

A Positive Study on Wage Level and Marginal Labor Productivity

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I. Introduction

Since the wage means an income for the laborer, the increase or decrease in wage is not only directly connected with the individual existence or living standard of laborers, but also connected with the economic development of a nation, because the wage is, if it is viewed from the point of national economy, the source of consumption, demand, saving or investment. On the other hand, for the men of enterprise, the wage constitutes a part of production cost, and the increase or decrease in wage forms a factor of changes in prices. Furthermore, the increase or decrease in wage is related to the economic stability or growth. Therefore, the propriety of the wage level has a great influence on the economical and social aspects.

The most important task for the economy of our country is to realize the economic stability, and as a counter-measure therefore the establishment of a proper wage level is very important.

In the theory concerning the decision of wages as is generally accepted in the modern economics, there is the so-called theory of marginal productivity. According to this theory, the wage is decided by the marginal productivity of labour. Of

course, this theory of marginal productivity is not only applicable to the wages, but also applicable to the decision of prices of producers, goods in general. Therefore, if we were able to actually measure the marginal productivity of such producers, goods as labor or capital, we may be able to obtain an objective standard for deciding a proper level of wages or profit. Many scholars paid their attentions to this fact, and studied this matter in various ways. Among them, the most famous one is P.H. Dauglas' production function. By utilizing the statistics of U.S.A., Canada and Australia, he actually measured the productivity of labor and capital by means of macro-approach. The writer attempted to actually measure the marginal productivity of labor in our country by availing of Dauglas' production function. It goes without saying that there are not so many statistical data available, and the knowledge of the writer in this field is not enough, and there may be many mistakes which the writer does not realize. Therefore, the writer does results of research by the writer, and to make a bit of contribution towards the study of this field by my fellow researchers.

II. Production Function

Goods or services are produced by the combination of labor and capital, However, with regard to the relation between labor and capital which is a means of production in the method of production, there have been many views. In general, according to the modern economic theory which is based on the theory of marginal productivity, it is supposed that there is a considerably elastic substitutive relation between labor necessary for producing a certain amount of output and capital. In other words, when the rate of wages is fallen, there is a tendency that the capital is substituted by labor.

However, as for individual enterprise, it may be a more realistic view that there is a fixed combined relation between labor and capital. In other words, in order to produce a certain amount of output, it is considered that a certain combined relation between the corresponding labor and capital is uniformly decided from the technical point of view. In fact, Leontief positively analyzed the reality in this way. Therefore, in this case, there is not any substitutive relation between labor and capital, even though the wage level or the rate of interest is changed. In general, such a method of production or production function is called "limitational production function". However, this limitational relation which is viewed from the point of individual enterprise or individual industry cannot be said that it is proper when we observe it as a whole to include different kinds of enterprise or different kinds

of industry, the production function of which is different.

When it is viewed as a whole, the combined relation between labor and capital in each enterprise or industry is constant, and according to the relative change in the rate of wages or interest, i.e. in case an industry which relatively requires a great amount of capital is extended, or in case an industry which relatively requires a great number of laborers is extended, it is possible that the enterprises of different kind or the industries of different kind may be substituted each other. Therefore, if the existence of limitational production function for each enterprise or industry is recognised, the substitutive relation between capital and labor as a whole can be enough secured in case many enterprises or industries which have different rate of combination between capital and labor, i.e. the degree of organic composition of capital, are observed as a whole.

According to the traditional view of econometrics, the production function is presumed as follows:

$$Y=f(L, K)=AL^{\alpha}K^{\beta} \dots\dots\dots (1)$$

In the expression (1), Y is the value of output, L is the amount of employed labor, K is the stock amount of capital composed of various items such as circulating capital and fixed capital, etc. It is promised here that Y and K are measured by the unit of P , price level, and L is measured by the unit of w , rate of wages. The production function such as the expression (1) is called P.H. Douglas' production function, and the expression is generally recognised by means of a positive experience as a comparatively proper one.

Now, if the value of output (Y) is differentiated by labor (L) and capital (K), the following equation can be obtained:

$$\frac{\partial Y}{\partial L} = \alpha \cdot \frac{Y}{L} \quad \therefore \alpha = \frac{\partial Y}{\partial L} \div \frac{Y}{L} \dots\dots\dots (2)$$

$$\frac{\partial Y}{\partial K} = \beta \cdot \frac{Y}{K} \quad \therefore \beta = \frac{\partial Y}{\partial K} \div \frac{Y}{K} \dots\dots\dots (3)$$

is, when capital (K) is constant, the relative effect to the value of output (Y), i.e. the production elasticity to the value of output of labor in case the labor (L) alone increases; and β is the production elasticity to the value of output of capital.

In case the total of production elasticity of labor (α) and production elasticity of capital (β) becomes 1, i.e. $\alpha + \beta = 1$, we say that the production function of the expression (1) is constant returns to the production scale; and in case $\alpha + \beta > 1$, increasing returns; and in case $\alpha + \beta < 1$, the law of decreasing returns is applied. For example, $\alpha + \beta = 1$ means that, when labor and capital are doubled

respectively, Y is also doubled.

In general, since the law of decreasing returns is overwhelming in agriculture, the law of decreasing returns, i.e. $\alpha + \beta < 1$, is realized in the production function as shown in the expression (1). On the contrary, in manufacturing industry which is comparatively free from the restriction of land, the law of constant returns is almost overwhelming.

III. Calculation Method of Production Function

Then, we must consider how Dauglas' production function, i.e. $Y = AL^\alpha K^\beta$ is calculated. If a system which is completed with an equation can be called a single equation system, the presumption of the structure of a single equation system, parameter, can be done by applying the classical minimum multiplication. The presumptive calculation method of the structure of production function, parameter, A , α and β is as follows:

$$Y = AL^\alpha K^\beta \dots \dots \dots (1)$$

If the expression (1) is made into logarithm,

$$\log Y = \log A + \alpha \log L + \beta \log K \dots \dots \dots (2)$$

Thus, the structure of production function, parameter, A , α and β may be solved through the expression (2). Now, we will consider the method by which the parameter, a_{10} , a_{12} , a_{13} , $\dots \dots a_{1k}$ of the following equation of

$$x_1 = a_{10} + a_{12}x_2 + a_{13}x_3 + \dots \dots a_{1k}x_k \dots \dots \dots (3)$$

can be presumed by the minimum multiplication. In this case the available data is the following data of time series:

| t | 1 | 2 | 3 | $\dots \dots \dots n$ |
|----------|----------|----------|----------|-----------------------|
| x_1 | x_{11} | x_{12} | x_{13} | $\dots \dots x_{1n}$ |
| x_2 | x_{21} | x_{22} | x_{23} | $\dots \dots x_{2n}$ |
| x_3 | x_{31} | x_{32} | x_{33} | $\dots \dots x_{3n}$ |
| \vdots | \vdots | \vdots | \vdots | \vdots |
| x_k | x_{k1} | x_{k2} | x_{k3} | $\dots \dots x_{kn}$ |

These data of time series are plotted by n dots, $P_1, P_2, P_3, \dots \dots P_n$ in k -dimensional space. (The dot, P_i is expressed by beta $(x_{1i}, x_{2i}, x_{3i}, \dots \dots x_{ki})$, and represents the data of i point of time). If the presumption of the parameter of the expression (3) by the minimum multiplication is explained geometrically, it means that a plain of

$$x'_1 = a_{10} + a_{12}x_2 + a_{13}x_3 + \dots \dots + a_{1k}x_k \dots \dots \dots (4)$$

which minimizes the total of the double times of distances from each dot to n

dots in the k -dimensional space which is obtained by such means as mentioned above is made curve fitting. Such a plane is called Regression Plane or Regression Equation. In order to make such regression equation curve fitting, Q which can be defined as

$$Q = \sum_{t=1}^n u_t^2 = \sum_{t=1}^n (x_{1t} - x'_{1t})^2 = \sum_{t=1}^n (x_{1t} - a_{10} - a_{12}x_{2t} - \cdots - a_{1k}x_{kt})^2 \cdots \cdots (5)$$

must be minimized. In order to make it so,

$a_{10}, a_{12}, a_{13}, \cdots, a_{1k}$ which satisfy

$$\begin{aligned} -\frac{1}{2} \frac{Q}{a_{12}} &= \sum_{t=1}^n (x_{1t} - a_{10} - a_{12}x_{2t} - \cdots - a_{1k}x_{kt}) = 0 \\ -\frac{1}{2} \frac{Q}{a_{12}} &= \sum_{t=1}^n (x_{1t} - a_{10} - a_{12}x_{2t} - \cdots - a_{1k}x_{kt})x_{2t} = 0 \\ -\frac{1}{2} \frac{Q}{a_{13}} &= \sum_{t=1}^n (x_{1t} - a_{10} - a_{12}x_{2t} - \cdots - a_{1k}x_{kt})x_{3t} = 0 \\ &\cdots \cdots \cdots \\ -\frac{1}{2} \frac{Q}{a_{1k}} &= \sum_{t=1}^n (x_{1t} - a_{10} - a_{12}x_{2t} - \cdots - a_{1k}x_{kt})x_{kt} = 0 \end{aligned} \quad \cdots (6)$$

should be computed. Therefore, the following normal equation should be solved:

$$\begin{aligned} a_{10}n + a_{12} \sum_{t=1}^n x_{2t} + \cdots + a_{1k} \sum_{t=1}^n x_{kt} &= \sum_{t=1}^n x_{1t} \\ a_{10} \sum_{t=1}^n x_{2t} + a_{12} \sum_{t=1}^n x_{2t}^2 + \cdots + a_{1k} \sum_{t=1}^n x_{2t}x_{kt} &= \sum_{t=1}^n x_{1t}x_{2t} \\ a_{10} \sum_{t=1}^n x_{3t} + a_{12} \sum_{t=1}^n x_{2t}x_{3t} + \cdots + a_{1k} \sum_{t=1}^n x_{3t}x_{kt} &= \sum_{t=1}^n x_{1t}x_{3t} \\ &\cdots \cdots \cdots \\ a_{10} \sum_{t=1}^n x_{kt} + a_{12} \sum_{t=1}^n x_{2t}x_{kt} + \cdots + a_{1k} \sum_{t=1}^n x_{kt}^2 &= \sum_{t=1}^n x_{1t}x_{kt} \end{aligned} \quad \cdots (7)$$

In other words, $a'_{10}, a'_{12}, a'_{13}, \cdots, a'_{1k}$ which can be obtained by solving the normal equation of (7) are the presumption value of the minimum multiplication of the parameter of the equation (4), $a_{10}, a_{12}, a_{13}, \cdots, a_{1k}$. From the normal equation of (7), we can obtain

$$\begin{aligned} a_{10} &= -\frac{\sum x_1}{n} - \frac{\sum x_2}{n} a_{12} - \frac{\sum x_3}{n} a_{13} - \cdots - \frac{\sum x_k}{n} a_{1k} \\ &= \bar{x}_1 - a_{12}\bar{x}_2 - a_{13}\bar{x}_3 - \cdots - a_{1k}\bar{x}_k \end{aligned}$$

If the above is inserted into No. 1 equation of (7)

$$\begin{aligned} (\bar{x}_1 - a_{12}\bar{x}_2 - a_{13}\bar{x}_3 - \cdots - a_{1k}\bar{x}_k) \sum x_1 + a_{12} \sum x_2 \\ x_1 + \cdots + a_{1k} \sum x_1 \sum x_k = \sum x_1 x_k \end{aligned}$$

If the above is a ranged,

$$a_{12}(\sum x_2 x_1 - x_2 \sum x_1) + \dots + a_{1k}(\sum x_1 x_k - \bar{x}_k \sum x_1) = \sum x_1 x_1 - \bar{x}_1 \sum x_1$$

Therefore,

$$a_{12} \sum x_2 x_1 + \dots + a_{1k} \sum x_1 x_k = \sum x_1 x_1$$

Thus, we can obtain the following expression:

$$\left. \begin{aligned} & a_{12} \sum_{t=1}^n x_{2t}^2 + a_{13} \sum_{t=1}^n x_{2t} x_{3t} + \dots + a_{1k} \sum_{t=1}^n x_{2t} x_{kt} = \sum_{t=1}^n x_{1t} x_{2t} \\ & a_{12} \sum_{t=1}^n x_{2t} x_{3t} + a_{13} \sum_{t=1}^n x_{3t}^2 + \dots + a_{1k} \sum_{t=1}^n x_{3t} x_{kt} = \sum_{t=1}^n x_{1t} x_{3t} \\ & \dots \\ & a_{12} \sum_{t=1}^n x_{2t} x_{kt} + a_{13} \sum_{t=1}^n x_{3t} x_{kt} + \dots + a_{1k} \sum_{t=1}^n x_{kt}^2 = \sum_{t=1}^n x_{1t} x_{kt} \\ & a_{10} = \bar{x} - a_{12} \bar{x}_2 - a_{13} \bar{x}_3 - \dots - a_{1k} \bar{x}_k \end{aligned} \right\} \dots (8)$$

However,

$$\begin{aligned} x_{1i} &= \frac{\sum_{t=1}^n x_{it}}{n} \quad (i=1 \dots k) \\ \sum_{t=1}^n x_{pt} x_{qt} &= \sum_{t=1}^n (x_{pt} - \bar{x}_{pt})(x_{qt} - \bar{x}_{qt}) = \sum_{t=1}^n x_{pt} x_{qt} - n \bar{x}_p \bar{x}_q \\ & \quad (p=1 \dots k, \quad q=1 \dots k) \end{aligned}$$

\bar{x}_i expresses the average value x_i , and $\sum_{t=1}^n x_{pt} x_{qt}$ is common dispersion of x_p and x_q , and n times of $M^0_{pq} = \frac{1}{n} \sum x_{pi} x_{qi}$

If we use the procession we can express the expression of (8) in the form of more compact expression. Thus,

$$\left. \begin{aligned} & n \begin{pmatrix} M^0_{22} & M^0_{23} & \dots & M^0_{2k} \\ M^0_{32} & M^0_{33} & \dots & M^0_{3k} \\ \dots & \dots & \dots & \dots \\ M^0_{k2} & M^0_{k3} & \dots & M^0_{kk} \end{pmatrix} \begin{pmatrix} a_{12} \\ a_{13} \\ \vdots \\ a_{1k} \end{pmatrix} = n \begin{pmatrix} M^0_{12} \\ M^0_{13} \\ \vdots \\ M^0_{1k} \end{pmatrix} \\ & a_{10} = \bar{x}_1 - a_{12} \bar{x}_2 - a_{13} \bar{x}_3 - \dots - a_{1k} \bar{x}_k \end{aligned} \right\} \dots (9)$$

However,

$$M^0_{pq} = \frac{1}{n} \sum_{t=1}^n x_{pt} x_{qt} \quad (p=1 \dots k, \quad q=1 \dots k)$$

The presumption value of $a_{12}, a_{13}, \dots, a_{1k}, a'_{12}, a'_{13}$ can be obtained by solving k simultaneous equations in the expression (8). However, if they are expressed by using the procession, they become as follows:

If the expression (9) is solved,

$$\begin{bmatrix} a'_{12} \\ a'_{13} \\ \vdots \\ a'_{1k} \end{bmatrix} = \begin{bmatrix} M^0_{22} & M^0_{23} & \cdots & M^0_{2k} \\ M^0_{32} & M^0_{33} & \cdots & M^0_{3k} \\ \vdots & \vdots & \ddots & \vdots \\ M^0_{k2} & M^0_{k3} & \cdots & M^0_{kk} \end{bmatrix} \begin{bmatrix} -1 \\ M^0_{12} \\ M^0_{13} \\ \vdots \\ M^0_{1k} \end{bmatrix} \quad (10)$$

IV. Calculation of Production Function in the Manufacturing Industry of our Country

It is a well-known fact that the study of econometrics is very difficult in our country due to the want of statistical data. Therefore, in calculating the production function, the want of statistical data also causes a great difficulty. With regard to the value of output in the manufacturing industry, the data were obtained from the national income prepared by the Bank of Korea. (Refer to Table 2) With regard to the number of employees, the data were obtained from the industrial census data and the report on the sample research of mining and manufacturing industry prepared by the Industrial Bank of Korea. (Refer to Table 1) Since the data concerning capital were not available, and moreover, they were not reliable even though they could be obtained, the stock amount of capital of manufacturing industry was substituted with the amount of electricity by industry prepared by Korea Electric Power Corp. Although such a method is unavoidable in the under-developed countries in producing statistical data, the method is also employed in Japan, and furthermore, it has an advantage to deduct the value of idle capital. In using the data of time series, it is better to use more data of time series.

However, owing to the lack of statistical data concerning number of employees, it was inevitable to restrict the period from 1958 to 1962. Moreover, during the period of 1959 to 1961, there was no material available, and the period was covered by inevitable presuming the data basing on the data of 1958, 1960 and 1962. In doing so, the number of employees in each month was multiplied by the average number of days of employees engaging in manufacturing industry in each month of 1962, which was prepared by the Bank of Korea, to obtain the amount of labor invested. Through such a method as mentioned above, the following result was obtained. Of course, it goes without saying that such a method is a way of preparing data which is lacking in accuracy, but this was inevitable.

The result of production function in the manufacturing industry of our country, which was presumed in such a way as mentioned above, is as follows:

$$Y=0.067289 \quad L^{0.57195} \quad K^{0.42329}$$

Table 1

| 1958 | No. of (1) employees | No. of non-pay employees (2) | (1) + (2) | No. of work days per month (3) | (1) + (2) 2(3) |
|------|-------------------------|---------------------------------|-----------|-----------------------------------|-------------------|
| 1 | 199,765 | 17,128 | 216,893 | 35.3 | |
| 2 | 201,783 | 17,128 | 218,911 | 23.8 | |
| 3 | 214,663 | 17,128 | 231,791 | 23.7 | |
| 4 | 216,831 | 17,128 | 233,959 | 25.3 | |
| 5 | 218,672 | 17,128 | 235,800 | 25.4 | |
| 6 | 215,913 | 17,128 | 233,041 | 25.0 | |
| 7 | 215,886 | 17,128 | 233,014 | 25.0 | |
| 8 | 218,326 | 17,128 | 235,454 | 25.1 | |
| 9 | 227,276 | 17,128 | 244,404 | 25.5 | |
| 10 | 232,182 | 17,128 | 249,310 | 24.9 | |
| 11 | 228,137 | 17,128 | 245,265 | 26.2 | |
| 12 | 225,141 | 17,128 | 242,269 | 25.9 | |

$$\Sigma=70,801,506$$

Note: Figures in Jan., Feb., and Mar. are the adjusted figures (based on the data of 1960).

| 1958 | No. of (1) employees | No. of non-pay employees (2) | (1) + (2) | No. of work days per month (3) | (1) + (2) × (3) |
|------|-------------------------|---------------------------------|-----------|-----------------------------------|--------------------|
| 1 | 218,401 | 25,682 | 244,088 | 25.0 | |
| 2 | 221,329 | 25,682 | 247,011 | 24.8 | |
| 3 | 236,848 | 25,682 | 262,530 | 25.3 | |
| 4 | 240,240 | 25,682 | 265,922 | 25.8 | |
| 5 | 236,145 | 25,682 | 264,484 | 25.8 | |
| 6 | 231,743 | 25,682 | 257,425 | 25.5 | |
| 7 | 234,211 | 25,682 | 259,883 | 25.5 | |
| 8 | 238,802 | 25,682 | 264,484 | 25.8 | |
| 9 | 250,506 | 25,682 | 276,190 | 26.0 | |
| 10 | 255,100 | 25,682 | 280,782 | 25.2 | |
| 11 | 251,768 | 25,682 | 277,254 | 26.1 | |
| 12 | 249,572 | 25,682 | 265,254 | 25.9 | |

$$\Sigma=81,076,108$$

| 1962 | (1)'=(1)+(2) | (3) | (1)'×(3) |
|------|--------------|------|----------|
| 1 | 263,412 | 25.6 | |
| 2 | 261,149 | 24.2 | |
| 3 | 274,921 | 24.2 | |
| 4 | 281,832 | 24.7 | |
| 5 | 288,508 | 25.0 | |
| 6 | 288,110 | 24.4 | |
| 7 | 287,217 | 24.7 | |
| 8 | 291,254 | 25.8 | |
| 9 | 300,353 | 24.7 | |
| 10 | 305,570 | 25.1 | |
| 11 | 303,507 | 25.1 | |
| 12 | 304,535 | 25.3 | |

 $\Sigma=85,939,928$

Table 2

| | (Y) Value of output | (L) No. of employees | (K) Used amount of electricity | log Y | log L | log K |
|------|---------------------|----------------------|--------------------------------|---------|---------|----------|
| 1958 | 12.24 | 70.80 | 356 | 1.08778 | 1.85003 | 2.59988 |
| 1959 | 13.45 | 75.94 | 456 | 1.12874 | 1.88047 | 2.65896 |
| 1960 | 14.32 | 81.08 | 468 | 1.15534 | 1.90945 | 2.67029 |
| 1961 | 14.35 | 83.51 | 484 | 1.15685 | 1.92174 | 2.68485 |
| 1962 | 16.99 | 85.94 | 670 | 1.23019 | 1.93420 | 2.82607 |
| | 71.35 | 357.27 | | 5.75948 | 9.45589 | 13.44005 |
| | 14.27 | 79.454 | | 1.15190 | 1.89918 | 2.68801 |

Note: Value of output: One billion won

Number of employees: One million persons

Used amount of electricity: One million KWH

Table 3

| | (log Y—log \bar{Y}) | (log Y—log \bar{Y}) |
|------|------------------------|------------------------|
| 1958 | -0.06412 | 0.004111 |
| 1959 | -0.02316 | 0.000536 |
| 1960 | 0.00404 | 0.000016 |
| 1961 | 0.00495 | 0.000025 |
| 1962 | 0.07829 | 0.006129 |

 $\Sigma=-0.010818$

| | $(\log L - \log L)$ | $(\log L - \log L)^2$ |
|------|---|---|
| 1958 | -0.04915 | 0.002416 |
| 1959 | -0.01871 | 0.000350 |
| 1960 | 0.01871 | 0.000105 |
| 1961 | 0.02256 | 0.000509 |
| 1962 | 0.03502 | 0.001226 |
| | | $\Sigma = 0.004606$ |
| | $(\log K - \log K)$ | $(\log K - \log K)^2$ |
| 1958 | -0.08813 | 0.007767 |
| 1959 | -0.02905 | 0.000844 |
| 1960 | -0.01772 | 0.000314 |
| 1961 | -0.09316 | 0.000010 |
| 1962 | 0.13806 | 0.019061 |
| | | $\Sigma = 0.027996$ |
| | $\frac{(\log Y - \log Y)}{(\log L - \log L)}$ | $\frac{(\log Y - \log X)}{(\log K - \log K)}$ |
| 1958 | 0.003051 | 0.005651 |
| 1959 | 0.000433 | 0.000673 |
| 1960 | 0.000041 | -0.000072 |
| 1961 | 0.000112 | -0.000016 |
| 1962 | 0.002742 | 0.010809 |
| | | $\Sigma = 0.006479$ |
| | | $\Sigma = 0.017045$ |
| | $(\log L - \log L)$ | $(\log K - \log K)$ |
| 1958 | | 0.004332 |
| 1959 | | 0.000139 |
| 1960 | | -0.000182 |
| 1961 | | -0.000071 |
| 1962 | | 0.004835 |

$$\Sigma = 0.009053$$

$$n \cdot M^0_{11} = (\log \bar{Y} - \log \bar{Y})^2 = 0.010817$$

$$n \cdot M^0_{12} = (\log \bar{Y} - \log \bar{Y})(\log L - \log L) = 0.006479$$

$$n \cdot M^0_{13} = (\log \bar{Y} - \log \bar{Y})(\log K - \log \bar{K}) = 0.017045$$

$$n \cdot M^0_{22} = (\log L - \log \bar{L})^2 = 0.004606$$

$$n \cdot M^0_{23} = (\log L - \log \bar{L})(\log K - \log \bar{K}) = 0.009083$$

$$n \cdot M^0_{33} = (\log K - \log \bar{K})^2 = 0.027996$$

$$\begin{aligned}
 n \begin{pmatrix} M_{22}^0 & M_{23}^0 \\ M_{32}^0 & M_{33}^0 \end{pmatrix} &= \begin{pmatrix} 0.004606 & 0.009083 \\ 0.009083 & 0.027996 \end{pmatrix} \\
 n \begin{pmatrix} M_{12}^0 \\ M_{13}^0 \end{pmatrix} &= \begin{pmatrix} 0.006479 \\ 0.017045 \end{pmatrix} \\
 \begin{vmatrix} \alpha' \\ \beta' \end{vmatrix} &= \begin{pmatrix} M_{22}^0 & M_{23}^0 - 1 \\ M_{32}^0 & M_{33}^0 \end{pmatrix} \begin{pmatrix} M_{12}^0 \\ M_{13}^0 \end{pmatrix} \\
 &= \begin{pmatrix} 0.571950483 \\ 0.423273816 \end{pmatrix}
 \end{aligned}$$

$$\therefore \alpha' = 0.57195$$

$$\beta' = 0.42327$$

$$\log A = \log \bar{Y} - \alpha \log \bar{Y} - \beta \log \bar{K}$$

$$= 1.15190 - (0.57195 \times 1.89918) - (0.42327 \times 2.68801)$$

$$= -1.17209$$

$$A = 0.067289$$

$$Y = 0.067289 \quad L^{0.57195} \quad K^{0.42327}$$

V. Conclusion

With regard to the problem as to whether the production function in the manufacturing industry of our country as was obtained in Paragraph 4, i.e. $Y = 0.067289 L^{0.57195} K^{0.42327}$, is reasonable as a realistic expression, nothing can be said about it with a certainty without further study.

However, the fact that the production function of India in 1952 was presumed as $Y = 0.68 L^{0.53} K^{0.50}$ shows that the above calculation is not so erroneous. In other words, the above mentioned production function of India was calculated on the basis of 320 firms in India, and since there are many common points in developing countries, the production elasticity of labor of our country, 0.57, and that of India, 0.53, are similar. Hence the presumption made in this article is, it may be said, not a complete failure.

Moreover professor P.H. Dauglas presumed, in the time series from 1899 to 1922, the production function in the manufacturing industry of U.S.A. as $Y = 1.35 L^{0.63} K^{0.30}$, and the standard error of production elasticity of labor and production elasticity of capital was presumed as α is 0.15 and β is 0.05. Furthermore, he presumed in his study on South America $Y = 55.25 L^{0.65} K^{0.31}$.

According to the examples, the marginal productivity of labor of our country, in the production function, was sought to obtain the following result:

Table 4

(Unit: won)

| | Average value of the marginal productivity of labor per month | | Average amount of wages per month |
|------|---|-------------|-----------------------------------|
| | Price in 1955 | Fixed Price | |
| 1958 | 2, 576. 27 | 3, 637. 94 | 2, 170 |
| 1959 | 2, 629. 53 | 3, 986. 52 | 2, 350 |
| 1960 | 2, 619. 36 | 4, 300. 00 | 2, 300 |
| 1961 | 2, 547. 46 | 4, 870. 74 | 2, 830 |
| 1962 | 2, 557. 73 | 5, 635. 37 | 2, 990 |

Note: Average amount of wages per month was cited from the monthly amount of wages of the employees engaging in the manufacturing industry in the Monthly Statistical Review of the Bank of Korea.

$$\frac{\partial Y}{\partial L} = \alpha \frac{Y}{L}$$

$$= 0.57195 \times \frac{14.21}{79.454}$$

$$= 0.10272263 \text{ (Unit: one billion won)}$$

$$\therefore \text{Daily marginal productivity per person} = 102.72 \text{ won}$$

$$102.72 \times 25.1 \text{ (Average work days per month)} = 2,578.27 \text{ won}$$

$$102.72 \times 25.6 \text{ (")} = 2,629.62 \text{ won}$$

$$102.72 \times 25.5 \text{ (")} = 2,619.36 \text{ won}$$

$$102.72 \times 24.8 \text{ (")} = 2,547.46 \text{ won}$$

$$102.72 \times 24.9 \text{ (")} = 2,557.73 \text{ won}$$

$$2,578.27 \text{ won} \times 1.411 \text{ (Price index except grain)} = 3,637.94 \text{ won}$$

$$2,629.62 \text{ won} \times 1.516 \text{ (")} = 3,986.52 \text{ won}$$

$$2,619.36 \text{ won} \times 1.642 \text{ (")} = 4,302.99 \text{ won}$$

$$2,547.46 \text{ won} \times 1.912 \text{ (")} = 4,870.74 \text{ won}$$

$$2,557.73 \text{ won} \times 2.215 \text{ (")} = 5,665.37 \text{ won}$$

Since the notion of marginal productivity of labor is the differential of production function to labor amount,

$$\begin{aligned} \text{marginal productivity of labor} &= \frac{\partial Y}{\partial L} \\ &= \alpha \cdot \frac{\partial Y}{\partial L} \end{aligned}$$

In the above table, assuming that $\alpha = 0.57195$ is constant, the marginal productivity per month of each year was sought. In converting it into a fixed price, it was deflated into the price index of whole commodities except grain as was published by the Bank of Korea,

However, what should be noted here is that the fixed price and the monthly amount of wages cannot be compared each other. It is because both the data published by the Bank of Korea and the data presumed in this article are not so reliable, and furthermore, even though the data are reliable enough, those of the Bank of Korea are related to the amount of wages of employees engaging in manufacturing industry, while those which are presumed in this article are related not only to the number of employees engaging in manufacturing industry, but also to the number of employees of other than the manufacturing industry, (Of course, this is derived from the fact that the data were limited). Is it an excess ambition of the writer to grasp the tendency of general level as a whole despite of such fact as mentioned above?

Anyway, in the above list, we see that (1) the productivity of labor is very low, (2) wage level of our country is considerably low, compared with the marginal productivity of labor. (Since the productivity of labor is five years on an average, we should be careful for comparison) Such a fact is, even though no statistical data are referred, generally recognized as pointed out by prof. Nurkse in compulsory saving in the developing countries. In this connection, it may be safely asserted that the above mentioned result is a statistical positive analysis for such general view.