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THE DECOMPOSITION BY FACTORS IN DIRECT AND INDIRECT REQUIREMENTS: WITH APPLICATIONS TO ESTIMATING THE POLLUTION GENERATION

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Gim and Kim (1998) showed the general relation between the two different notions of direct and indirect input requirements of commodity i to support a unit of final demand of commodity j, γ^f_{ij}, and that to produce a unit of gross output of commodity j, γ^g_{ij}, in the open static input-output model. In this paper, we show that γ^g_{ij} can be decomposed into the direct and the technical indirect effects, γ^f_{ij} into the direct, the technical indirect, and the interrelated indirect effects, and the element of the Leontief inverse c_{ij} into four different parts as the final demand, the direct, the technical indirect, and the interrelated indirect effects.

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I. INTRODUCTION

Jeong (1982, 1984) pointed out and showed that the notion of direct and indirect input requirements to support a unit of final demand and that to produce a unit of gross output are not the same. Recently, Gim and Kim (1998) proposed and showed the general relation between γ^f_{ij} and γ^g_{ij}, which represent the direct and indirect input requirements of commodity i to support a unit of

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final demand of commodity j and the direct and indirect input requirements of commodity i to produce a unit of gross output of commodity j, respectively, in the open static input-output model. The relations were shown to be $\gamma^f_{ii} = c_{ii}\gamma^g_{ii}$ and $\gamma^f_{ij} = c_{ii}\gamma^g_{ij}$, $i \neq j$, where the element of the Leontief inverse c_{ij} represents the direct and indirect output requirements to support a unit of final demand.[1]

In this paper, we expand γ^g_{ij} and show that it consists of two fundamental components as the direct effect and the technical indirect effect which comes from only the purely technical relation between inputs and output. In consequence, we show that the cumulative indirect effect can be decomposed into two different parts as the technical indirect effect and the interrelated indirect effect which only means the interrelated interdependence effect excluding the technical indirect effect from the total cumulative indirect effect. We also demonstrate that the notions of γ^g_{ij} and the decomposition[2] we obtained in this study can be used for analyzing how the total amount of pollution is generated; that is, by what factors and by what amount from each factor.

II. THE DECOMPOSITION OF THE DIRECT AND INDIRECT REQUIREMENTS

Since the Leontief inverse C is equal to $(I-A)^{-1}$, where I is the identity matrix and A is the technical coefficient matrix, it follows that

$$C(I-A) = I. \tag{1}$$

Also, the Leontief inverse can be expressed in the power series form (Waugh, 1950)

$$C = I + A + A^2 + \cdots + A^{k+1} + \cdots \tag{2}$$

[1] Oosterhaven and Stelder (2002) introduced the new concept of net multiplier which accepts sectoral outputs as entries instead of exogenous final demand to avoid double-counting impacts and overestimation of the importance of a sector's output. This net multiplier is found by multiplying the traditional Leontief multiplier by the sectoral final output ratios. An alternative net multiplier is also proposed by de Mesnard (2002).

[2] The decomposition in this paper is based on factors and is not directly related to input-output structural decomposition analysis(SDA). However, see Rose and Casler (1996) for various publications on SDA.

and has been viewed as a term that consists of three different parts: the final demand(I), the direct effect(A), and the cumulative indirect effect($A^2+A^3+\cdots$).

The specific expression for each element c_{ij} can be obtained from (2) and is given by

$$c_{ij}=\delta_{ij}+a_{ij}+\Big(\sum_{r_1=1}^{n}a_{ir_1}a_{r_1j}+\sum_{r_1=1}^{n}\sum_{r_2=1}^{n}a_{ir_2}a_{r_2r_1}a_{r_1j}+\cdots+\sum_{r_1=1}^{n}\sum_{r_2=1}^{n}\cdots\sum_{r_k=1}^{n}a_{ir_k}a_{r_kr_{k-1}}\cdots a_{r_2r_1}a_{r_1j}+\cdots\Big), \tag{3}$$

where

$$\delta_{ij}=\begin{cases}1, & i=j\\ 0, & i\neq j,\end{cases} \tag{4}$$

for $i,j=1,2,\ldots,n$.

Now, the elements of the total requirements matrix for a unit of final demand $\Gamma^f=(\gamma^f_{ij})$ are given by

$$\gamma^f_{ii}=c_{ii}-1,\quad \gamma^f_{ij}=c_{ij},\quad i\neq j, \tag{5}$$

and in Gim and Kim (1998), the elements of the total requirements matrix for a unit of gross output $\Gamma^g=(\gamma^g_{ij})$ are shown to be

$$\gamma^g_{ii}=1-\frac{1}{c_{ii}}=a_{ii}+\sum_{j=1,j\neq i}^{n}\frac{c_{ij}}{c_{ii}}a_{ji}, \tag{6}$$

$$\gamma^g_{ij}=\frac{c_{ij}}{c_{ii}}=(a_{ij}+\sum_{k=1,k\neq i,k\neq j}^{n}\frac{c_{ik}}{c_{ii}}a_{kj})(1-a_{jj})^{-1},\quad i\neq j, \tag{7}$$

for $i,j=1,2,\ldots,n$. To obtain a specific expression and decomposition for each γ^g_{ij} in terms of a_{ij}, we describe two approaches below: one by expanding (6) and (7) and the other by considering the interrelated interdependence of the sectors.

Starting with $n=2$ and considering only through the second-round indirect effect($k=2$ in (2)) for convenience of illustration, the elements of the Leontief inverse c_{11} and c_{21}, for example, can be obtained from (3) as

$$c_{11}=1+a_{11}+(a_{12}a_{21}+a_{12}a_{22}a_{21})+(a_{11}a_{11}+a_{11}a_{11}a_{11}+a_{12}a_{21}a_{11}+a_{11}a_{12}a_{21});$$
$$c_{21}=a_{21}+(a_{21}a_{11}+a_{21}a_{11}a_{11})+(a_{22}a_{21}+a_{22}a_{21}a_{11}+a_{21}a_{12}a_{21}+a_{22}a_{22}a_{21}).$$

From (6) and (7) we obtain $\gamma^g_{11}=a_{11}+(c_{12}/c_{11})a_{21}$ and $\gamma^g_{21}=a_{21}(1-a_{11})^{-1}$, and the expansion of (1) yields four equations, one of which is $-c_{11}a_{12}+c_{12}(1-a_{22})=0$. From this equation, we obtain c_{12}/c_{11}, and using the fact that $(1-a_{ii})^{-1}=1+a_{ii}+a_{ii}^2+\cdots$ for $(1-a_{ii})>0$, we have

$$\begin{aligned}\gamma^g_{11}&=a_{11}+a_{12}(1-a_{22})^{-1}a_{21}\\&=a_{11}+a_{12}a_{21}+a_{12}a_{22}a_{21}+a_{12}a_{22}^2a_{21}+\cdots;\\\gamma^g_{21}&=a_{21}(1-a_{11})^{-1}\\&=a_{21}+a_{21}a_{11}+a_{21}a_{11}^2+a_{21}a_{11}^3+\cdots.\end{aligned}$$

For γ^g_{11}, the term a_{11} is the direct input requirement(direct effect) and the other terms $(a_{12}a_{21}, a_{12}a_{22}a_{21}, \ldots)$ indicate the purely technical relation between inputs and output; that is, they are indispensable indirect effect in production. More specifically, a_{21} induces the indirect input requirements; it is connected technically with the element a_{12}, and it is also technically connected with a_{22} which, in turn, is connected with a_{12}. In consequence, $a_{12}a_{21}+a_{12}a_{22}a_{21}+\cdots$ represent the total amount of the indirect effect, which we will call the technical indirect effect. Comparing c_{11} with γ^g_{11}, we see that c_{11} contains an additional part $(a_{11}a_{11}+a_{11}a_{11}a_{11}+a_{12}a_{21}a_{11}+a_{11}a_{12}a_{21})$. We will call this the interrelated indirect effect. Similar interpretation can be given for γ^g_{21}.

For $n=3$, we have, from (3) and after rearranging,

$$\begin{aligned}c_{11}=1+a_{11}&+(a_{12}a_{21}+a_{12}a_{22}a_{21}+a_{13}a_{32}a_{21}+a_{13}a_{31}+a_{12}a_{23}a_{31}+a_{13}a_{33}a_{31})\\&+(a_{11}a_{11}+a_{11}a_{11}a_{11}+a_{12}a_{21}a_{11}+a_{13}a_{31}a_{11}+a_{11}a_{12}a_{21}+a_{11}a_{13}a_{31});\end{aligned}\tag{8}$$

$$\begin{aligned}c_{21}=a_{21}&+(a_{21}a_{11}+a_{21}a_{11}a_{11}+a_{23}a_{31}a_{11}+a_{23}a_{31}+a_{21}a_{13}a_{31}+a_{23}a_{33}a_{31})\\&+(a_{22}a_{21}+a_{22}a_{21}a_{11}+a_{21}a_{12}a_{21}+a_{22}a_{22}a_{21}+a_{23}a_{32}a_{21}+a_{22}a_{23}a_{31}).\end{aligned}\tag{9}$$

The expressions for γ_{11}^{g} and γ_{21}^{g} can be obtained from (6) and (7) as

$$\gamma_{11}^{g} = a_{11} + \frac{c_{12}}{c_{11}} a_{21} + \frac{c_{13}}{c_{11}} a_{31}, \tag{10}$$

$$\gamma_{21}^{g} = (a_{21} + \frac{c_{23}}{c_{22}} a_{31})(1 - a_{11})^{-1}, \tag{11}$$

and the expansion of (1) gives nine equations, two of which are

$$-c_{11} a_{12} + c_{12}(1 - a_{22}) - c_{13} a_{32} = 0, \tag{12}$$

$$-c_{11} a_{13} - c_{12} a_{23} + c_{13}(1 - a_{33}) = 0. \tag{13}$$

Dividing (12) and (13) by c_{11} and rearranging, we obtain a linear system of equations

$$(1 - a_{22})\frac{c_{12}}{c_{11}} - a_{32}\frac{c_{13}}{c_{11}} = a_{12},$$
$$-a_{23}\frac{c_{12}}{c_{11}} + (1 - a_{33})\frac{c_{13}}{c_{11}} = a_{13},$$

and solving for c_{12}/c_{11} and c_{13}/c_{11}, we get

$$\frac{c_{12}}{c_{11}} = [a_{12}(1 - a_{33}) + a_{13}a_{32}][(1 - a_{22})(1 - a_{33}) - a_{23}a_{32}]^{-1}, \tag{14}$$

$$\frac{c_{13}}{c_{11}} = [a_{13}(1 - a_{22}) + a_{12}a_{23}][(1 - a_{22})(1 - a_{33}) - a_{23}a_{32}]^{-1}. \tag{15}$$

Notice that $(1 - a_{22})(1 - a_{33}) - a_{23}a_{32}$ in (14) and (15) can not be a negative value or zero. It follows that $a_{22} + a_{33} - a_{22}a_{33} + a_{23}a_{32} < 1$, and this enables us to write

$$[(1 - a_{22})(1 - a_{33}) - a_{23}a_{32}]^{-1}$$
$$= 1 + (a_{22} + a_{33} - a_{22}a_{33} + a_{23}a_{32}) + (a_{22} + a_{33} - a_{22}a_{33} + a_{23}a_{32})^{2} + \cdots. \tag{16}$$

Substituting (16) in (14) and (15), and then substituting these two equations in (10), and simplifying and collecting terms with only through the second-round indirect effect, we obtain

$$\gamma_{11}^{g} = a_{11} + (a_{12}a_{21} + a_{12}a_{22}a_{21} + a_{13}a_{32}a_{21} + a_{13}a_{31} + a_{12}a_{23}a_{31} + a_{13}a_{33}a_{31}). \quad (17)$$

Similarly, solving for c_{23}/c_{22}, using an appropriate set of equations which can be obtained from the expansion of (1), and simplifying (11), we obtain

$$\gamma_{21}^{g} = a_{21} + (a_{21}a_{11} + a_{21}a_{11}a_{11} + a_{23}a_{31}a_{11} + a_{23}a_{31} + a_{21}a_{13}a_{31} + a_{23}a_{33}a_{31}). \quad (18)$$

Hence, as in the case with $n=2$, both γ_{11}^{g} and γ_{21}^{g} consist of two parts: the direct effect and the second part of (17) and (18), respectively, which we will call the technical indirect effect. Also, by comparing, we see that c_{11} and c_{21} contain an additional part; the last part in (8) and in (9), respectively. We will call this the interrelated indirect effect. In this manner, the expression for any other γ_{ij}^{g} can be obtained. In fact, it will turn out to be the expression given in (19).

For the above approach the amount of work increases rapidly as the size of the matrix and the number of the round effect increases. However, without expanding (6) and (7) the general expression for γ_{ij}^{g} can also be obtained by considering the interindustry interdependence, which reflects the complete technical relation of production.

For γ_{ij}^{g} with n sectors, a_{ij} is the direct input requirement and the other elements in jth column of A, $a_{1j}, a_{2j}, \ldots, a_{(i-1)j}, a_{(i+1)j}, \ldots, a_{nj}$, induce the technical indirect input requirements. Each one of these $a_{r_1 j}$, $r_1 = 1, 2, \ldots, i-1, i+1, \ldots, n$, is technically connected with the elements in the corresponding column r_1(determined according to the first subscript of $a_{r_1 j}$), and hence it should be premultiplied by the elements in that corresponding column r_1 of A. Consequently, the resultant terms have the form $a_{r_2 r_1}a_{r_1 j}$, $r_2 = 1, 2, \ldots, n$, and the terms with $r_2 = i$ will become the first-round indirect effect($a_{ir_1}a_{r_1 j}$, where $r_1 \neq i$). For the terms with $r_2 \neq i$, further premultiplication is necessary since each element $a_{r_2 r_1}$ is technically connected with the elements in column r_2 of A. Then, after the second premultiplication, the resultant terms have the form

$a_{r_3r_2}a_{r_2r_1}a_{r_1j}$, $r_3=1,2,\ldots,n$, and the terms with $r_3=i$ will become the second-round indirect effect ($a_{ir_2}a_{r_2r_1}a_{r_1j}$, where $r_1\neq i$, $r_2\neq i$). For the terms with $r_3\neq i$, further premultiplication is needed since each element $a_{r_3r_2}$ is technically connected with the elements in column r_3 of A. In like manner, the process can be continued to obtain the third-round indirect effect, the forth-round indirect effect, and on and on. Then, the sum of the direct input requirement a_{ij}, the first-round indirect effect, the second-round indirect effect, and so on, yields γ^g_{ij}. The general expression for each of the elements of Γ^g with n sectors can be formulated and is given by

$$\gamma^g_{ij}=a_{ij}+\Big(\sum_{\substack{r_1=1\\r_1\neq i}}^{n}a_{ir_1}a_{r_1j}+\sum_{\substack{r_1=1\\r_1\neq i}}^{n}\sum_{\substack{r_2=1\\r_2\neq i}}^{n}a_{ir_2}a_{r_2r_1}a_{r_1j}+\cdots+\sum_{\substack{r_1=1\\r_1\neq i}}^{n}\sum_{\substack{r_2=1\\r_2\neq i}}^{n}\cdots\sum_{\substack{r_k=1\\r_k\neq i}}^{n}a_{ir_k}a_{r_kr_{k-1}}\cdots a_{r_2r_1}a_{r_1j}+\cdots\Big), \tag{19}$$

where k stands for kth-round and for $i,j=1,2,\ldots,n$.

Then, (19) can be written as

$$\gamma^g_{ij}=a_{ij}+t_{ij},$$

where t_{ij} is the second part in (19). We call t_{ij} the technical indirect effect. As a consequence, each element of the Leontief inverse c_{ij} can be written in the following form:

$$c_{ij}=\delta_{ij}+a_{ij}+t_{ij}+r_{ij} \tag{20}$$

where δ_{ij} is given in (4), a_{ij} is the element of the technical coefficient matrix A, t_{ij} is the technical indirect effect defined above, and r_{ij} is given by

$$r_{ij}=a_{ii}a_{ij}+\Big(\sum_{r_2=1}^{n}a_{ir_2}a_{r_2i}a_{ij}+\sum_{\substack{r_1=1\\r_1\neq i}}^{n}a_{ii}a_{ir_1}a_{r_1j}\Big)$$
$$+\Big(\sum_{r_2=1}^{n}\sum_{r_3=1}^{n}a_{ir_3}a_{r_3r_2}a_{r_2i}a_{ij}+\sum_{\substack{r_1=1\\r_1\neq i}}^{n}\sum_{r_3=1}^{n}a_{ir_3}a_{r_3i}a_{ir_1}a_{r_1j}$$

$$+\sum_{\substack{r_1=1\\ r_1\neq i}}^{n}\sum_{\substack{r_2=1\\ r_2\neq i}}^{n} a_{ii}a_{ir_2}a_{r_2r_1}a_{r_1j})+\cdots, \tag{21}$$

for $i,j=1,2,\ldots,n$. That is, r_{ij} is just the difference between the third part of the right-hand side of (3) and the second part of the right-hand side of (19) $(r_{ij}=(c_{ij}-\delta_{ij}-a_{ij})-t_{ij})$. We call r_{ij} the interrelated indirect effect. It represents the *interrelated* interdependence indirect effect which consists of terms that are not technically connected in production.

Since $\gamma^f_{ij}=c_{ij}-\delta_{ij}$, the expression for each element of Γ^f can be obtained from (20) and can be written as

$$\gamma^f_{ij}=a_{ij}+t_{ij}+r_{ij}, \tag{22}$$

for $i,j=1,2,\ldots,n$. As a consequence, the difference between the two different notions γ^f_{ij} and γ^g_{ij} is the interrelated indirect effect r_{ij} given in (21), and the total cumulative indirect effect in (3) is decomposed into two parts as t_{ij} and r_{ij}.

Ⅲ. SIMPLER FORMS AND MATRIX NOTATION FOR THE DECOMPOSITION

Denoting I, A, T, and R as the identity matrix, the direct effect, the technical indirect effect, and the interrelated indirect effect, and letting their elements as δ_{ij}, a_{ij}, t_{ij}, and r_{ij}, respectively, we have

$$\Gamma^g=A+T; \tag{23}$$

$$\Gamma^f=A+T+R; \tag{24}$$

$$C=I+A+T+R. \tag{25}$$

Note, however, the computations of t_{ij} using the second part in (19) and r_{ij} using (21) might be tedious. Below we present simpler forms for t_{ij} and r_{ij} in terms of γ^f_{ij} and γ^g_{ij}, since γ^f_{ij} and γ^g_{ij} can be also computed from (5), (6), and (7). Combining (23) and (24) gives

$$R=\Gamma^{f}-\Gamma^{g}.$$

Hence, from (5), (6), and (7),

$$r_{ii}=(c_{ii}-1)-(1-\frac{1}{c_{ii}})=\frac{(c_{ii}-1)^{2}}{c_{ii}}=(c_{ii}-1)(1-\frac{1}{c_{ii}})=\gamma_{ii}^{f}\gamma_{ii}^{g} \tag{26}$$

for the diagonal elements and

$$r_{ij}=c_{ij}-\frac{c_{ij}}{c_{ii}}=(c_{ii}-1)\frac{c_{ij}}{c_{ii}}=\gamma_{ii}^{f}\gamma_{ij}^{g} \tag{27}$$

for the nondiagonal elements, and using (23), we have

$$t_{ij}=\gamma_{ij}^{g}-a_{ij}, \quad i, j=1,2,\ldots,n. \tag{28}$$

Moreover, if we denote K as the diagonal matrix that contains only the elements of the diagonal of C, then from (5), (6), and (7), $\Gamma^{g}=K^{-1}\Gamma^{f}$. Hence, the above can also be written as

$$T=K^{-1}\Gamma^{f}-A \quad \text{and} \quad R=(I-K^{-1})\Gamma^{f}. \tag{29}$$

Ⅳ. APPLICATION TO ESTIMATING THE POLLUTION GENERATION

In this section we apply the results obtained in the previous sections to the environmental Leontief model(Leontief, 1970)[3], which is an input-output model augmented by pollution-generation and pollution-abatement sectors. For the case with the tolerated level of pollution(the amount not eliminated) are given exogenously as a negative variable $-d_p$ (Leontief, 1970; Miller and Blair, 1985), the well-known augmented Leontief model for $n=3$ can be written as

$$\begin{pmatrix} 1-a_{11} & -a_{12} & -a_{1p} \\ -a_{21} & 1-a_{22} & -a_{2p} \\ -a_{p1} & -a_{p2} & 1-a_{pp} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_p \end{pmatrix}=\begin{pmatrix} d_1 \\ d_2 \\ -d_p \end{pmatrix}, \tag{30}$$

[3] The decomposition can also be applied to the waste input-output model for a single region(Nakamura and Kondo, 2002; Kagawa *et al.*, 2003) and to multi-regional input-output models for waste(Kagawa *et al.*, 2004).

where a_{p1} is the output of pollutant p per unit of output of commodity 1, a_{1p} is the input of commodity 1 per unit of eliminated pollutant p, a_{pp} is the output of pollutant p per unit of eliminated pollutant p, x_p represents the total amount of pollutant p eliminated, and d_p represents the amount of pollutant p not eliminated. Then, in this pollution model the Leontief inverse C_P becomes

$$C_P = (I - A_p)^{-1} = \begin{pmatrix} c_{11} & c_{12} & c_{1p} \\ c_{21} & c_{22} & c_{2p} \\ c_{p1} & c_{p2} & c_{pp} \end{pmatrix},$$

and by (5), (6), and (7), Γ_P^f and Γ_P^g the total requirements matrix for a unit of final demand and that for a unit of gross output for this Leontief model, become

$$\Gamma_P^f = \begin{pmatrix} c_{11}-1 & c_{12} & c_{1p} \\ c_{21} & c_{22}-1 & c_{2p} \\ c_{p1} & c_{p2} & c_{pp}-1 \end{pmatrix} \text{ and } \Gamma_P^g = \begin{pmatrix} 1-\dfrac{1}{c_{11}} & \dfrac{c_{12}}{c_{11}} & \dfrac{c_{1p}}{c_{11}} \\ \dfrac{c_{21}}{c_{22}} & 1-\dfrac{1}{c_{22}} & \dfrac{c_{2p}}{c_{22}} \\ \dfrac{c_{p1}}{c_{pp}} & \dfrac{c_{p2}}{c_{pp}} & 1-\dfrac{1}{c_{pp}} \end{pmatrix}.$$

When the example given in Miller and Blair (1985, p.247) is applied to the linear system of (30), we obtain the following specific pollution-activity-augmented Leontief model:

$$\begin{pmatrix} 0.85 & -0.25 & -0.10 \\ -0.20 & 0.95 & -0.20 \\ -0.05 & -0.04 & 1.00 \end{pmatrix} \begin{pmatrix} 100.0 \\ 100.0 \\ 6.0 \end{pmatrix} = \begin{pmatrix} 59.4 \\ 73.8 \\ -3.0 \end{pmatrix}.$$

Here, the technical coefficient matrix for this pollution model is given as

$$A_P = \begin{pmatrix} 0.15 & 0.25 & 0.10 \\ 0.20 & 0.05 & 0.20 \\ 0.05 & 0.04 & 0.00 \end{pmatrix}.$$

The amount of pollution tolerated by society, $-d_p$, is entered as -3.0, and the term $x_p = 6$ is a measure of the total amount of pollution eliminated; that is, the total amount being produced is 9 units. Since it is presumed that the pollution-abatement sector does not generate pollution in the process of eliminating pollution from other sectors and the final demand, the term $1-a_{pp}$ is 1.00. Then,

$$C_P=\begin{pmatrix} 1.2684 & 0.3420 & 0.1952 \\ 0.2828 & 1.1378 & 0.2558 \\ 0.0747 & 0.0626 & 1.0200 \end{pmatrix},$$

$$\Gamma_P^f=\begin{pmatrix} 0.2684 & 0.3420 & 0.1952 \\ 0.2828 & 0.1378 & 0.2558 \\ 0.0747 & 0.0626 & 0.0200 \end{pmatrix}, \text{ and } \Gamma_P^g=\begin{pmatrix} 0.2116 & 0.2696 & 0.1539 \\ 0.2485 & 0.1211 & 0.2248 \\ 0.0732 & 0.0614 & 0.0196 \end{pmatrix}.$$

Using (26), (27), and (28) or using (29), we have

$$T_P=\begin{pmatrix} 0.0616 & 0.0196 & 0.0539 \\ 0.0485 & 0.0711 & 0.0248 \\ 0.0232 & 0.0214 & 0.0196 \end{pmatrix} \text{ and } R_P=\begin{pmatrix} 0.0568 & 0.0724 & 0.0414 \\ 0.0342 & 0.0167 & 0.0310 \\ 0.0015 & 0.0012 & 0.0004 \end{pmatrix}.$$

It can be checked that

$$C_P=I+A_P+T_P+R_P,$$
$$\Gamma_P^f=A_P+T_P+R_P,$$

and

$$\Gamma_P^g=A_P+T_P$$

hold.

For the analysis of the economic impact, the term $C_{P(31)}=0.0747$, for example, is the direct and indirect output units of pollutant p to support one dollar's worth of final demand of commodity 1. However, by the results of the decomposition, it can be viewed that the total amount of pollution generation of pollutant p (0.0747) is generated by the direct effect(0.05), the technical indirect effect(0.0232), and the interrelated indirect effect(0.0015). Also, Γ_P^g enables us to estimate the total pollution generation associated with only the interindustry technical relation between inputs and output. Hence, $\Gamma_{P(31)}^g=0.0732$, for example, can be interpreted notionally as the direct and indirect input units of pollutant p to produce one dollar's worth of gross output of commodity 1.

This amount is actually generated by the two effects: the direct(0.05) and the technical indirect effect(0.0232). Note that $\Gamma^{g}_{P(33)}=0.0196$ are the only indirect input units of pollutant p to abate one unit of eliminated pollutant p. In this case, however, there is no direct input units by the presumption.

V. AN APPLICATION TO ESTIMATING THE WASTES GENERATION: THE CASE OF KOREA

In this section we again apply the results obtained in the previous sections on a practical problem to the environmental Leontief model, which is an input-output model augmented by waste-generation. The waste-generation data shown in Table 1 is obtained from the ministry of the environment of Korea (2002). The data includes domestic and industrial wastes. Industrial wastes consist of combustible, incombustible and specified wastes, but domestic wastes by industry is excluded from this data. The sectors in this data and the sector classification in the 2000 input-output tables of Korea (2003) have been rearranged to make 15 sectors out of the 28 basic sectors(see Appendix A), for the convenience of illustration. The transactions are at producers' prices in one million Korean Won. The amount of wastes is measured in kg per day, which is generated to produce one million Korean Won.

The augmented Leontief model for $n=16$ can be written in compact matrix form as

$$\left(\begin{array}{cccc:c} & & & & 0 \\ & & & & 0 \\ & (I-A) & & & \vdots \\ & & & & 0 \\ \hdashline -a_{w1} & -a_{w2} & \cdots & -a_{w15} & 1 \end{array}\right)\left(\begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_{15} \\ \hdashline x_w \end{array}\right)=\left(\begin{array}{c} d_1 \\ d_2 \\ \vdots \\ d_{15} \\ \hdashline 0 \end{array}\right), \tag{31}$$

where a_{wj} is the output of waste w per unit of output of commodity j and corresponds to the last line in Table 1, x_w represents the total amount of waste w generated, and d_i represents the final demand of commodity i. We denote the above expanded coefficient matrix given in (31) as $(I-A_w)$.

Computing $C_w=(I-A_w)^{-1}$ and the corresponding matrices Γ^{g}_{w}, Γ^{f}_{w}, and T_w and showing only the bottom rows of these matrices gives the following table:

[Table 1] Input and Waste-generation Coefficients, 2000

	Sectors															
Sectors	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1. Agriculture, Forestry and Fishery	.048	.002	.398	.015	.026	.003	.000	.000	.000	.000	.000	.002	.021	.000	.002	0
2. Mining	.000	.000	.000	.000	.002	.217	.114	.021	.000	.000	.206	.004	.000	.000	.000	0
3. Food and Beverages	.102	.000	.132	.019	.001	.003	.000	.000	.000	.000	.000	.000	.111	.000	.005	0
4.Textiles and Leather	.003	.000	.000	.339	.007	.003	.002	.001	.001	.014	.000	.001	.002	.001	.004	0
5. Lumber, Paper and Publishing	.007	.008	.017	.011	.394	.007	.015	.004	.008	.018	.001	.018	.008	.004	.017	0
6. Petroleum and Chemicals	.096	.077	.040	.158	.078	.345	.120	.043	.054	.082	.092	.043	.027	.114	.038	0
7. Non-metallic Mineral Products	.000	.000	.006	.001	.003	.004	.175	.008	.016	.008	.001	.092	.001	.000	.001	0
8. Metal Products and Machinery	.007	.015	.015	.008	.011	.019	.031	.495	.059	.161	.011	.167	.002	.003	.008	0
9. Electronic and Electric Equipment	.003	.004	.001	.001	.003	.002	.004	.028	.451	.064	.012	.046	.003	.014	.009	0
10. Transportation Equipment (and Other Manufactured Products)	.004	.017	.003	.005	.002	.001	.006	.003	.001	.280	.001	.006	.004	.025	.013	0
11. Electric, Gas and Water Services	.003	.038	.007	.014	.022	.021	.040	.024	.007	.008	.113	.002	.019	.007	.012	0
12. Construction	.001	.002	.000	.001	.000	.000	.001	.001	.000	.000	.030	.000	.003	.002	.019	0
13. Commerce, Restaurants and Hotels	.012	.007	.035	.029	.031	.017	.021	.020	.032	.031	.005	.025	.038	.008	.054	0
14. Transportation and Communications	.011	.018	.013	.018	.026	.012	.031	.013	.011	.012	.006	.013	.050	.187	.028	0
15. Finance and Public Service	.079	.178	.060	.084	.109	.068	.101	.071	.086	.071	.067	.140	.164	.152	.163	0
16. Waste	.017	.399	.374	.918	1.678	.166	38.241	3.116	.220	1.612	1.916	.083	.047	.123	1.305	0

[Table 2] Data Obtained from the Bottom Rows of C_w, Γ^g_w, T_w, and R_w

	j = 1	2	3	4	5	6	7	8
c_{wj}	0.7308	1.3642	1.6568	2.5885	4.1031	1.7024	47.7400	7.9449
γ^g_{wj}	0.7308	1.3642	1.6568	2.5885	4.1031	1.7024	47.7400	7.9449
t_{wj}	0.7138	0.9652	1.2828	1.6705	2.4251	1.5364	9.4990	4.8289
r_{wj}	0	0	0	0	0	0	0	0

	j = 9	10	11	12	13	14	15	16
c_{wj}	3.3693	5.5162	3.2693	6.5139	0.9789	1.1532	2.2951	1
γ^g_{wj}	3.3693	5.5162	3.2693	6.5139	0.9789	1.1532	2.2951	0
t_{wj}	3.1493	3.9042	1.3533	6.4309	0.9319	1.0302	0.9901	0
r_{wj}	0	0	0	0	0	0	0	0

Note that the inputs from the other sectors to waste abatement are not available and so the values of the elements in the last column of the matrix $(I-A_w)$ in (31) are all zero. This causes $r_{wj}=0$ for all j in this particular case. As we discussed in the previous example, the "waste multipliers" c_{wj} give an indication of the effects on wastes generation. Moreover, due to the results obtained in this paper, Γ^g_w enables us to estimate the total waste generation associated with only the purely technical relation between inputs and output. Hence, $\gamma^g_{w1}=0.7308$, for example, can be interpreted notionally as the direct and indirect input units of waste w to produce one million Korean Won worth of gross output of commodity 1. This amount is actually generated by the two effects: the direct(0.017) and the technical indirect effect(0.7138). Consequently, an implication of the above results is that by decomposing wastes generation into factors, one can establish more effective methods for abatement of wastes generation by approaching and examining factor by factor.

Before leaving this section, we further point out that the matrix Γ^g can also be applied in the calculation of energy intensity(or the total energy requirement) measured in physical units for a unit of gross output. Up to date, the Leontief inverse has been used to compute the energy intensity for a unit of each sector's gross output. Recently, however, it has been shown that no identical meaning can be placed between the notion of total energy requirement for a unit of final demand and that for a unit of gross output. Hence, the concept of Γ^g and the decomposition of it into two different effects will provide us more elaborate analysis on the results and meaningful values, which coincide with the definition of energy intensity.

Ⅵ. CONCLUSIONS

In this paper, we showed that γ_{ij}^{g} can be decomposed into two different parts as the direct and the technical indirect effects, γ_{ij}^{f} into three different parts as the direct, the technical indirect, and the interrelated indirect effects, and the element of the Leontief inverse c_{ij} into four different parts as the final demand, the direct, the technical indirect, and the interrelated indirect effects. By applying the decomposition results on a pollution and waste generation models, we were able to obtain (or at least to expect) how the total amount of pollution and waste generation were produced; that is, by what factors(such as the direct, the technical indirect, and the interrelated indirect effects) and by what amount from each factor. The decomposition results can also be applied in many different areas such as in environmental and energy input-output models, intensity analysis of resources, impact analysis for one unit of output, etc.

Appendix A: Sector Classifications (15 Sectors)

15 Commodity Sectors	28 Commodity Sectors
1. Agriculture, Forestry and Fishery	1. Agriculture, Forestry and Fishery
2. Mining	2. Mining
3. Food and Beverages	3. Food and Beverages
4. Textiles and Leather	4. Textiles and Leather
5. Lumber, Paper and Publishing	5. Lumber and Wood Products 6. Paper, Printing and Publishing
6. Petroleum and Chemicals	7. Petroleum and Coal Products 8. Chemical Products
7. Non-metallic Mineral Products	9. Non-metallic Mineral Products
8. Metal Products and Machinery	10. Primary Metal Products 11. Metal Products 12. General Industrial Machinery
9. Electronic and Electric Equipment	13. Electronic and Electric Equipment 14. Measuring, Medical and Optical Instruments
10. Transportation Equipment and Other Manufactured Products	15. Transportation Equipment 16. Other Manufactured Products
11. Electric, Gas and Water Services	17. Electric, Gas and Water Services
12. Construction	18. Construction
13. Commerce, Restaurants and Hotels	19. Commerce 20. Restaurants and Hotels
14. Transportation and Communications	21. Transportation and Warehousing 22. Communications and Broadcasting
15. Finance and Public Service	23. Finance and Insurance 24. Real Estate and Rental 25. Public Administration and Defense 26. Education and Medical Service 27. Social and Other Services 28. Others

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