route. Equivalent situations can arise where trades are not direct but are mediated by complicated sets of partners. All such cases must be detected, and eliminated by suitable transfers of trade among regions.

III. AN EXAMPLE

The computer program used in the analysis was written in FORTRAN IV for the IBM PDP/10 Model. With minor modification to fit the computer facility at the James Cook University in Townsville, Australia, the program has determined the competitive equilibrium pattern of trade and prices for a single commodity for twenty regions,[6] using hypothetical linear demand and supply

6. As an example in this study, the 20 regions are based on major meat export works in principal beef cattle areas in Australia, and they are: Cairns, Townsville, Rockhampton, Marybor-

curves and transportation costs. All operational gradients were reduced to unity plus or minus one-sixteenth of one per cent. About *9 minutes* of machine time was required for the complete execution time compilation of the program read from cards to finding the final solution. Storing the main program on tape would reduce this time considerably. Further minor modifications in the program might also improve the efficiency of the convergence process as well as the computation time.

The present program consists of over 1,000 statements, therefore some core space is used up for the program *per se.* In addition, there are 22 vectors with the number of elements equal to the number of regions. There are four square matrices with each dimension equal to the number of regions. With the twenty regions, hence, about 2,040 words should be reserved to store these vectors and matrices. (That is, $(22)\times(20)+(20)\times(20)\times(4)=2{,}040$ words).

The linear supply and demand curves are hypothesized for 20 regions and given in Table I. For the transportation costs, the coordinates of the trading point for each region are identified from the grid map coordinate system, and they are also shown in Table I and Figure 1.[7] The actual transportation costs between the trading partners were calculated internally in the program using a relationship having a linear variable cost, 0.00154 times the direct distance on this coordinate system and a fixed cost of 0.73768 cents per pound weight.

The initial prices and quantities produced before trade and the equilibrium prices and quantities consumed after trade are shown in Table II. The quantities traded and the trading routes are also shown in Table II. The relevant price gradients before and after trade are shown in Table III. The final trading pattern is shown in Figure 2.

IV. CONCLUSIONS

There are many possibilities for extension and modification of the program. For instance, the hypothetical linear demand and supply curves for each region can easily be altered. Additional economic relationships could be included, for example to permit shifts in the demand curve or the supply curve. The problem of interregional shipment of an intermediate product (or a factor of production) can be considered as a shift in the supply curves for the final product for the importing and exporting regions.

Even competing products of producers or competing commodities for con-

ough, Roma, Brisbane (Queensland) Grafton, Dubbo, Newcastle, Sydney, Wagga Wagga (N.S.W.)
Melbourne (Victoria)
Peterborough, Adelaide (S.A.)
Darwin (N.T.)
Wyndham, Broome, Geraldton, Perth, Albany (W.A.)

7. For an application with empirical data of 48 regions, see H.K. Sohn and A.B. Larsonrs *op.cit.*

sumers can be considered as demand or supply shifters and forced to converge simultaneously and in competition with all goods being analyzed. Dynamic factors, for example, incorporating a production cycle into a model, appear to present great difficulties, since the model in this paper is essentially a static and partial equilibrium approach. However, most interregional models so far developed have also been static, thus this is not a special disadvantage of the gradient method.

Advantages of the static model presented in this paper are that it is simple and intuitive, with more prospects for simulated results than quadratic programming or other elaborate mathematical programming. It also takes less space in the computer and convergence appears to be reasonably fast.

A difficulty which might appear is lack of data.[8] In addition, if regions cannot be subdivided or if resource endowments of regions, factor supplies and input and intermediate product shipments cannot be ascertained and accommo-

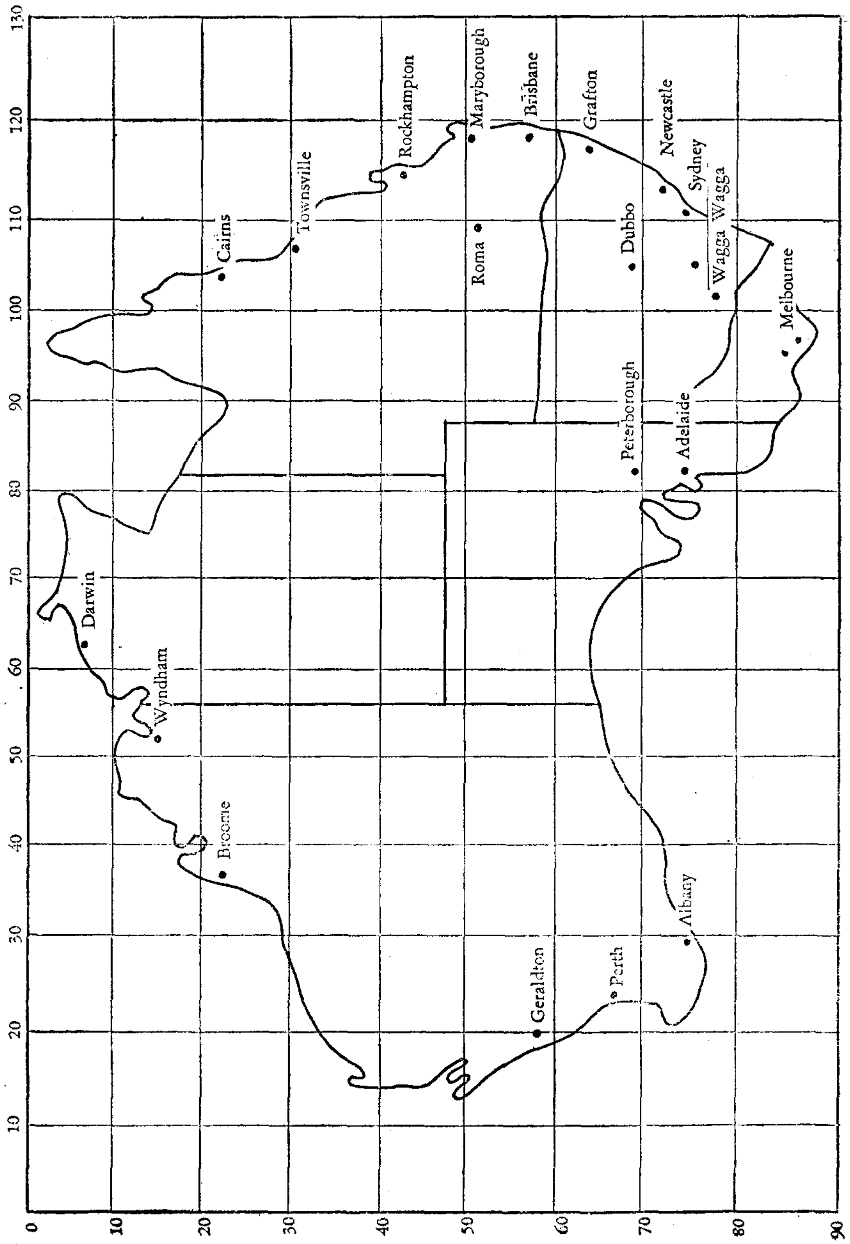
Table 1. Demand and Supply Curves and (South and East) **Coordinates of Trading Points**

Region	Demand	Supply	South	East
1. Cairns	19.5−0.45Q	4.5+1.95Q	22	104
2. Townsville	21.0−0.35Q	7.5+1.70Q	31	106
3. Rockhampton	16.0−1.50Q	5.0+0.65Q	43	115
4. Roma	17.0−1.50Q	5.5+0.85Q	52	109
5. Maryborough	18.0−2.00Q	10.0+2.00Q	52	119
6. Brisbane	16.0−1.55Q	10.5+1.45Q	57	119
7. Grafton	15.4−0.40Q	4.5+1.85Q	64	118
8. Dubbo	15.5−0.35Q	6.5+2.40Q	69	105
9. Newcastle	22.0−0.40Q	12.0+2.40Q	73	113
10. Sydney	20.0−0.50Q	16.0+0.05Q	75	111
11. Wagga Wagga	17.5−0.95Q	8.5+0.40Q	78	102
12. Melbourne	22.5−0.20Q	10.0+0.85Q	84	95
13. Adelaide	17.6−1.85Q	6.5+1.15Q	74	82
14. Peterborough	15.5−1.85Q	9.0+0.95Q	69	82
15. Albany	17.5−1.85Q	9.0+0.95Q	75	29
16. Perth	24.0−0.20Q	16.0+1.25Q	66	24
17. Geraldton	20.0−0.65Q	8.0+1.20Q	57	20
18. Broome	18.0−0.65Q	14.5+0.90Q	22	36
19. Wyndham	20.0−0.65Q	6.5+0.40Q	14	52
20. Darwin	20.0−1.00Q	8.0+0.20Q	5	62

Source: Hypothetical Data by the Author.

8. For a limited study with empirical data in demand schedule for beef in Australia, see H.K. Sohn's "Demand Schedule for Beef in Australia" (unpublished mimeograph), James Cook University of North Queensland, 1973.

Figure 1. Grid Map for 20 Regions in Mainland Australia



dated into the model, then some of the advantages of this program might be vitiated.

Table II. Prices and Quantities Consumed, By Region, Before and After Trade, and Amounts Imported or Exported
(ITERATION NO. 650)

REGION	CONSUMPTION				ESTIMATED PRODUCTION*	TRADE	
	Before Trade		After Trade			Imports (in thousand pounds)	Exports (in thousand pounds)
	Price (in cents)	Quantity (in thousand pounds)	Price (in cents)	Quantity (in thousand (punods)			
1. Cairns	16.69	6.25	15.68	8.49	5.73	2.76	—
2. Townsville	18.70	6.59	15.69	15.18	4.82	10.36	—
3. Rockhampton	8.33	5.12	14.93	0.72	15.28	—	14.56
4. Roma	9.66	4.89	14.94	1.37	11.10	—	9.73
5. Maryborough	14.00	2.00	14.94	1.53	2.47	—	0.94
6. Brisbane	13.16	1.83	14.95	0.68	3.07	—	2.39
7. Grafton	13.46	4.84	14.96	1.11	5.66	—	4.55
8. Dubbo	14.35	3.27	14.96	1.53	3.52	—	1.99
9. Newcastle	20.57	3.57	15.71	15.72	1.54	14.18	—
10. Sydney	16.36	7.27	15.71	8.57	—	8.57	—
11. Wagga Wagga	11.17	6.67	14.98	2.66	16.20	—	13.54
12. Melbourne	20.12	11.90	15.73	33.86	6.74	27.12	—
13. Adelaide	10.76	3.70	14.97	1.43	7.37	—	5.94
14. Peterborough	11.21	2.32	14.96	0.29	6.27	—	5.98
15. Albany	11.88	3.04	14.97	1.37	6.28	—	4.91
16. Perth	22.90	5.52	15.72	41.40	—	41.40	—
17. Geraldton	15.78	6.49	15.71	6.60	6.43	0.17	—
18. Broome	16.53	2.26	15.66	3.60	1.28	2.32	—
19. Wyndham	11.64	12.86	14.89	7.86	20.98	—	13.12
20. Darwin	10.00	10.00	14.87	5.13	34.36	—	29.23

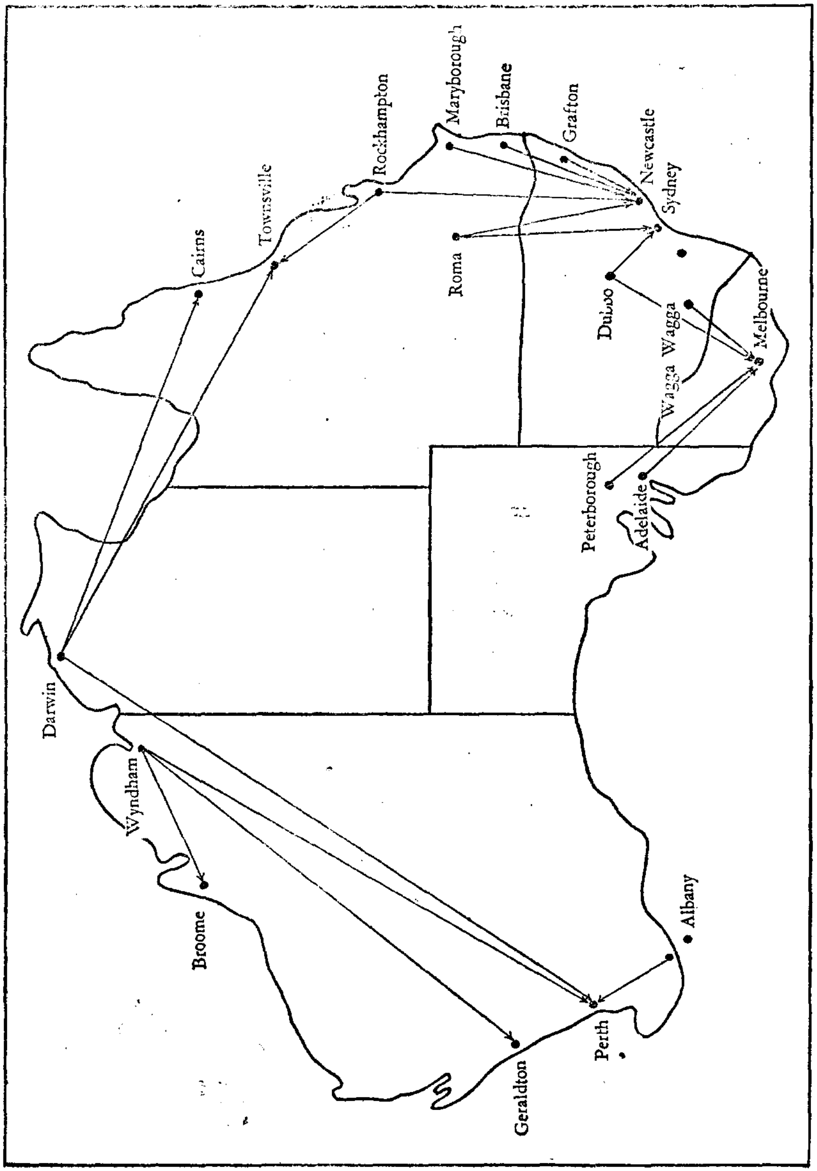
*Estimated Production was derived as equal to the quantity consumed at the equilibrium after trade minus the import amount of the region plus the export amount of the region.

Table III. Trading Partners in Final Solution, and Price Gradient Between Each Set of Partners.

Trading		Amount Shipped (in thousand lbs.)	Price Gradient
From	To		Before Trade*
Rockhampton	Townsville	9.75	3.22
Rockhampton	Newcastle	4.81	3.50
Roma	Newcastle	1.49	3.30
Roma	Sydney	8.24	2.59
Maryborough	Newcastle	0.94	2.56
Brisbane	Newcastle	2.39	2.72
Grafton	Newcastle	4.55	2.67
Dubbo	Sydney	0.33	1.42
Dubbo	Melbourne	1.66	2.40
Wagga Wagga	Melbourne	13.54	2.99
Adelaide	Melbourne	5.94	3.06
Peterborough	Melbourne	5.98	2.99
Albany	Perth	4.91	3.32
Wyndham	Perth	10.63	3.36
Wyndham	Geraldton	0.17	2.04
Wyndham	Broome	2.32	2.21
Darwin	Cairns	2.76	2.59
Darwin	Townsville	0.61	2.95
Darwin	Perth	25.86	3.59

*After trade, all price gradients will be equal to 1.00.

Figure 2. Trading Pattern within Australian Mainland using Hypothetical Data



Transportation Cost Matrix
(in cents per pound)

FROM \ TO	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0.000	0.752	0.774	0.785	0.789	0.796	0.806	0.810	0.817	0.820	0.824	0.834	0.825	0.818	0.879	0.878	0.878	0.842	0.819	0.807
2		0.00	0.761	0.770	0.776	0.782	0.792	0.796	0.803	0.806	0.810	0.821	0.814	0.807	0.874	0.875	0.876	0.846	0.825	0.816
3			0.000	0.754	0.753	0.760	0.770	0.781	0.784	0.787	0.795	0.808	0.807	0.802	0.879	0.832	0.886	0.864	0.844	0.838
4				0.000	0.753	0.755	0.761	0.765	0.771	0.773	0.779	0.791	0.791	0.787	0.866	0.870	0.875	0.859	0.843	0.840
5					0.000	0.745	0.756	0.772	0.771	0.775	0.736	0.799	0.804	0.800	0.881	0.886	0.890	0.874	0.856	0.851
6						0.000	0.749	0.766	0.764	0.768	0.779	0.793	0.800	0.798	0.879	0.885	0.890	0.876	0.860	0.856
7							0.000	0.759	0.754	0.758	0.770	0.785	0.795	0.794	0.876	0.882	0.889	0.880	0.865	0.863
8								0.000	0.751	0.751	0.752	0.765	0.774	0.773	0.855	0.863	0.870	0.866	0.855	0.856
9									0.000	0.742	0.756	0.770	0.785	0.786	0.867	0.875	0.883	0.880	0.868	0.869
10										0.000	0.752	0.766	0.782	0.783	0.864	0.872	0.881	0.879	0.868	0.869
11											0.000	0.752	0.769	0.771	0.850	0.859	0.868	0.871	0.863	0.866
12												0.000	0.763	0.768	0.840	0.850	0.860	0.869	0.864	0.870
13													0.000	0.745	0.819	0.828	0.837	0.845	0.841	0.848
14														0.000	0.820	0.827	0.835	0.839	0.834	0.841
15															0.000	0.754	0.769	0.820	0.838	0.857
16																0.000	0.753	0.808	0.829	0.848
17																	0.000	0.797	0.820	0.841
18																		0.000	0.765	0.786
19																			0.000	0.758
20																				0.000