

## FI-BREAK MODEL OF US INFLATION RATE: LONG-MEMORY, LEVEL SHIFTS, OR BOTH?\*

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*This paper presents a new time series model, called the FI-BREAK model, which is used to describe US inflation, and incorporates long memory and occasional level shifts at a priori unknown locations. It is demonstrated that, even in the presence of such level shifts, the long memory parameter of the FI-BREAK model can be estimated reasonably accurately. For US inflation, it is found that the proposed model's in-sample fit and out-of-sample forecasts are superior over those of single-feature models with long memory or level shifts.*

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### I. INTRODUCTION AND MOTIVATION

A popular application of long-memory time series models concerns some economic time series, such as inflation rates or certain financial volatility series. There is substantial evidence to suggest that inflation rates have long memory, a feature which can be captured by a fractionally integrated  $I(d)$  model, see Hassler and Wolters (1995) and several papers cited in Baillie (1996). Alternatively, Bos *et al.* (1999) demonstrated that US inflation rates may perhaps

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have long memory, because of the presence of neglected breaks in the series rather than because they are actually  $I(d)$  alone. Indeed, occasional level shifts may mimic the effect of a long-lasting shock, and hence one might be inclined to believe that the data did have long memory. It appears from studies such as Diebold and Inoue (2001) and Granger and Hyung (2004) that apparent long memory can also be caused by neglected occasional level shifts.

For out-of-sample forecasting, it is important as to whether one opts for one or the other model, since the forecast generating equations are completely different. Hence, it seems of practical relevance to examine which model is more appropriate for a given time series, that is, a long-memory model or a model with occasional level shifts or, as demonstrated in this paper, a hybrid model with both features.

Hyung and Franses (2001) compared time series models with structural breaks and models with long memory for US inflation rates in terms of out-of-sample forecasting and found that these two types of models are difficult to distinguish. They suggest that a joint BREAK model and  $I(d)$  model is able to capture all long memory components of inflation rates. There are studies which incorporate level shifts in a long memory model, where the location of the shifts is determined from the outset, see for example Bos, Franses and Ooms (1999, 2002). However, it might be more reasonable to allow the data to reveal where these shifts occur. There are various ways to detect breaks in time series data, however, we will use a model which jointly incorporates both long memory and occasional level shifts. Hyung and Franses (2005) propose such a model, the FI-BREAK model, where FI means 'fractionally integrated'.

In this paper, evidence of long memory in US inflation rates is examined with regard to correlation with occasional breaks. After small sample properties of the FI-BREAK model are examined for estimation of the long memory parameter, this model is used to see if US inflation has a long memory, has occasional level shifts or both, in terms of in-sample estimation and out-of-sample forecastability.

The outline of the paper is as follows. Section II discusses the FI-BREAK model and a method to estimate its parameters. In Section III, a model is considered for US inflation, and its fit is compared with models containing long memory feature or models with only occasional level shifts. In an out-of-sample forecasting exercise, it is found that the FI-BREAK model performs better than these two single-feature models. In Section IV, conclusions are presented.

## II. THE FI-BREAK MODEL

Hyung and Franses (2005) exploit the possibility that the occasional structural break model (or simply BREAK model) and autoregressive fractionally integrated model (or simply ARFI model) can be summarized into a single joint model. The following representation of a FI-BREAK model is considered:

$$\begin{aligned} \alpha(L)(1-L)^d y_t &= x_t \\ x_t &= m_t + \varepsilon_t \end{aligned} \tag{1}$$

and

$$q_t = \frac{m_t - m_{t-1} + q_{t-1} \varepsilon_{t-1}}{\gamma + (\varepsilon_t + \dots + \varepsilon_{t-s+1})^2},$$

where  $\alpha(L) = (1 - \alpha_1 L - \dots - \alpha_p L^p)$ . For simplicity, it is assumed that  $\varepsilon_t \sim$  i.i.d.  $(0, \sigma_\varepsilon^2)$ , although, if necessary, this assumption can be relaxed.

The FI-BREAK model “nests” various possible useful models. When  $d=0$  and  $\gamma \rightarrow \infty$ , the model in (1) becomes an AR(p) model. Indeed, as  $\gamma \rightarrow \infty$ ,  $q_t = 0$  for all  $t$ , implies that  $m_t$  is constant for all  $t$ . Furthermore, if  $d=0$ , then  $\alpha(L)y_t = \varepsilon_t$ . Next, when  $0 < d < 1$  and  $\gamma \rightarrow \infty$ , the model becomes an ARFI(p,d) model, that is,  $\alpha(L)(1-L)^d y_t = \varepsilon_t$ . When  $d=0$  and  $0 < \gamma < \infty$ , the model in (1) becomes an endogenous smooth break model (See Smith (2003)) such that  $\alpha(L)y_t = m_y + \varepsilon_t$  with  $m_t = m_{t-1} + q_{t-1} \varepsilon_{t-1}$ . Finally, when  $0 < d < 1$  and  $0 < \gamma < \infty$ , the FI-BREAK model combines the I( $d$ ) model and the break model. In addition, if one suspects that  $d \geq 1$ , proper differences could be used, and then one of the models above could be used.

For estimation of the parameters  $\varphi' = (\alpha_1, \dots, \alpha_p, d, \gamma, m_0)$  in the FI-BREAK model, Beran’s (1995) approximate maximum likelihood (AML) estimator is modified for invertible and possibly nonstationary ARFIMA models, to allow for occasional level changes. The AML estimator for the FI-BREAK model is consistent and asymptotically normal. The resulting estimator amounts to minimizing the sum of squared residuals

$$Q_T(\varphi) = \sum_{t=1}^T e_t^2(\varphi)$$

where the residuals  $e_t(\varphi)$  are computed as

$$e_t(\varphi) = y_t - m_t + \sum_{j=1}^{t+b-1} \pi_j y_{t-j}$$

where  $m_t = m_{t-1} + q_t(\varphi)e_{t-1}$ ,  $q_t(\varphi) = (e_t + \dots + e_{t-s+1})^2 / \{\gamma + (e_t + \dots + e_{t-s+1})^2\}$ . The  $\pi_j$ 's are the autoregressive coefficients in the infinite order FI-BREAK representation as given in  $(1 - \alpha_1 L - \dots - \alpha_p L^p)(1 - L) = \sum_{j=0}^{\infty} \pi_j L^j$ .  $m_0$  is treated as a parameter to be estimated. If  $\gamma \rightarrow \infty$ , it can be shown that the estimate of  $m_0$  is consistent. When  $\gamma < \infty$ , the influence of  $m_0$  decays as  $t$  increases and it will not have any effect as to the asymptotic distribution of the other parameters, as explained by Smith (2003).

This section examines the finite sample performance of the estimator  $d$  in the FI-BREAK model. We generate level shifts discretely as in Granger and Hyung (2004), where they show that the estimated long memory parameters are closely related with the number of breaks and size of breaks. The simulations reveal how well this FI-BREAK model can approximate discrete shifts and estimate the long memory parameter reasonably accurately even in the presence of such level shifts. If level shifts are not considered properly, the estimate of  $d$  reveals upward bias from the true value. We simulate  $T$  observations from four different DGPs of long-memory processes with occasional breaks:

$$(1 - L)^d y_t = m_t + \varepsilon_t. \quad (5)$$

Discrete level shifts are generated using

$$m_t = m_{t-1} + q_t \eta_t$$

where  $q_t$  follows an i.i.d. binominal distribution, that is,

$$q_t = \begin{cases} 1, & \text{with probability } \delta \\ 0, & \text{with probability } 1 - \delta \end{cases}$$

For simplicity, it is assumed  $\varepsilon_t \sim \text{n.i.d.}(0, 1)$ , and  $\eta_t \sim \text{n.i.d.}(0, \sigma_\eta^2)$ , where “ $n$ ” denotes the normal distribution. If  $\sigma_\eta^2$  increases, the size of jumps in the simulations is likely to be large.

In order to see small sample properties, it is assumed that the expected number of breaks,  $E\{\sum_{t=1}^T a_{it}\} = T\delta$ , is fixed. Strictly speaking, if  $\delta > 0$ , a break component  $m_t$  becomes essentially an I(1) process in the limit (as  $T \rightarrow \infty$ ), and the series  $y_t$  is I( $d'$ ) where  $d' > 1$ . In this paper, this case is not explored further. The expected number of breaks is set equal to 3, that is,  $T\delta = 3$ , and hence it does not increase with the sample size: when  $T = 300$ ,  $\delta = 0.01$ , when  $T = 600$ ,  $\delta = 0.005$ , and when  $T = 1500$ ,  $\delta = 0.002$ . The estimated values of the long memory parameter in a process with occasional breaks are closely related to the number of breaks and the size of the breaks, regardless of the size of the sample. See Granger and Hyung (2004). In the simulation, it is expected to have similar long memory properties, despite the increasing sample size if  $T\delta$  is fixed.

The four data generating processes are (DGP A) with  $d = 0.1$  and  $\sigma_\eta^2 = 0.1$ , which provides data with mild breaks and weak long memory, (DGP B) with  $d = 0.1$  and  $\sigma_\eta^2 = 0.5$ , where the breaks are visually obvious, (DGP C) with  $d = 0.4$  and  $\sigma_\eta^2 = 0.5$ , which gives data with obvious breaks and evident long memory, and (DGP D) with  $d = 0.4$  and  $\sigma_\eta^2 = 0.1$ , giving data with evident long memory but weak evidence of breaks.

The number of simulation runs is set at 250. The DGPs do not contain an AR part, and models without any AR dynamics are therefore considered.<sup>1</sup> The  $d$  parameter is estimated in the FI-BREAK model, while imposing that  $\gamma > 0$  and  $d \geq 0$ , enables to capture breaks and long memory within a model. For comparison purposes,  $d$  is also estimated in an ARFI model, that is, a model that neglects the breaks.

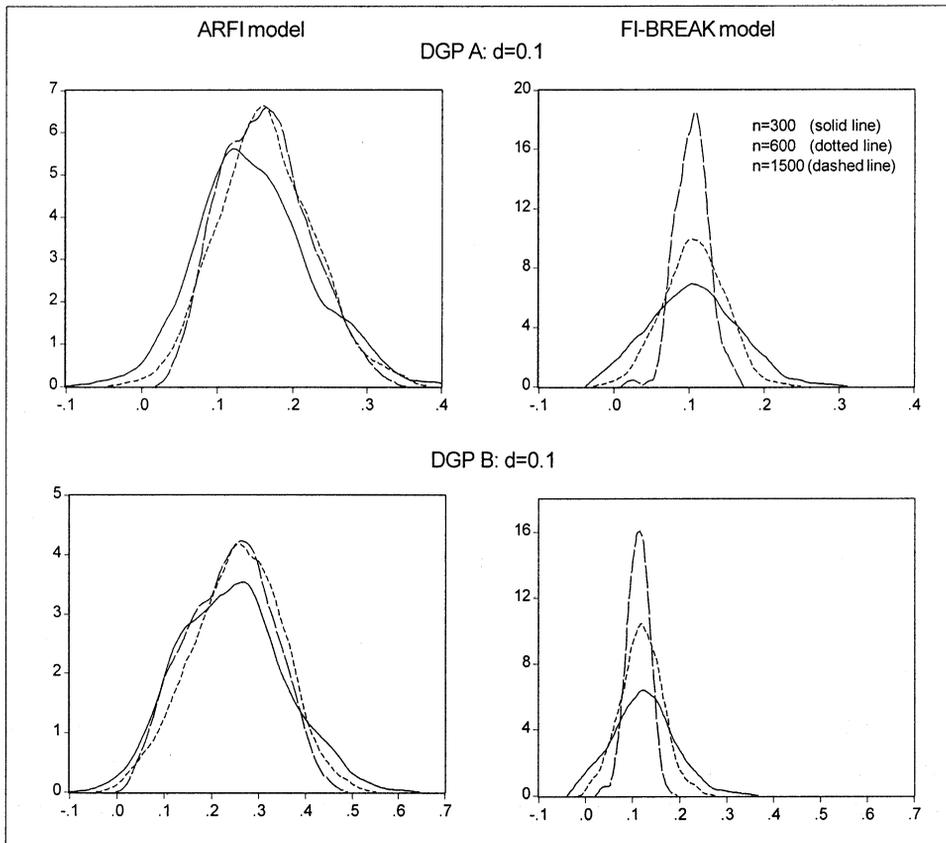
We depict kernel estimates of the density of the estimated  $d$  values for the ARFI and FI-BREAK models as in Figures 1 and 2. The bandwidth is selected by Silverman's rule, and an Epanechnikov kernel is used. A clear upward bias

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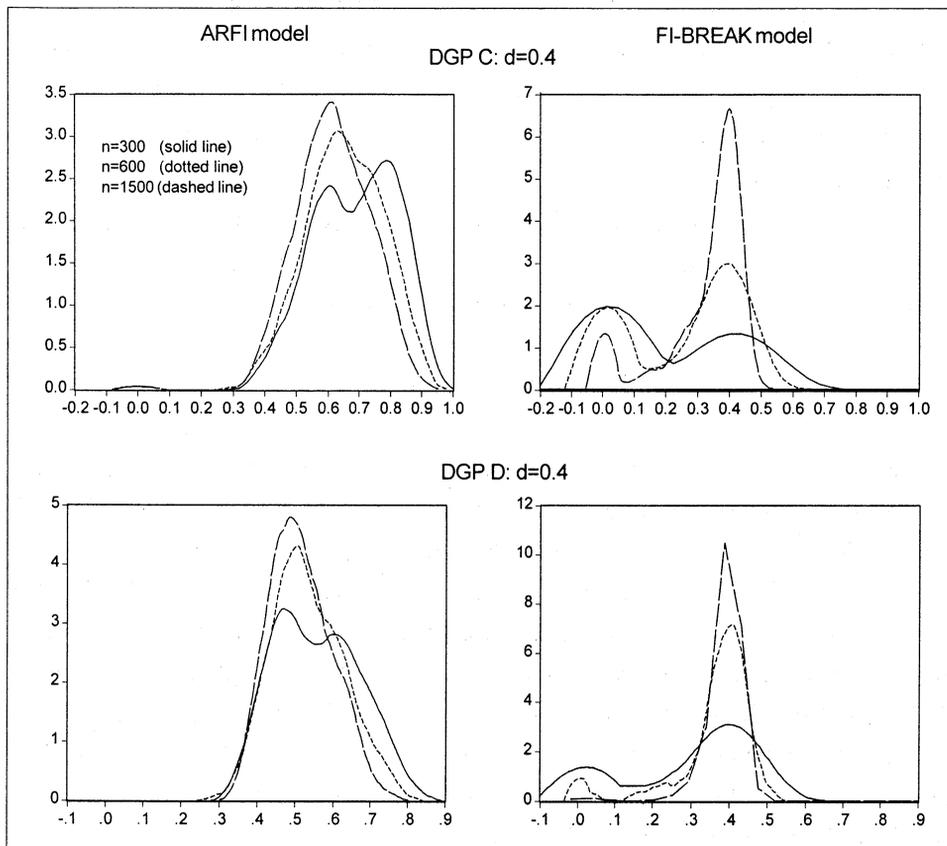
<sup>1</sup> This simulation study uses the same types of DGPs as Hyung and Franses (2005). In many ways the two sets of results complement, rather than conflict with each other. Hyung and Franses (2005) select AR order using AIC or BIC, where often the FI-BREAK model ends with  $\gamma \rightarrow \infty$  and results in the estimated values of  $d$  of the FI-BREAK model larger than the values in this paper.

in the estimated values of  $d$  is observed in the ARFI model, which is of course due to the neglected level shifts. As expected, when the sample size increases, but the expected number of breaks does not change, the estimated  $d$  values roughly remain the same. The FI-BREAK model appears to be reasonably successful in estimating the long memory parameter  $d$  in the presence of level shifts. Additionally, Table 1 shows that the increase of sample size from 300 to 1500 provides substantial evidence of improvement in estimating  $d$  with the FI-BREAK model.

[Figure 1] Kernel Density of the Distributions of Estimated Memort Parameter  $d$  in ARFI models and FI-BREAK model



[Figure 2] Kernel Density of the Distributions of Estimated Memort Parameter  $d$  in ARFI models and FI-BREAK model



[Table 1] Estimation of long memory parameter  $d$

Sample size	DGP A		DGP B	
	ARFI	FI-BREAK	ARFI	FI-BREAK
$T = 300$	.149 (.074)	.106 (.056)	.242 (.108)	.123 (.064)
$T = 600$	.166 (.062)	.107 (.040)	.256 (.092)	.119 (.043)
$T = 1500$	.164 (.056)	.104 (.022)	.240 (.087)	.112 (.024)
	DGP C		DGP D	
	ARFI	FI-BREAK	ARFI	FI-BREAK
$T = 300$	.689 (.130)	.215 (.301)	.558 (.110)	.298 (.171)
$T = 600$	.652 (.125)	.257 (.182)	.535 (.094)	.364 (.105)
$T = 1500$	.617 (.117)	.327 (.128)	.516(.081)	.408 (.266)

Note: The entries are values averaged over 250 replications with sample size 300, 600 and 1500. The numbers of each entry concern the estimates of  $d$ . The values in parentheses are the standard deviations across simulated data.

For DGP C ( $d=0.4$ ), the estimation of  $d$  in the FI-BREAK model improves quickly in accuracy as sample size increases. However, the empirical distribution of estimated  $d$  values is still bi-modal. Indeed, with a sample size of 300, for 120 replications, an estimated value of  $\gamma$  approximately equal to 0 is obtained, this implies  $q_t=1$  for all  $t$  and  $m_t$  is a unit root process. Once  $\gamma=0$ , the long memory component in  $y_t$  is captured by  $m_t$ , thus one obtains an estimated value of  $d$  close to zero.

Overall, it is concluded from the simulation results that the AML estimation method for the FI-BREAK model appears reliable in moderate sample sizes, particularly in the case of weak long memory components.

### III. US INFLATION

Attention is now turned to the question in the title of this paper. To answer this question, we will evaluate the empirical merits of the AR model, BREAK model, ARFI model and FI-BREAK model. It is important to note that only AR type models, for the sake of estimation convenience, are considered. The monthly Consumer Price Index series from the *U.S. city average (All items)*, was obtained from <http://www.economagic.com>. The sample period covers 1951:01 - 2003:05 and the base years are 1982 - 1984. The series is seasonally adjusted. The monthly inflation rate is constructed by taking 100 times the first differences of the logarithmic transformed series. Then, annual rate is calculated by taking 12 times the monthly inflation rate.

#### 3.1. In-sample fit

We estimate the parameters of the FI-BREAK model, as well as for the AR model ( $d=0$  and  $\gamma \rightarrow \infty$ ), ARFI model ( $\gamma \rightarrow \infty$ ), and BREAK model ( $d=0$ ).<sup>2</sup> Evidently, the ARFI model is close to (or “nests”) the AR model, and the smoothed BREAK model nests the AR model, however, the ARFI model and the BREAK model are not nested.

As explained below, a recursive method is adopted for forecasting purposes. We search for and fix the appropriate autoregressive dynamic structures of each

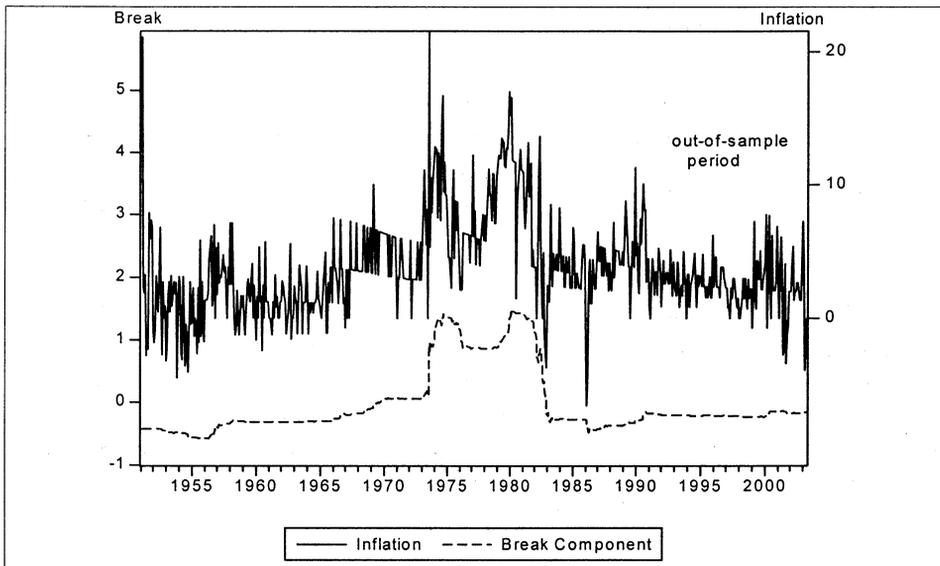
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<sup>2</sup> The sample mean is subtracted from the original observation prior to estimation of the FI-BREAK and ARFI models (See Beran (1994)). For consistency of comparison, the same procedure is imposed for estimation of the AR and BREAK models.

model, using BIC following a general-to-specific approach for the first subsample period, 1951:01 - 1990:12. The estimation results of 149 different sample periods are available for each model; however, this paper only presents results of the full sample period in Table 2, which contains the sum of squared residuals (SSR), AIC, BIC and log likelihood for the selected models. The BIC selects the ARFI model as the best, while the AIC favors the AR model.

Figure 3 presents plots of inflation and the estimated break component  $m_t$  using the proposed FI-BREAK model. Level shifts around the two oil crises are found. Interestingly, the estimated timing of the breaks is remarkably similar to that proposed by Bos et al. (1999), where they set the timing of breaks from the outset. In allowing for a long memory component, the FI-BREAK model suggests the presence of 3 evident breaks. It is important to note that the relative size of breaks in 1976 and 1980 are less than half of that of breaks in 1973 and 1982.

[Figure 3] CPI Inflation: U.S. city average (All items)



From Table 2 it is revealed that neglecting long memory suggests the presence of approximately 16 breaks. Hence, the estimated number of breaks shows a strong upward bias if one does not filter the long memory. Finally, for the long memory parameter  $d$ , the ARFI model produces 0.371, which is very similar to the estimate of Bos et al. (1999) for US inflation. However, if the

break component is included, the degree of persistence decreases to 0.327, which is consistent with Dittman and Granger (2002).

[Table 2] Estimation results for various models for US inflation

	AR	BREAK	ARFI	FI-BREAK
$\gamma$	-	36.013 (0.763)	-	172.996 (1.886)
$d$	-	-	0.371 (0.025)	0.327 (0.030)
$\alpha_1$	0.320 (0.039)	0.271 (0.042)	-	-
$\alpha_2$	0.171 (0.039)	0.137 (0.041)	-	-
$\alpha_3$	-	-	-	-
$\alpha_4$	-	-	-	-
$\alpha_5$	0.108 (0.037)	-	-	-
$\alpha_6$	-	-	-	-
$\alpha_7$	0.114 (0.038)	-	-	-
$\alpha_8$	-	-	-	-
$\alpha_9$	0.180 (0.036)	0.130 (0.039)	0.148 (0.038)	0.137 (0.038)
$m_0$	-	-1.058 (0.398)	-	-0.429 (1.329)
$\sum_{t=1}^T q_t$	-	15.77	-	3.12
SSR	3798.08	3860.05	3881.59	3857.60
Loglikelihood	-1439.80	-1444.81	-1446.53	-1444.61
AIC	1.834	1.845	1.846	1.843
BIC	1.877	1.882	1.867	1.871

Note: The sample period is 1951:01 - 2003:05, hence 629 observations. The AR structures of each model are selected using BIC. The values in parentheses are the standard errors for the estimated coefficients.  $\sum_{t=1}^T q_t$  measures the expected number of breaks for the given sample.

### 3.2. Out-of-sample forecasting

This section compares the out-of-sample forecasting performance of the four models, where the forecast horizon is set at 1, 3, 12 and 24 months. Table 3 presents the results of cumulative forecasts. We select 1990:12 as the first forecast origin. All models are considered for the first  $T(= 480)$  observations

and forecasts are made for the horizon  $T+h$ ,  $h=1, 3, 12$ , and  $24$ , using in-sample parameter estimates, where  $T$  is the number of observations in the first subsample period, 1951:01 - 1990:12. When one observation is added, the parameters are re-estimated and forecasts are made for time  $T+1+h$ . This recursive method is repeated until the end of the sample. Hence the  $h$ -step-ahead forecasts provide predictions for  $150-h$  out-of-sample periods. Note that the AR structure is not re-specified and is kept fixed, as in Table 2.

[Table 3] Cumulative out-of-sample forecasts

I. Mean Absolute Forecast Errors

h-step	AR	BREAK	ARFI	FI-BREAK
1	1.557 (.000)	1.505 (.021)	1.541 (.000)	1.445
3	2.929 (.023)	2.868 (.116)	3.017 (.002)	2.683
12	9.515 (.050)	10.225 (.005)	10.796 (.015)	7.438
24	21.690 (.000)	22.249 (.008)	25.173 (.002)	13.212

II. Root Mean Squared Forecast Errors

h-step	AR	BREAK	ARFI	FI-BREAK
1	2.155 (.000)	2.074 (.030)	2.093 (.011)	2.020
3	3.995 (.016)	3.805 (.090)	4.008 (.000)	3.603
12	12.399 (.027)	12.079 (.009)	13.004 (.006)	9.469
24	25.768 (.000)	24.625 (.007)	28.184 (.000)	16.401

Note: The out-of-sample period is 1991:1 - 2003:5 with 149 observations. The specifications of each model are the same as those in Table 2. The values in parentheses are the  $p$ -values from the test statistics of Diebold and Mariano (1995), which tests the null of equal forecast accuracy as compared with the FI-BREAK model.

We generate cumulative forecasts of inflation. The mean of absolute forecast errors (MAFE) and the root mean squared forecast errors (RMSFE), are computed and shown in Table 3. Clearly, the values of the FI-BREAK model are the smallest for all horizons. Based on the MAFE and RMSFE criteria, FI-BREAK, BREAK, AR and ARFI emerge in this order as the best forecasting models, as the prediction horizon increases from 1 to 24 months. Using the

Diebold-Mariano (1995) test, we examine the hypothesis that there is no difference in the prediction accuracy between each model against the FI-BREAK model. The  $p$ -values in Table 3 reveal that equal forecast accuracy with the FI-BREAK model for the given horizon is rejected at the 5% significance level for almost all cases. In conclusion, it is observed that the FI-BREAK model performs much better than the ARFI model and BREAK model in terms of out-of-sample forecasting.

### 3.3. Non-seasonally adjusted inflation data and TRAMO/SEATS

In the previous sub-sections, seasonally adjusted data is used. To see the robustness of the FI-BREAK model in the pre-filtering seasonal adjustment, the same analysis is applied to the non-seasonally adjusted CPI. For comparison, the TRAMO/SEATS<sup>3</sup> procedure with automatic model selection is applied. We examine whether this *ad hoc* method, TRAMO/SEATS, can detect the same breaks and beat the FI-BREAK model.

For consistency of analysis, some restrictions are imposed on the TRAMO/SEATS procedure.<sup>4</sup> This is selected to fit the ARIMA model to the level of the inflation rate series, fix only (seasonal) difference orders as 0 and search for the best ARMA model. The automatic detection for additive outlier, temporary change and level shift is also selected. We use only part of the previous sample period from 1953:06 to 2003:05, because the TRAMO/SEATS procedure can adjust up to 600 observations, and set 1990:12 as the first forecast origin.

With a full sample period (1953:06 - 2003:05) in Table 4, the ARFI model is the best model for in-sample fit, except for TRAMO/SEATS. The diagnostics of the TRAMO/SEATS procedure are the following: SSR = 4564, AIC = 2.041, BIC = 2.0627, which favor TRAMO/SEATS over all other models in Table 4. Out-of-sample performance of the procedure is tested from 1991:01 to 2003:06

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<sup>3</sup> The TRAMO (Time Series Regression with ARIMA Noise, Missing Observations and Outliers) and SEATS(Signal Extraction in ARIMA Time Series) procedures were developed by Gomez and Maravall (1996). TRAMO is a program for estimation and forecasting of regression models with possibly nonstationary ARIMA errors and any sequence of missing values, and SEATS is a program for estimation of unobserved components in time series following the ARIMA model based method.

<sup>4</sup> When a fully automatic model selection procedure is permitted, the results are slightly different but qualitatively similar. The estimation results of TRAMO/SEATS are available on request.

in Table 5. For the TRAMO/SEATS, the same automatic model selection procedure is maintained with the same restrictions. Although the TRAMO/SEATS procedure may improve forecasting precision over the short-run forecasting horizon, it is not significantly better than the FI-BREAK model. For long run forecasting, it is found that FI-BREAK is superior over all other models in terms of predictability.

[Table 4] Estimation results with non-seasonally adjusted data

	AR	BREAK	ARFI	FI-BREAK
$\gamma$	-	34.080 (0.7688)	-	273.23 (2.8717)
$d$	-	-	0.2895 (0.0295)	0.2736 (0.0330)
$\alpha_1$	0.2436 (0.0394)	0.1922 (0.0434)	-	-
$\alpha_2$	0.1829 (0.0391)	0.1279 (0.0429)	-	-
$\alpha_3$	-	-	-	-
$\alpha_4$	-	-	-	-
$\alpha_5$	-	-	-	-
$\alpha_6$	-	-	-	-
$\alpha_7$	-	-	-	-
$\alpha_8$	0.1313 (0.0373)	-	-	-
$\alpha_9$	-	-	-	-
$\alpha_{10}$	-	-	-	-
$\alpha_{11}$	0.1166 (0.0392)	-	0.1273 (0.0404)	0.1113 (0.0413)
$\alpha_{12}$	0.1887 (0.0394)	0.1463 (0.0417)	0.2362 (0.0400)	0.2186 (0.0413)
$m_0$	-	-1.0581 (0.5514)	-	-0.1220 (0.3216)
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$\sum_{t=1}^T q_t$	-	19.87	-	2.49
SSR	5161.30	5242.67	5139.25	5146.92
Loglikelihood	-1470.96	-1475.55	-1469.71	-1470.14
AIC	2.1944	2.2066	2.1833	2.1882
BIC	2.2391	2.2439	2.2131	2.2254

Note: The sample period is 1953:06 - 2003:05 with 600 observations. The autoregressive structures of each model are selected by BIC. The values in parentheses are the standard errors for the estimated coefficients.  $\sum_{t=1}^T q_t$  can be interpreted as the expected number of breaks for the given sample.

**[Table 5]** Cumulative Out-of-sample Forecasts with non-seasonally adjusted data

## I. Mean Absolute Forecast Errors

h-step	AR	BREAK	ARFI	TRAMO/SEATS	FI-BREAK
1	2.073 (.000)	2.016 (.000)	1.905 (.215)	1.736 (.112)	1.871
3	4.226 (.000)	3.989 (.000)	3.906 (.007)	3.333 (.166)	3.624
12	10.229 (.007)	10.184 (.017)	11.431 (.004)	9.904 (.042)	8.372
24	22.933 (.000)	21.339 (.029)	26.726 (.000)	22.660 (.040)	17.227

## II. Root Mean Squared Forecast Errors

h-step	AR	BREAK	ARFI	TRAMO/SEATS	FI-BREAK
1	2.658 (.000)	2.584 (.000)	2.440 (.000)	2.347 (.621)	2.396
3	5.373 (.000)	5.191 (.000)	4.980 (.023)	4.508 (.483)	4.708
12	12.741 (.015)	12.478 (.044)	13.605 (.002)	13.136 (.022)	10.487
24	26.749 (.000)	24.733 (.096)	29.885 (.000)	29.855 (.027)	20.159

Note: The out-of-sample period is 1991:1 - 2003:05 with 149 observations. The model specifications of each model are fixed as the selected specification for in-sample estimation. The values in parentheses are the  $p$ -values from the test statistics of Diebold and Mariano (1995), which tests the null of equal forecast accuracy with the FI-BREAK model for the given horizon.

## IV. CONCLUSION

In this paper, the usefulness of a new time series model called the FI-BREAK model, which can capture both long memory and level shifts, is examined. This model is applied to US inflation. For in-sample fit and out-of-sample forecasts, this new model performs as well as or better than single-feature models, such as AR, ARFI, BREAK and TRAMO/SEATS. Further research should be made by using the proposed FI-BREAK model to analyze other presumably long memory variables.

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