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Cheap Talk by Two Senders in the Presence of Network Externalities*

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We develop a model of cheap talk with two senders in the presence of network externalities, such that their utility functions are increasing in the network size. We first show that, if there is no noise in the private information received by each sender, the full information is revealed by the harshest cross-checking strategies, that is, strategies to punish the senders unless their messages exactly coincide. Then, we prove that, even with a small noise, cross-checking strategies cannot induce full revelation if the utility functions of senders are linear in the network size, whereas full revelation is possible if utility functions are strictly concave. Finally, we show that a CARA (constant absolute risk aversion) utility function of senders is the necessary and sufficient condition for the existence of a fully revealing equilibrium, which is supported by the cross-checking strategy with a positive confidence interval independent of each sender's private information.

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I. Introduction

Is a recommendation letter of a professor credible? Is a car dealer selling used cars trustworthy? Is a lawyer's legal advice reliable? Since the publication of the seminal paper by Crawford and Sobel (1982), hereafter abbreviated as CS, it has

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been widely accepted that the partial (not full) information of the informed can be transmitted to the uninformed if their interests are similar enough.

However, in many cases, an uninformed party refers not to a single informed party but to multiple informed parties. Then, a natural question arises as to whether or not the uninformed can really elicit more accurate information by doing so. Why do universities require multiple letters of recommendation from applicants? Why do major academic journals make it a rule for multiple referees to review an unsolicited article? Why do wealthy people hire more than one attorney at a time? Whay do the independent opinions of Siskel and Ebert, two famous movie critics, appear side by side?

One obvious solution is to elicit more accurate information from the informed. This scenario is okay if the informed always provides honest opinions. However, if the interests of the informed are not aligned with those of the uninformed, the solution provided above can only offer a partial answer, especially when university professors, article referees, attorneys, and movie critics have some common interests with the students, article authors, the opposite legal party, and movie producers/ directors. This is because the answer does not take into account the effect on the incentive of the informed to misrepresent their information. Therefore, a more satisfactory solution should be able to address how the presence of other speakers discipline the incentive of a speaker to distort information.

Many articles have been published on information transmission by multiple informed parties, whose interests possibly differ with the interest of the uninformed.[1] Each article provides different models basically addressing the question of whether or not the uninformed is able to obtain more accurate information by multiple informed parties than by a single informed party. However, all of the analyses are based on the preference assumption that the uninformed's actions favored both by the informed and the uninformed are increasing in the state of nature, which is the private information of the informed. In other words, in all of the models, one essential ingredient for credible communication is that the informed with different private information must have different preferences over the actions of the uninformed.

[1] A short list, if not comprehensive, includes Gilligan and Krehbiel (1989), Austen-Smith (1993), Krishna and Morgan (2001), Battaglini (2002), Ambrus and Takahashi (2008), Li (2010), Galeotti *et al*. (2013), McGee and Yang (2013), Ambrus and Lu (2014), and Li *et al*. (2016). In particular, Ambrus and Takahashi (2008) assume that the state space is multi-dimensional and bounded, both of which critically depart from our model that assumes a unidimensional and unbounded state space. Ambrus and Lu (2014) extend the analysis to the unbounded state space, but they exclude the possibility of network externalities by assuming that the biases of the senders are finite. Galeotti *et al*. (2013) consider multiple senders who send messages to some or all of the others. Thus, each player can be both a sender and a receiver. However, there is no network externality assumed in the paper, either. The model proposed by McGee and Yang (2013) is very close to our model, but again, no network externality is assumed.

In this paper, we assert that such a condition on preferences is not necessary for credible communication in the presence of multiple informed parties. As a matter of fact, situations where such a condition is not satisfied abound. For instance, suppose there is an experience good, the quality of which is not learned before a consumer purchases one. An uninformed consumer who has to decide whether to buy one may refer to informed consumers for the quality. If the purchasing decision by the latter does not affect the utility of existing consumers at all, that is, no consumer externalities are involved, existing consumers who are referred to will have no incentive to garble their own information about the quality. In this case, it would not be surprising that all references were truth-revealing.[2] However, if it does affect the utility of other existing consumers, that is, there are network externalities, they might have an incentive to exaggerate the quality of the good to boost the demand for it. In fact, it was often observed that old-time Mac-users alleged the ultra-superioity of Macintosh even though they knew of the inconvenience brought about by the limited network size exceeding the benefit from the relative quality advantage after the advent of the window system. In this situation, a consumer may wonder if word-of-mouth (WoM) communication can be a reliable source of information regarding the quality of an experience good with network externalities.[3]

In this paper, we show that even in such a situation where there is no room for coordination between an informed party and an uninformed party, truthful revelation is possible if the latter solicits references from multiple informed parties. The intuitive reason for this is that, in this case, the uninformed has a means of probabilistically checking the truth of the message from one informed party, which is the message from the other informed parties. Furthermore, the situation is like a coordination game among informed parties. Even though coordination is, in fact, realized by the action of the uninformed, the communication messages of the informed parties are a vehicle of implementing coordination, and more fundamentally, correlation among them is the genuine source of their coordination. That is, in this situation, WoM communication is not a way of coordination between an informed party and an uninformed party, but a way of coordination among informed parties with correlated private information.

To support the truth-revealing outcome as an equilibrium, we will use a specific form of strategy of the uninformed, which we shall call "cross-checking strategies." By a cross-checking strategy, we mean a strategy to reward senders if their messages are similar to one another and otherwise punish them.

We consider two cases: the noiseless information case and the noisy information case. If there is no noise in the private information received by each sender, the

[2] This argument will be more convincing if the reputational effect is taken into account.

[3] While we were revising this paper, an anonymous referee informed us of the existence of the literature on cheap talk with transparent motives. See Section 2 for more details.

cross-checking strategy takes the following form, that is, the uninformed believes that either one of the informed parties is fibbing as far as their messages are not exactly the same, and then takes a punishing action that is harsh to both of them; otherwise, the uninformed believes them literally and takes the optimal action given the updated posterior belief. If there is some noise in their information, the cross-checking strategy takes a rather complicated form. If the messages sent by the informed are observed not to be too far apart, more specifically, to be within a certain distance, the uninformed believes them, thereby taking a rewarding action, i.e., choosing the maximum amount of senders' messages, and otherwise punish them by choosing the minimum amount of their messages.

We first show that if there is no noise in private information that each sender receives, the full information is revealed by the harshest cross-checking strategies, that is, strategies to punish the senders unless their messages exactly coincide. Then, we show that, with the introduction of even a small noise, the cross-checking strategy cannot induce full revelation if the utility functions of the senders are linear in the network size. The difficulty in this case arises mainly because even a small noise makes off-the-equilibrium messages vanish completely under the normal distribution of the noise. Even if a message is too high or too far from the other messages, it is a possible event, although the likelihood is very low. Thus, the uninformed receiver cannot believe that it is a consequence of a sender's lying. This makes it difficult to sufficiently penalize a sender who sends a higher message than the true value. However, we show that if the utility functions are strictly concave, full revelation is possible with the cross-checking strategy. In this case, the strict concavity of utility functions can make the penalty from inflating the message exceed the reward from it, so that it can discipline senders who are tempted to lie. We also show that senders having a constant absolute risk aversion (CARA) utility function is the necessary and sufficient condition for the existence of a fully separating equilibrium with the cross-checking strategy.

The rest of the paper is organized into sections. In Section 2, we briefly review the related literature. In Section 3, we introduce the model. In Section 4, we analyze the noiseless case, in which both senders receive exactly the same information. In Section 5, we analyze the noisy case. Section 6 presents the concluding remarks and an avenue for future research.

II. Related Literature

Seidmann (1990) and Gibbons (1989) previously noticed that cheap talk can influence the receiver's equilibrium actions even if all the types of the sender share a common preference ordering over the actions of the receiver. In a setting with one sender, Seidmann (1990) shows that whether the receiver is himself privately informed, or his action is multi-dimensional, the sender's types may disagree in their preferences over distributions of actions generated by the distribution of the receiver's types or over the pair of actions by the receiver due to their different trade-offs between the actions.

Gibbons (1989) presents a model that is closest to ours. He analyzes a model of conventional arbitration, in which the employer and the union simultaneously submit offers and then the arbitrator imposes a settlement. He also obtains the truth-revealing result that the parties' offers perfectly reveal their private information to the arbitrator. The crucial difference of his model from ours is that the parties observe the same noisy signal of the underlying state variable and that the arbitrator himself receives a direct correlated signal. This feature of correlation between senders' information and the receiver's information drives his result of perfect communication.

In a series of papers on legislative decisions (Gilligan and Krehbiel, 1989; Austen-Smith, 1993; Epstein, 1998), the authors explore the informational role of the committee. Gilligan and Krehbiel (1989) and Epstein (1998) both consider models of legislative organization and two committee members with diverse preferences (presumably from different parties). Gilligan and Krehbiel assert that if the committee preferences are symmetric about the floor's ideal point, floor members can obtain better information on the bill reported to the floor when two committee members with diverse preferences both agree to support the bill. Epstein shows that the argument proposed by Gilligan and Krehbiel does not hold under asymmetric committee preferences. Meanwhile, Austen-Smith (1983) considers a model, in which an informed House multiplely refers legislation to two committees with diverse preferences. He shows that more information can be communicated under multiple referrals than under a single referral.

Recent studies on cheap talk with transparent motives include, for example, Chakraborty and Harbaugh (2010) and Lipnowski and Ravid (2018). By transparent motives, they mean that the informed sender does not care about the state but only about the receiver's action. In that sense, senders in our model with network externalities also have transparent motives. The authors of both papers show that cheap talk can be informative even if the sender has a transparent motive. One important difference from our model is that their models are about cheap talk with one sender, not about cheap talk with multiple senders. In Charkraborty and Harbaugh (2010), the informativeness of cheap talk relies on the multi-dimensionality of the state variable, which implies that the receiver cares about multiple issues rather than one issue, unlike in our model. Moreover, while we assume that the state space and the receiver's action space are unbounded, Lipnowski and Ravid (2018) use a different assumption that the state space is compact (and an implicit assumption that the action space is compact). Moreover, they do not consider the noisy information case which is central to our analysis.

Meanwhile, Farrell and Saloner (1985) explore the role of communication in an industry with network externalities. They consider a situation, in which potential users with independent private information on valuations of alternative technologies can engage in cheap talk with each other about which technology to adopt. They find that communication eliminates excess inertia where the preferences of the users coincide, while it increases inertia where their preferences differ. Their model also assumes that all potential users are informed of their valuations on technologies before they purchase one without any explicit explanation of how they obtained the information. In their model, the role of cheap talk by potential consumers is to announce their intentions of which technology to purchase, whereas in our model, it is made by existing consumers in order to inform the potential consumer of their valuation on the product.

Communication via W0M has been modelled by several authors. In Ellison and Fudenberg (1995), decision-making agents ask several other randomly chosen individuals from the population about their current choice and payoff, so that the former can make their own choices between two alternatives, based on the latter's reports, assuming that they are truth-telling. As they assumed that each player's payoff is not influenced by the actions chosen by others, it seems natural, in their model, not to pay heed to the incentives of the informed consumers to be honest. Satterthwaite (1979) addresses the question of how information about sellers flows among consumers. However, his analysis is also based on the assumption of naive speakers, who always speak honestly, and naive listeners, who always take messages seriously.

III. Model

We develop a model of cheap talk with two senders. There are two senders or speakers S_i, $i \in N \equiv \{1,2\}$ and one receiver or listener R. The state of nature, θ, is a random variable with probability distribution function, $F(\theta)$, and density function, $f(\theta)$, supported on $\Theta \equiv \mathbb{R}$. For example, senders are consumers using the same computer of quality, θ. Here, R can be interpreted as a large organization, such as a university or a company that is going to decide to buy a number of same computers.

For simplicity, we assume that θ is uniformly distributed over $\Theta = \mathbb{R}$,[4] i.e., senders have no information about θ or no bias *a priori*. Only S_i's observe a noisy signal on the state of nature $v_i \in V = \mathbb{R}$ where $v_i = \theta + \varepsilon_i$, ε_i is stochastically independent with θ, and ε_i's are i.i.d. We assume that ε_i follows

[4] Note that we are assuming an improper prior distribution.

a normal distribution with its mean zero and the variance σ^2.[5]

The game proceeds as follows. First, the state of nature θ is realized and then senders receive their respective signal v_i without knowing θ. After observing private information v_i, S_i's send a payoff-irrelevant signal (cheap talk) $m_i \in M = \mathbb{R}$ to R simultaneously.[6] Then, receiving a vector of signals $m = (m_1, m_2) \in M^2$, R updates his posterior belief about v_1 and v_2, $b_1(m)$ and $b_2(m)$, and then forms his belief about θ $b(m)$ by using $b_1(m)$ and $b_2(m)$, where $b_i, b : M^2 \to \mathbb{R}$,[7] based on which he chooses an action $a \in A(=\mathbb{R})$ that is a network size.[8] A strategy of the receiver determines the senders' payoffs as well as his own payoff.

The payoff to S_i is given by a continuously differentiable function $U^{S_i} : A \to \mathbb{R}$ for all i, and the payoff to R is given by the twice continuously differentiable function $U^R : A \times \Theta \to \mathbb{R}$. Throughout the paper, we assume that (1) $U^{S_i}(a) = u(a)$ where $u' > 0$, $u'' \leq 0$, i.e., increasing in a, and (2) $U^R(a, \theta) = -(a - \theta)^2$. The receiver's utility function implies that it has a unique maximum in a for all θ and the maximizer of U^R, denoted by $a^R(\theta)$, is strictly increasing in θ.[9] The utility functions of senders that are increasing in a mean that the decision of R involves a network externality. The monotonic increase of $a^R(\theta)$ in θ means that the receiver will want to buy more units of high θ, which can be interpreted as quality. The asymmetry between the utility function of senders and the receiver comes from the feature that only the receiver (consumer) pays the price. That is, while the uninformed consumer wants to purchase more units as the quality is higher, the informed consumers who already purchased one want the uninformed consumer to buy as many as possible regardless of the quality, because of the network externality.

A strategy for S_i specifies a signaling rule given by a measurable function $s_i : V \to M$. A strategy for R is an action rule given by a function $\alpha : M^2 \to A$.

The equilibrium concept that we employ is that of weak Perfect Bayesian equilibrium (wPBE). An equilibrium of this game consists of a vector of a signaling

[5] Alternatively, we can assume that S_i observes the true value of θ with probability $1 - \varepsilon_i$ and observes something else with ε_i.

[6] Given that the cheap talk messages of senders, m_i, are payoff-irrelevant by the definition of cheap talk, the payoffs of the players (U^{S_i} and U^R) which are described below should not depend on m_i. Kartik (2009) considers messages with lying costs. As a lying message of a sender affects the payoff of the sender, it is not cheap talk. In our model, cheap talk affects the payoffs of players not directly, but only through the belief of the receiver.

[7] The belief b could be defined by a function μ, which corresponds to a probability distribution to each pair of messages (m_1, m_2), but we prefer our notation mainly because of its simplicity and intuitiveness. Thus, $\mu(\theta = v \mid m) = 1$ in the standard notation can be denoted simply by $b(m) = v$ in our notation.

[8] By network size, we mean the number of products which is the same as, or at least compatible to the product that the senders are using.

[9] If $\theta < 0$, $a^R(\theta) < 0$.

rule for S_i, an action rule of R, and a system of beliefs $((s_i^*(v_i))_{i=1}^2, \alpha^*(m), (b_i(m))_{i=1}^2, b(m))$ such that

(2-I) $s_i^*(v_i) \in \text{argmax}_{m_i} \int_{-\infty}^{\infty} U^{S_i}(\alpha^*(m_i, s_j^*(v_j)))h(v_j \mid v_i)dv_j$,

where $h(v_j \mid v_i)$ is the conditional density function of v_j given v_i, for $j \neq i$.

(2-II) $\alpha^*(m) \in \text{argmax}_a U^R(a, b(m))$.

(2-III) R's posterior belief $b_i(m)$ is consistent with the Bayes' rule on the equilibrium path and $b(m)$ is an unbiased estimator of $b_1(m)$ and $b_2(m)$.[10]

Henceforth, we simply use the notation of $h(v_j)$ for the density function conditional on v_i and $H(v_j)$ for the corresponding distribution function by suppressing v_i. Prior to characterizing the equilibria, we adapt some standard definitions often used in the literature.

Definition 1 *An equilibrium is communicative iff there exist two different vectors of observations* v, v'*, such that* $s^*(v) \neq s^*(v')$ *and* $\alpha^*(m) \neq \alpha^*(m')$*, where* $m = s^*(v)$*,* $m' = s^*(v')$*. An equilibrium is uncommunicative (or babbling) otherwise.*

Definition 2 *A communicative equilibrium is fully-revealing iff* $s^*(v) \neq s^*(v')$ *for any* v, v'*, such that* $v \neq v'$*. In particular, if* $s^*(v) = v$*, a fully-revealing equilibrium is a truth-revealing equilibrium.*[11]

Definition 3 *A message vector* m *induces an action* a *iff* $a = \alpha^*(m)$.

In this paper, we will restrict our attention to symmetric equilibria, such that $s_i^*(v_i) = s_j^*(v_j)$ if $v_i = v_j$, for all $j \neq i$. Let the symmetric equilibrium strategy be denoted by $s^*(\cdot)$. Then, the definition of the fully-revealing equilibrium is reduced to $s^*(v_i) \neq s^*(v_i')$ for any v_i, v_i' such that $v_i \neq v_i'$.

Observe that, in this model, unlike the CS model, if only one informed party can engage in cheap talk, the message he sent cannot be credible at all. In the CS model, the payoff function of the sender (S) as well as that of the receiver is single-peaked,

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[10] Given that R is not informed of three values, v_1, v_2 and θ, he must form all of the three beliefs. However, the beliefs that he can infer from the weak consistency requirement of wPBE are only about v_1 and v_2, not about θ, because the value of θ is not known to senders, either. Instead, R can obtain an estimator for θ from the two observations or the beliefs $b_1(m_1)$ and $b_2(m_2)$. However, given that θ is not a type of sender (because senders do not know the value), the definition of wPBE does not impose any requirement for the estimator. It could be any weighted average of the messages, $\lambda m_1 + (1-\lambda)m_2$ where $\lambda \in [0,1]$, in particular, m_1 or m_2 by ignoring one message, if one requires the unbiasedness of the estimator at the very least. All of them are perfect unbiased estimators of θ. Our $b(m)$ is a summary statistic that can be obtained after two separate processes, the inference process and the estimation process based on the inference.

[11] Given that even fully revealing strategies, which are $v_i \neq s_i^*(v_i)$, reveal the truth in equilibrium, those strategies are literally truth-revealing. Thus, in fact, the words "fully-revealing" and "truth-revealing" could be exchangeable.

so that, given θ, the favorite actions to S and R do not differ very much. This implies that, for some low θ, both S and R prefer one action to another, whereas the reverse is true for some other high θ. In other words, there is room for coordination between S and R and, in effect, cheap talk enables such coordination to occur by conveying the message whether θ is high or low. In this model, however, the assumption of single-peaked preferences is violated and all the types of S prefer a higher level of the receiver's action a. Thus, S would like to pretend to have observed as highest v as possible to induce R's highest action possible, regardless of his type.

We now summarize with

Proposition 1 *If* $n=1$*, there exists no communicative equilibrium.*

Proof. Suppose, in equilibrium, that there exist two different observations v, v', such that $m \neq m'$ and $a \neq a'$, where $m = s^*(v)$, $m' = s^*(v')$, $a = \alpha^*(m)$ and $a' = \alpha^*(m')$. If we assume $a < a'$ without loss of generality, S who observes v will have an incentive to deviate to m' because $U^S(a,\theta) < U^S(a',\theta)$, $\forall \theta$.

However, if there is more than one sender, the above argument breaks down. Suppose there are two senders S_1, S_2 and the vector of messages (m_1, m_2) sent by them induces an action a, while (m_1', m_2') induces an action a' with $a < a'$. Then, we cannot conclude that S_i will prefer sending m_i' to m_i, because it does not necessarily induce a higher level of action a'. In other words, in the presence of more than a sender, one sender cannot be sure what message will be sent by the other sender.

In the next section, we conduct a formal analysis of cheap talk with two senders in the case that v_1 and v_2 are noiseless, i.e., $v_1 = v_2 = \theta$. In Section 5, the argument will be extended to the noisy case.

IV. Noiseless Case

We first consider the case that $\sigma^2 = 0$, so that $v_1 = v_2 = \theta$. As it is well-known, there always exists a babbling equilibrium, in which senders send a random message and the receiver ignores any vector of messages whatsoever and chooses $a = E(\theta)$. In this section, we aim to determine whether there can exist a communicative equilibrium as well, in particular, a truth-revealing equilibrium where each type of sender reveals its true information.[12]

[12] We do not want to propose an equilibrium selection criterion among multiple equilibria in this paper, because our main goal is to show the possibility that the first-best truth-revealing equilibrium

Proposition 2 *In the noiseless case* $(\sigma^2 = 0)$, *we have the following communicative truth-revealing equilibrium in this game:*

(i) $s_i^*(v_i) = v_i$,

(ii) $b(\mathbf{m}) = \begin{cases} m = m_1 = m_2 & \text{if } m_1 = m_2 \\ \underline{m} \equiv \min\{m_1, m_2\} - \eta & \text{if } m_1 \neq m_2, \end{cases}$

(iii) $\alpha^*(\mathbf{m}) = b$

for some $\eta > 0$.

As the proposition says, this equilibrium is supported by the posterior belief $b(m) = m$ if $m_1 = m_2 \equiv m$ and $b(m) = \underline{m}$ if $m_1 \neq m_2$ where $\underline{m}$ is lower than the minimum of m_1 and m_2.[13] The proof is straightforward. If a sender with v_i sends $m_i > v_i$, then $m_i > v_i = v_j = m_j$, given that the other sender sends a truthful message $m_j = v_j$. Given that $m_i \neq m_j$, $U^{S_i}(m_i) = u(\underline{m}) < U^{S_i}(v_i) = u(v_i)$, because $\underline{m} = v_i - \eta < v_i$. Clearly, a sender does not have an incentive to send $m_i < v_i$. Moreover, if the messages are the same, the receiver must believe them as the value of θ, i.e., $b(\mathbf{m}) = m$ if $m_1 = m_2 = m$, because senders do not lie in equilibrium and there is no noise in their information. If the messages differ ($m_1 \neq m_2$), any belief can be possible, because it is off the equilibrium path, so $b(\mathbf{m}) = \underline{m}$ is also a perfectly legitimate belief. However, we can justify the belief as follows. After observing the messages, the receiver will have two scenarios in mind. If either sender deviated from the equilibrium, it can be inferred that he is the one who sent a higher message, considering that both senders want to inflate their information, not to deflate it. If both senders deviated, which is much less likely, the true value of θ should be lower than the minimum of the messages.

Readers may wonder whether the off-the-equilibrium belief $b(\mathbf{m}) = \min\{m_1, m_2\}$, without subtracting η, can sufficiently support the above truth-revealing strategies as an equilibrium. In fact, the strategies can be an equilibrium with the off-the-equilibrium belief, but only in a weak sense in that S_i is indifferent between being honest and lying in the equilibrium.[14] Under our belief,

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can exist, not to single out the most plausible equilibrium outcome by a possibly disputable criterion. One possible selection criterion might be to resort to the focal point argument proposed by Schelling (1963) by asking which equilibrium between the complete babbling equilibrium and the fully-revealing equilibrium is more prominent.

[13] Although this cross-checking strategy is similar to the strategy constructed by Krishna and Morgan (2001), there is a slight difference. Our model is closer to the case that senders have like biases in Krishna and Morgan (2001), but in the model, they used the off-the-equilibrium belief $b(m) = \min\{m_1, m_2\}$.

[14] This equilibrium can be eliminated by a stronger refinement such as a variation of trembling-hand perfect equilibrium in extensive form games which is adapted to games with continuum strategy space although the perfectness concept was originally defined for finite games. To illustrate, first discretize the message set of each sender to $\tilde{M} = \{\cdots, \theta - 2\delta, \theta - \delta, \theta, \theta + \delta, \theta + 2\delta, \cdots\}$ for some small

S_i strictly prefers honesty over a case of exaggerating the information.

V. Noisy Case

In Section 3, we assumed that $v_i = \theta + \varepsilon_i$, where $Var(\varepsilon_i) = \sigma^2 > 0$. In this section, we analyze the noisy case.

For our purpose, let us concentrate on the following specific form of strategy profile:

(3-I) S_i with v_i announces $m_i = v_i$.

(3-II) R believes $b(\mathbf{m}) = \max\{m_1, m_2\}$ if $|m_1 - m_2| \le \rho$ and believes $b(\mathbf{m}) = \min\{m_1, m_2\}$ if $|m_1 - m_2| > \rho$ for some $\rho > 0$.[15]

(3-III) R chooses $\alpha(\mathbf{m}) = b$.

R's action rule given by (3-III) is called a "cross-checking strategy."[16] Note that there is no off-the-equilibrium message in this noisy case, because any message can occur even if both tell the truth, as long as ε_i follows a normal distribution over $(-\infty, \infty)$.

Now, consider the optimal strategy rules of senders. Sender 1 maximizes

$$U^{S_1}(m_1; v_1) = \int_{-\infty}^{m_1-\rho} u(v_2)h(v_2)dv_2 + \int_{m_1-\rho}^{m_1} u(m_1)h(v_2)dv_2$$
$$+\int_{m_1}^{m_1+\rho} u(v_2)h(v_2)dv_2 + \int_{m_1+\rho}^{\infty} u(m_1)h(v_2)dv_2 . \qquad (1)$$

The economic reasoning behind this formula goes as follows. Given that sender 2 announces truthfully, the first and last terms represent the punishments that sender 1 would receive when v_2 is very low ($v_2 < m_1 - \rho$) and when v_2 is very high ($v_2 > m_1 + \rho$), respectively. The second and the third terms indicate his utility when

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$\delta > 0$ and take a perturbed strategy $\sigma_i = (\cdots, \varepsilon_k^n, \cdots, \varepsilon_k, 1 - \frac{2\varepsilon_k}{1-\varepsilon_k}, \varepsilon_k, \varepsilon_k^2, \varepsilon_k^3, \cdots)$. Clearly, this is a totally mixed strategy converging to $m_i = \theta$, since $\sum_{n=1}^{\infty} \varepsilon_k^n = \frac{\varepsilon_k}{1-\varepsilon_k}$ and that $\frac{\varepsilon_k}{1-\varepsilon_k} \to 0$ as $\varepsilon_k \to 0$. Then, it is easy to see that $m_i = \theta$ (being honest) is weakly dominated by some inflation to $m_i' = \theta + \delta$, as the latter strategy is better when the other sender makes a mistake to $\theta + \delta$ with some positive probability. Therefore, $m_i = \theta$ cannot be a best response to the above perturbed strategy of the other. This implies that it cannot be a trembling-hand perfect equilibrium.

[15] As we put in Footnote 10, these on-the-equilibrium beliefs are perfectly legitimate beliefs about θ, as the weak consistency condition of wPBE requires $b_1(m_1)$ and $b_2(m_2)$ to be consistent with the equilibrium messaging strategies of senders, but does not impose any requirement on the estimator for θ based on $b_1(m_1)$ and $b_2(m_2)$.

[16] Navin Kartik commented on the monotonicity of our strategies. We could say that the receiver's strategy is monotonic if $\partial\alpha(m_1, m_2)/\partial m_i \ge 0$. It is not difficult to see that the cross-checking strategy is not monotonic. If we impose the monotonicity of R's strategy, there would be no equilibrium other than the babbling equilibrium in this game.

v_2 falls under a normal (reward) region. Then, we have

$$\frac{\partial U^{S_1}}{\partial m_1} = u(m_1-\rho)h(m_1-\rho) + u'(m_1)\int_{m_1-\rho}^{m_1} h(v_2)dv_2 + u(m_1)(h(m_1)-h(m_1-\rho))$$
$$+u(m_1+\rho)h(m_1+\rho) - u(m_1)h(m_1)$$
$$+\left[u'(m_1)\int_{m_1+\rho}^{\infty} h(v_2)dv_2 - u(m_1)h(m_1+\rho)\right]. \quad (2)$$

The first term is the loss from being punished by increasing his announcement marginally (when v_2 is very low), and the last term is the gain from avoiding punishment (when v_2 is very high). The remaining terms are just the effect of utility increases in normal cases due to the inflated announcement.

If $u(a)$ is linear, the first and last terms are cancelled out due to symmetry, so $\frac{\partial U^{S_1}}{\partial m_1} > 0$, for all m_1, but if u is strictly concave, the loss is larger than the gain in absolute values; thus, it may not be necessarily that $\frac{\partial U^{S_1}}{\partial m_1} > 0$ for all m_1.

Truthful revelation requires $\frac{\partial U^{S_1}}{\partial m_1}\Big|_{m_1=v_1} = 0$. This implies that

$$\left.\frac{\partial U^{S_1}}{\partial m_1}\right|_{m_1=v_1} = h(v_1-\rho)(u(v_1-\rho)-u(v_1)) + h(v_1+\rho)(u(v_1+\rho)-u(v_1))$$
$$+u'(v_1)\left(\int_{v_1-\rho}^{v_1} h(v_2)dv_2 + \int_{v_1+\rho}^{\infty} h(v_2)dv_2\right)$$
$$=0. \quad (3)$$

By using $h(v_1-\rho) = h(v_1+\rho)$ and $\int_{v_1}^{v_1+\rho} h(v_2)dv_2 = \int_{v_1-\rho}^{v_1} h(v_2)dv_2$, Equation (3) is reduced to

$$4h(v_1-\rho)\left[u(v_1) - \frac{u(v_1-\rho)+u(v_1+\rho)}{2}\right] = u'(v_1). \quad (4)$$

The equation above implies that the equilibrium value for ρ must balance the expected net loss from inflating the message, which is the left-hand side (LHS), with the direct gain from the inflated message, which is the right-hand side (RHS).

If $u(\cdot)$ is linear, LHS is zero, implying that a sender always has an incentive to inflate his message, as it incurs no net penalty in expected terms.

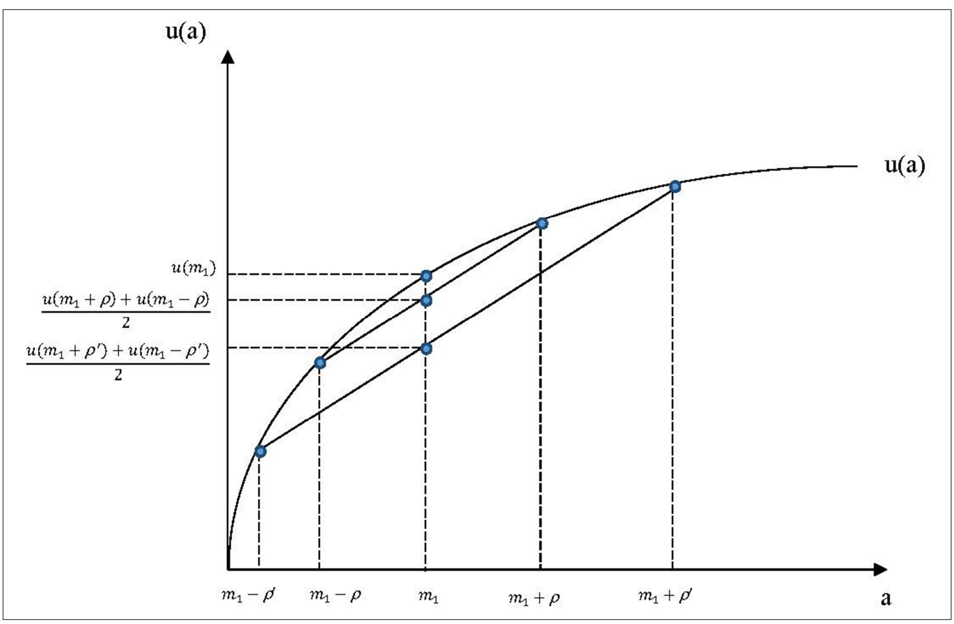
Proposition 3 *In the noisy case, there is no communicative equilibrium with the cross-checking strategy if the utility function* $u(a)$ *is linear.*

If sender 1 increases m_1, this leads to the gain due to a transition from the punishment interval to the reward interval $h(m_1+d)(u(m_1+d)-u(m_1))$ and the loss due to a transition from the reward interval to the punishment interval $h(m_1-d)(u(m_1-d)-u(m_1))$. The two conflicting effects resulting from region changes are cancelled out and thus, only the positive effect of inflating information remains. Surprisingly, introducing even a small noise would overturn the truth-revealing equilibrium with the cross-checking strategy. Even a small noise would make all messages possible in equilibrium by vanishing any off-the-equilibrium path thereby making it difficult to punish a sender who sends a high message by cross-checking strategy.

If $u''(\cdot)<0$, however, Equation (4) can have a solution for ρ (which is independent of v_1), as $\frac{u(v_1-\rho)+u(v_1+\rho)}{2}-u(v_1)<0$ due to the concavity of u and $u'(v_1)>0$. We denote the solution by ρ^*.

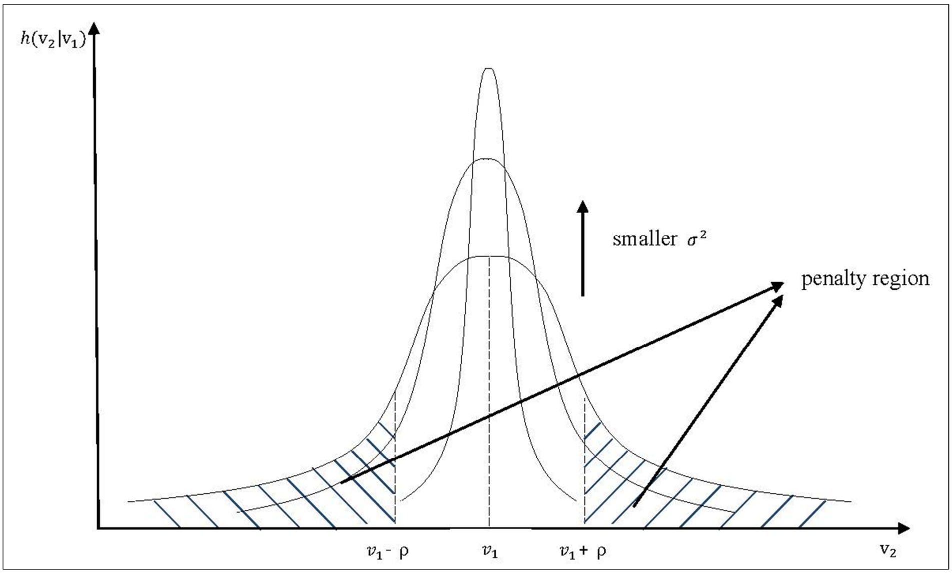
The effect of an increase in σ^2 on ρ^* is ambiguous. First, note that the RHS of Equation (4) depends on neither σ^2 nor ρ. Now, suppose σ^2 gets larger, i.e., the probability that m_1 and $m_2(=v_2)$ fall outside the non-punishment region gets higher. Then, to maintain the expected loss (LHS) equal to the gain (RHS), one must choose a larger ρ^* to recover the penalty probability to the original lower level. In other words, when the information is less accurate, the receiver must use a more lenient strategy that allows a wider confidence interval.

[Figure 1] The effect of an increase in $\rho(\rho'>\rho)$



Yet, increasing ρ also has other effects: it not only lowers the penalty probability, but also increases the net loss from the penalty strategy itself, because the expected utility from increasing m_1, $\frac{u(m_1+\rho)+u(m_1-\rho)}{2}$, decreases in ρ (See Figure 1.) If this effect dominates the former effect on the penalty probability, the expected loss due to an increase in ρ could be larger. Thus, in this case, ρ^* must be adjusted to a lower level if σ^2 is larger.

[Figure 2] The effect of a fall in σ



Now, consider the limiting case of σ^2. Given any fixed ρ, if σ^2 keeps falling, the penalty probability approaches zero, while the loss remains the same (because the loss is independent of σ^2) (See Figure 2.) Therefore, the expected net loss from inflating m converges to zero, implying that senders will have an incentive to inflate their messages and that no communicative equilibrium occurs for low σ^2.

Example

Let $u(a)=1-e^{-a}$. Note that $u'(a)=e^{-a}>0$ and $u''(a)=-e^{-a}<0$. Equation (4), which characterizes the first order condition of the incentive compatibility constraint, can be written as

$$4h(v_1-\rho)\left[(1-e^{-v_1})-\frac{1-e^{-v_1+\rho}+1-e^{-(v_1+\rho)}}{2}\right]=e^{-v_1}. \tag{5}$$

This is reduced to

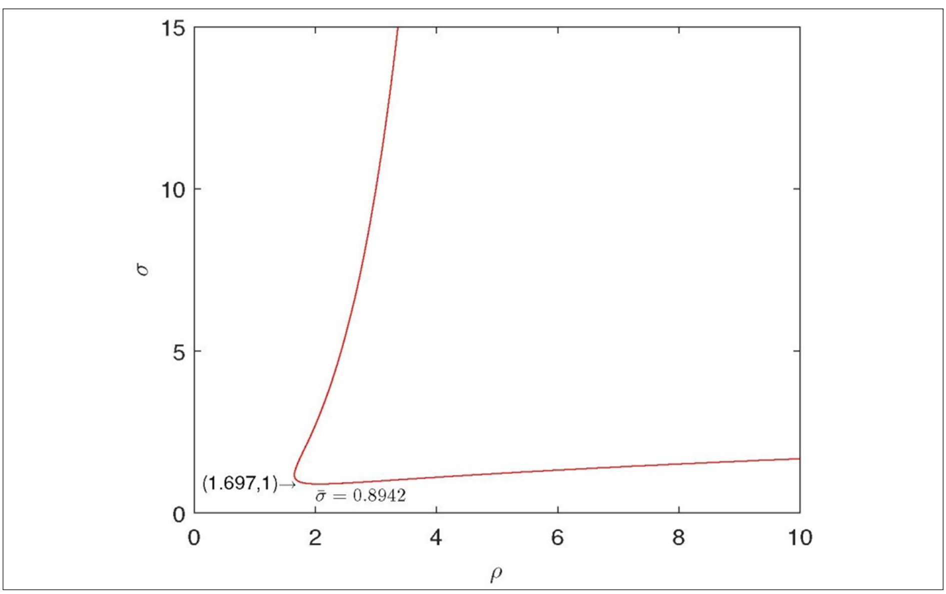
$$4h(v_1-\rho)\left(\frac{e^{\rho}+e^{-\rho}}{2}-1\right)=1, \tag{6}$$

where $\frac{e^{\rho}+e^{-\rho}}{2}\geq 1$ with equality if $\rho=0$. This determines the equilibrium confidence interval ρ. Moreover, given that $v_2(=v_1+\varepsilon_2-\varepsilon_1)$ has the distribution of $N(v_1,2\sigma^2)$, we have

$$\begin{aligned}
&H(v_1-\rho)=Prob(v_2\leq v_1-\rho)\\
&=\Phi\left(\frac{v_1-\rho-v_1}{\sqrt{2}\sigma}\right)\\
&=\Phi\left(\frac{\rho}{\sqrt{2}\sigma}\right),
\end{aligned}$$

where $\Phi(x)=\frac{1}{2}[1+erf(\frac{x}{\sqrt{2}})]$. Note that this probability does not depend on v_1, and neither does ρ^*. As shown in Figure 3, for this particular utility function, (i) there exists $\bar{\sigma}(>0)$, such that the first order condition is satisfied for some $\rho^*(\sigma)$ whenever $\sigma\geq\bar{\sigma}$, (ii) $\rho^*(\sigma)$ is increasing in σ for most of the values of σ, and (iii) there is no $\rho^*(\sigma)$ for very low values of σ. The appendix also shows that this solution satisfies the second-order condition and global optimality.

[Figure 3] Optimal ρ^* for various values of σ



The following proposition slightly generalizes this numerical example.

Proposition 4 *In the noisy case, there exists* $\bar{\sigma} > 0$*, such that for any* $\sigma \geq \bar{\sigma}$*, there exists a truth-revealing equilibrium for some* $\rho > 0$*, which is independent of* v_1 *and* v_2 *if the utility function* u *is any negative affine transformation of* e^{-a}*, i.e.,* $u(a) = \gamma - \beta e^{-a}$ *where* $\beta > 0$.

This proposition says that the utility function $u(a) = \gamma - \beta e^{-a}$ satisfies the differential equation given by (4) for some ρ, which is independent of v_i, implying that under this utility function, there is a possibility that there exists ρ^* that characterizes the cross-checking strategy and that this does not depend on v_1 and v_2. This utility function enables senders to reveal the truth by making the punishment larger than the reward when a sender inflates his information.

One drawback of this proposition, however, is that the existence of the truth-revealingequilibrium is not guaranteed if σ^2 is very low. The following proposition strengthens the result.

Proposition 5 *In the noisy case, there exists a truth-revealing equilibrium for some* $\rho > 0$*, which is independent of* v_1 *and* v_2 *if the utility function is* $u(a) = \gamma - \beta e^{-a/\sigma}$ *where* $\beta > 0$.

This proposition says that if $u(a) = \gamma - \beta e^{-a/\sigma}$, truth-telling is an equilibrium for *any* σ, i.e., no lower bound for σ exists for the truth-revealing equilibrium . The utility function reflects the reality that a sender's utility from consuming network goods is reduced by the noise of his information. Therefore, we can interpret a/σ as the effective network size, which is discounted by the noise of private information. In fact, this scaling has the effect of normalizing σ to one. This guarantees the existence of the optimal ρ^*, which turns out to be $\rho^* = 1.697\sigma$.

Is it still possible for there to be a truth-revealing equilibrium for a different form of utility function? The following proposition shows that it is not possible.

Proposition 6 *In the noisy case, if there exists a truth-telling equilibrium for very small* $\rho > 0$*, which is independent of* v_1 *and* v_2*, then the utility function must have the form of* $u(a) = \gamma - \beta e^{-ca}$*, where* $\beta > 0$ *and* $c = \frac{1}{2h(v_1 - \rho)\rho^2}$.

It says that an affine transformation of e^{-ca} for some $c > 0$, i.e., $u(a) = \gamma - \beta e^{-ca}$, is indeed a necessary condition as well as a sufficient condition for the existence of a fully revealing equilibrium. Therefore, we can conclude that it is possible to fully reveal the private information of the two senders with the cross-checking strategy only if the senders have a utility function of this form.

This result can be interpreted as a possibility theorem in the sense that truth-telling is possible in equilibrium if senders have this CARA utility function. It can also be interpreted as an impossibility theorem in the sense that truth-telling is possible, but only if senders have the CARA utility function. Considering the fact that the CARA function is a reasonable approximation to the real but unknown utility function,[17] we believe that this result is reassuring.

VI. Conclusion

We have shown that one sender can be disciplined by the presence of the other sender, so that each sender reveals its information truthfully for fear of being penalized by conveying false information. In reality, the information of the quality of a newly introduced experience good is diffused by WoM communication from existing users. This paper provides an explanation for why such WoM communication should convey reliable information on the quality of network goods.

Even though the arguments in this paper have been made within a limited context of WoM communication about the quality of an experience good, the general insight behind them can be carried over to enormous economic situations, in which multiple parties possess some information relevant to a certain decision-making. For instance, college professors may want more students of his own to be admitted to decent graduate schools, which can be thought of as network externalities. If a professor does not care about his reputation at all—and this is usually the case for a professor from abroad— he will always write the most favorable recommendation letters he can. This is the reason why most graduate schools do not believe references from foreign countries. Of course, this is classified as a kind of equilibrium (babbling equilibrium). However, apart from the reputational consideration, a professor—even a foreign professor—sometimes writes a very sincere and fair letter for fear that his student may be rejected simply because his evaluation is too different from another professor's evaluation.

Moreover, our result can be straightforwardly extended to n senders rather than two senders. With n senders, the cross-checking strategy will be of the form $\alpha^*(\mathbf{m}) = b$, where

$$b(\mathbf{m}) = \begin{cases} \max\{m_1, m_2, \cdots, m_n\} & \textit{if } \max\{|m_i - m_j| : i \neq j\} \leq \rho \\ \min\{m_1, m_2, \cdots, m_n\} & \textit{if } \max\{|m_i - m_j| : i \neq j\} > \rho, \end{cases}$$

for some $\rho > 0$, although the computations for the equilibrium value of ρ can be very complicated. This will be left to the readers.

An idea analogous with our insight, although using costly signals rather than cheap talk, has been discussed in the industrial-organization literature by Bagwell

[17] See Zuhair *et al*. (1992).

and Ramey (1991). Using a model that featured multiple incumbent firms facing a potential entrant, they demonstrated that one incumbent with unfavorable private information on the industry cost level could not pretend to be one with favorable information by deviating from its static Nash equilibrium price, as it could not coordinate its defection with the other incumbent sharing the information.

Some may suspect that our finding is not a good representation of the real world. The source of this suspicion is the assumption that the existence of the other speaker is common knowledge to both referees as well as the uninformed party. Thus, a more plausible scenario would be to assume that the number of referees is the private information of the uninformed party. This may be an interesting research agenda in future works.

Appendix

Proof of the Solution for the Example

(i) The Second-Order Condition of the Incentive Compatibility Constraint We have

$$\frac{\partial^2 U^{S_i}}{\partial m_i^2} = h'(m_i-\rho)(u(m_i-\rho)-u(m_i)) + h(m_i-\rho)(u'(m_i-\rho)-u'(m_i))$$
$$+h'(m_i+\rho)(u(m_i+\rho)-u(m_i)) + h(m_i+\rho)(u'(m_i+\rho)-u'(m_i))$$
$$+u''(m_i)\left(\int_{m_i-\rho}^{m_i} h(v_j)dv_j + \int_{m_i+\rho}^{\infty} h(v_j)dv_j\right)$$
$$+u'(m_i)(h(m_i)-(h(m_i-\rho)+h(m_i+\rho))), \tag{7}$$

and thus,

$$\left.\frac{\partial^2 U^{S_i}}{\partial m_i^2}\right|_{m_i=v_i} = h'(v_i-\rho)(u(v_i-\rho)-u(v_i+\rho))$$
$$+h(v_i-\rho)(u'(v_i+\rho)+u'(v_i-\rho)-2u'(v_i))$$
$$+u'(v_i)(-2h(v_i-\rho)+h(v_i)) + \frac{1}{2}u''(v_i). \tag{8}$$

By using $h(x) = \frac{1}{2\sqrt{\pi\sigma}} e^{-\frac{(x-v_i)^2}{4\sigma^2}}$ and $h'(x) = -\frac{x-v_i}{4\sqrt{\pi\sigma^3}} e^{-\frac{(x-v_i)^2}{4\sigma^2}}$, we obtain

$$\left.\frac{\partial^2 U^{S_i}}{\partial m_i^2}\right|_{m_i=v_i} = \frac{\rho}{4\sqrt{\pi\sigma^3}} e^{-\frac{\rho^2}{4\sigma^2}} (e^{-v_i-\rho} - e^{-v_i+\rho})$$
$$+\frac{1}{2\sqrt{\pi\sigma}} e^{-\frac{\rho^2}{4\sigma^2}} (e^{-v_i-\rho} + e^{-v_i+\rho} - 2e^{-v_i}))$$
$$+e^{-v_i}\left(-\frac{1}{\sqrt{\pi\sigma}} e^{-\frac{\rho^2}{4\sigma^2}} + \frac{1}{2\sqrt{\pi\sigma}}\right) - \frac{1}{2}e^{-v_i}$$
$$= e^{-v_i}\frac{1}{2\sqrt{\pi\sigma}}\left(\frac{\rho}{2\sigma^2}(e^{-\rho}-e^{\rho})e^{-\frac{\rho^2}{4\sigma^2}} + (e^{\rho}+e^{-\rho}-4)e^{-\frac{\rho^2}{4\sigma^2}} + 1 - \sqrt{\pi\sigma}\right)$$
$$= e^{-v_i}\frac{1}{2\sqrt{\pi\sigma}}\left(-\frac{\rho}{\sigma^2}e^{-\frac{\rho^2}{4\sigma^2}}\sinh\rho + (2\cosh\rho - 4)e^{-\frac{\rho^2}{4\sigma^2}} + 1 - \sqrt{\pi\sigma}\right). \tag{9}$$

Therefore, the second order condition requires

$$-\frac{\rho}{\sigma^2}e^{-\frac{\rho^2}{4\sigma^2}}\sinh\rho+(2\cosh\rho-4)e^{-\frac{\rho^2}{4\sigma^2}}+1-\sqrt{\pi}\sigma<0. \tag{10}$$

[Figure 4] Region where the second-order condition is satisfied

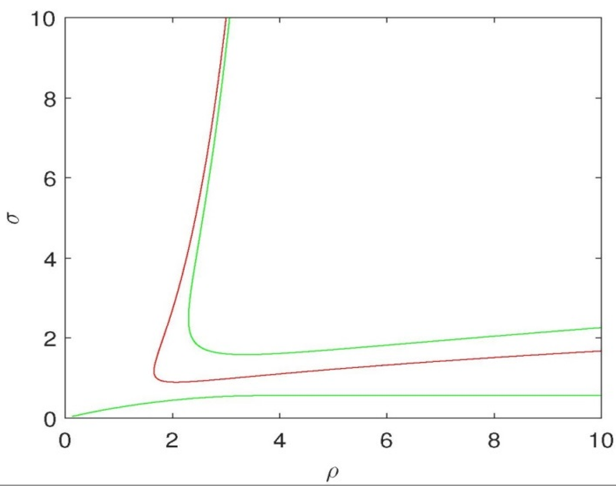


Figure 4 shows that (4) implies (10), i.e., if ρ satisfies the first-order condition, then it also satisfies the second-order condition. In Figure 4, the red curve is the region, in which the first-order condition is satisfied, and the interior of the green curve is the region, in which the second-order condition is met.

(ii) Global Optimality: As it is clear that a sender does not deviate to $m_i < v_i$, we check only the incentive to deviate to $m_i > v_i$.

Given that $u(a)=1-e^{-a}$, Equation (2) can be rearranged into

$$\frac{\partial U^{S_i}}{\partial m_i}=e^{-m_i}(1-e^{-\rho})h(m_i+\rho)-e^{-m_i}(e^{\rho}-1)h(m_i-\rho)$$
$$+e^{-m_i}\left[\int_{m_i-\rho}^{m_i}h(v_j)dv_j+\int_{m_i+\rho}^{\infty}h(v_j)dv_j\right]. \tag{11}$$

We show $\psi(m_i)\equiv(1-e^{-\rho})h(m_i+\rho)-(e^{\rho}-1)h(m_i-\rho)+\int_{m_i-\rho}^{m_i}h(v_j)dv_j+\int_{m_i+\rho}^{\infty}h(v_j)$ $dv_j<0$, $\forall m_i>v_i$.

Note that $(1-e^{-\rho})h(m_i+\rho)-(e^{\rho}-1)h(m_i-\rho)$ is decreasing in m_i when m_i $<v_i+\rho$ due to $h'(m_i-\rho)>0>h'(m_i+\rho)$. Clearly, $\int_{m_i-\rho}^{m_i}h(v_j)dv_j+\int_{m_i+\rho}^{\infty}$ $h(v_j)dv_j$ is also decreasing in m_i. Given that $\psi(v_i)=0$, it follows that $\psi(m_i)<0$, $\forall m_i\in(v_i,v_i+\rho)$, because $\frac{\partial\psi}{\partial m_i}<0$.

To check the behavior of $\frac{\partial\psi}{\partial m_i}$ when $m_i\geq v_i+\rho$, let us differentiate ψ with respect to m_i. Without loss of generality, we assume that $v_i=0$. By using

$h(x)=\frac{1}{2\sqrt{\pi\sigma}}e^{-\frac{(x-v_i)^2}{4\sigma^2}}$ and $h'(x)=-\frac{x-v_i}{4\sqrt{\pi\sigma^3}}e^{-\frac{(x-v_i)^2}{4\sigma^2}}$, we obtain

$$\frac{\partial\psi}{\partial m_i}=-(1-e^{-\rho})\frac{m_i+\rho}{4\sqrt{\pi\sigma^3}}e^{-\frac{(m_i+\rho)^2}{4\sigma^2}}+(e^{\rho}-1)\frac{m_i-\rho}{4\sqrt{\pi\sigma^3}}e^{-\frac{(m_i-\rho)^2}{4\sigma^2}}$$

$$+\frac{1}{2\sqrt{\pi\sigma}}e^{-\frac{m_i^2}{4\sigma^2}}-\frac{1}{2\sqrt{\pi\sigma}}e^{-\frac{(m_i-\rho)^2}{4\sigma^2}}-\frac{1}{2\sqrt{\pi\sigma}}e^{-\frac{(m_i+\rho)^2}{4\sigma^2}}$$

$$=\frac{1}{2\sqrt{\pi\sigma}}e^{-\frac{m_i^2}{4\sigma^2}}\left[-(1-e^{-\rho})\frac{m_i+\rho}{2\sigma^2}e^{-\frac{2\rho m_i+\rho^2}{4\sigma^2}}+(e^{\rho}-1)\frac{m_i-\rho}{2\sigma^2}e^{-\frac{-2\rho m_i+\rho^2}{4\sigma^2}}\right.$$

$$\left.+1-e^{-\frac{2\rho m_i+\rho^2}{4\sigma^2}}-e^{-\frac{-2\rho m_i+\rho^2}{4\sigma^2}}\right]$$

$$=\frac{1}{2\sqrt{\pi\sigma}}e^{-\frac{m_i^2+\rho^2}{4\sigma^2}}\left[-(1-e^{-\rho})\frac{m_i+\rho}{2\sigma^2}e^{-\frac{2\rho m_i}{4\sigma^2}}+(e^{\rho}-1)\frac{m_i-\rho}{2\sigma^2}e^{\frac{2\rho m_i}{4\sigma^2}}\right.$$

$$\left.+e^{\frac{\rho^2}{4\sigma^2}}-e^{-\frac{2\rho m_i}{4\sigma^2}}-e^{\frac{2\rho m_i}{4\sigma^2}}\right]$$

$$=\frac{1}{2\sqrt{\pi\sigma}}e^{-\frac{m_i^2+\rho^2}{4\sigma^2}}\phi(m_i), \tag{12}$$

where $\phi(m_i)\equiv-(1-e^{-\rho})\frac{m_i+\rho}{2\sigma^2}e^{-\frac{2\rho m_i}{4\sigma^2}}+(e^{\rho}-1)\frac{m_i-\rho}{2\sigma^2}e^{\frac{2\rho m_i}{4\sigma^2}}+e^{\frac{\rho^2}{4\sigma^2}}-e^{-\frac{2\rho m_i}{4\sigma^2}}-e^{\frac{2\rho m_i}{4\sigma^2}}$.

Observe that $\frac{\partial\psi}{\partial m_i}>0$ for all sufficiently large m_i and $\psi\to 0$ as $m_i\to\infty$. This leads us to conclude that $\psi(m_i)<0$, $\frac{\partial\psi}{\partial m_i}>0$ for all sufficiently large m_i. Now, if $\psi(m_i)>0$ for some $m_i\geq\rho$, there must be at least three solutions for $\frac{\partial\psi}{\partial m_i}=0$ in $(0,\infty)$. Now, ϕ can be further reduced to

$$\phi=(px+q)e^{x}-(rx+s)e^{-x}+t, \tag{13}$$

where $x=\frac{\rho m_i}{2\sigma^2}$, $p=e^{\rho}-1$, $q=-1$, $r=1$ and $t=e^{\frac{\rho^2}{4\sigma^2}}$ for $\sigma\geq\bar{\sigma}$ ($\bar{\sigma}$ is determined in Proposition 4).

Claim 1 *There exists ρ, such that (ρ,σ) for $\sigma\geq\bar{\sigma}$ satisfies the first-order condition and $\phi(x)=0$ has at most two solutions, such that $x>0$.*

This is reduced to show that $(px+p+q)e^{2x}=-rx+r-s$ has, at most, one zero such that $x>0$. In the case that $p+q\geq 0$, $(px+p+q)e^{2x}$ is increasing for $x>0$ and $-rx+r-s$ is decreasing for $x>0$. Hence, it has, at most, one zero such that $x>0$. Now, we can assume that $p+q<0$. Note that $p+q=e^{\rho}(r-s)$ and $p=e^{\rho}r$. Using this, we have

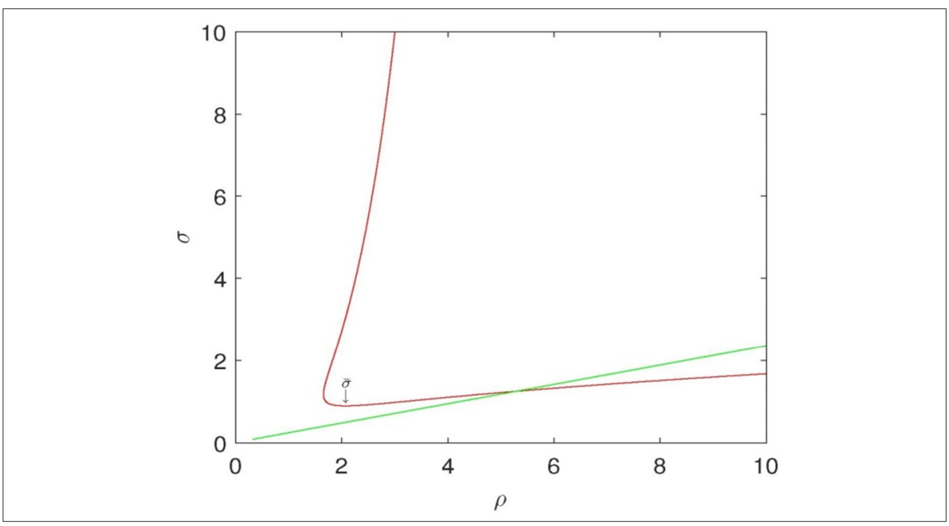
$$
\begin{aligned}
&(px+p+q)e^{2x}=-rx+r-s\\
&\Leftrightarrow (px+p+q)e^{2x}-rx-r+s=-2rx\\
&\Leftrightarrow (px+p+q)(e^{2x}-e^{-\rho})=-2rx\,.
\end{aligned}
$$

The y-intercept of $(px+p+q)(e^{2x}-e^{-\rho})$ is $(p+q)(1-e^{-\rho})$, which is negative. Here, $-2rx$ is decreasing for $x>0$ and passes $(0,0)$. Thus, it is enough to show that $(px+p+q)(e^{2x}-e^{-\rho})$ is convex for $x>0$, i.e., its second derivative is positive. (Then $(px+p+q)(e^{2x}-e^{-\rho})=-2rx$ would have only one solution.) Defining $\psi(x)=(px+p+q)(e^{2x}-e^{-\rho})$, then we have

$$
\begin{aligned}
&\psi'=p(e^{2x}-e^{-\rho})+2e^{2x}(px+p+q),\\
&\psi''=2pe^{2x}+2pe^{2x}+4e^{2x}(px+p+q)=8pe^{2x}+p+q\geq 9p+q\,,
\end{aligned}
$$

whenever $x>0$.

[Figure 5] Region where $9\frac{e^{\rho}-1}{\rho}-\frac{\rho(e^{\rho}-1)}{\rho}-1>0$ is satisfied



Thus, it is enough to show that $9p+q>0$ i.e. $9\frac{e^{\rho}-1}{\rho}-\frac{\rho(e^{\rho}-1)}{2\sigma^2}-1>0$. Note that the upper region of the green curve in Figure 5 satisfies $9p+q>0$. This completes the proof.

[Figure 6] Global optimality of $m_i = v_i$

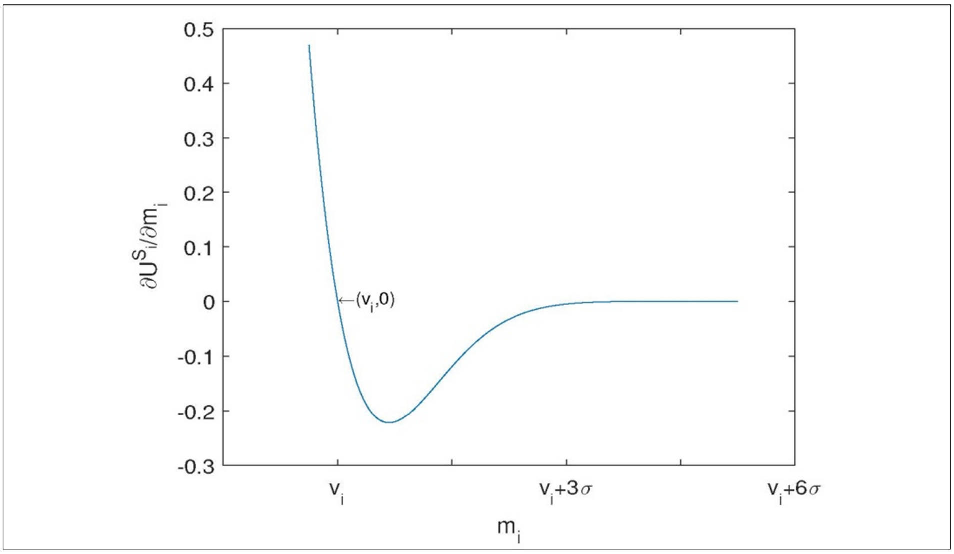


Figure 6 shows the global optimality of $m_i = v_i$, i.e., $\frac{\partial U^{S_i}}{\partial m_i} < 0$ for all $m_i > v_i$.

Proof of Proposition 4

We can rewrite $u(a)$ as follows:

$$u(a) = \gamma - \beta e^{-a} = \gamma\left(1 - \frac{\beta}{\gamma}e^{-a}\right) = \gamma(1 - e^{-(a - \ln\beta/\gamma)}) . \tag{14}$$

The graph of $u(a)$ is obtained simply by scaling the vertical axis and transition of the a-axis. This does not change the first-order condition and second-order condition given by Equations (6) and (10), respectively.

Proof of Proposition 5

Let $X = x/\sigma$. Given that $Var(X) = Var(x/\sigma) = 1$, the proof is immediate.

Proof of Proposition 6

By Taylor expansion, we have

$$u(v_1 - \rho) = u(v_1) - u'(v_1)\rho + u''(v_1)\frac{\rho^2}{2} - u'''(v_1)\frac{\rho^3}{6} + O(\rho^4),$$
$$u(v_1 + \rho) = u(v_1) + u'(v_1)\rho + u''(v_1)\frac{\rho^2}{2} + u'''(v_1)\frac{\rho^3}{6} + O(\rho^4).$$

Therefore, for very small $\rho > 0$, the first-order condition given by Equation (4) can be rewritten as

$$u''(v_1) = -\frac{1}{2h(v_1 - \rho)\rho^2} u'(v_1). \tag{15}$$

Note that the solution for the differential equation given by Equation (15) must be of the form $u(x) = \gamma - \beta e^{-cx}$ where $c = \frac{1}{2h(v_1 - \rho)\rho^2}$. Moreover, $u' > 0$ implies that $\beta > 0$.

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