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# DYNAMICS OF REAL EXCHANGE RATES AND TRADE BALANCE IN THE PRESENCE OF HETEROGENEOUS PREFERENCES AND INCOMPLETE FINANCIAL MARKETS

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*When agents across countries have home-biased preferences in a world of incomplete financial markets, an increase in the relative price of home-produced goods resulted from a permanent adverse home output shock makes the cost of living of home agents higher than that of foreign agents [appreciation of real exchange rate], reducing home relative consumption. Intertemporal utility maximizing agents with the degree of relative risk aversion greater than unity in the CES utility function would like to smooth their relative consumption by reducing exports and increasing imports, resulting in trade deficit.*

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## I. INTRODUCTION

The objective of this paper is twofold: one is to identify the effect of the change in the relative price of home-produced goods (resulting from permanent country-specific shocks) on the real exchange rate (relative cost of living across countries) and the other is, in turn, to investigate the effects of the change in real exchange rates on international macroeconomic variables such as relative consumption and trade balance in the presence of heterogeneous preferences across countries and incomplete financial markets.[1] In the framework of

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[1] International consumption synchronization, the determination of real exchange rate fluctuations

intertemporal utility maximization, we show that the magnitude of real exchange rate fluctuations, the degree of consumption synchronization across trading economies, and the magnitude of trade balance deviations resulting from country specific shocks are fundamentally determined by the parameters of the agents' CES utility function such as the degree of relative risk aversion and the share of home-produced goods in their consumption baskets.

Backus and Smith (1993) examine the effect of the relative price of home produced goods on relative consumption in the setting of intertemporal utility optimization. We employ a newly developed open economy macroeconomic framework featuring Dixit-Stiglitz type preference to distinguish the real exchange rate from the terms of trade and to show explicitly how the relative price of home produced goods is related to the real exchange rate and how real exchange rate changes influence relative consumption. In our model, the real exchange rate is defined as relative cost of living between countries in terms of the same currency (or as the relative purchasing power in home and foreign countries in terms of the same currency). The real exchange rate is introduced because country-specific shocks in a country are transmitted to other trading economies through the changes in their cost of living. Relative consumption between countries is affected by the changes in the relative cost of living, namely, the real exchange rate.

A commonly used assumption in macroeconomics literature is an identical and homothetic preference that represents all households' preferences in an economy. Wilson (1968), Stulz (1981), Townsend (1987), and Cole and Obstfeld (1991) identify that complete consumption risk sharing across agents (manifested in constant relative consumption) with respect to idiosyncratic shocks is the result of the assumption of agents' identical and homothetic preferences even when financial markets are incomplete. If agents' preferences are identical and homothetic, changes in the relative prices of goods caused by idiosyncratic shocks do not change the expenditure share on each individual good in the consumption basket. Therefore, relative cost of living and relative consumption would be always the same. When risk-averse agents have heterogeneous preferences, however, changes in the relative prices of goods due to idiosyncratic shocks alter the relative cost of living and therefore relative consumption.

If aggregate uncertainty in an economy is nil and if there are no problems of

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and international trade flows are the main research topics in international business cycles literature.

hidden information regarding the characteristics of agents and private information related to hidden actions of agents, the full set of Arrow-Debreu state contingent contracts or financial claims on products could be transacted to realize the first best Pareto-efficient consumption allocations of the economy intra-temporally as well as inter-temporally.[2] Aggregate uncertainty combined with asymmetric information in the financial markets, however, deters households from transacting contingent contracts and risky financial assets, bringing incompleteness into financial markets.[3] As a result, households' financial trading is concentrated mainly on a simplified structure of contracts for which the maturity and issue dates coincide, namely, nominal bonds. An implication for financial market incompleteness is that households' consumption is affected by the extent to which idiosyncratic shocks change their cost of living. When the consumption basket is home-biased in a world of incomplete financial markets, the increase in relative prices of home-produced goods resulted from country-specific shocks raises relative cost of living (real appreciation) and reduces relative consumption of home agents.

Obstfeld (1982) was the first to use intertemporal utility optimization approach to the determination of trade balance. The famous Harberger-Laursen-Metzler effect states that agents reduce their savings and current account when they experience a decrease in real income due to the worsening of terms of trade to smooth consumption over time. To the contrary, Obstfeld (1982) finds that the reversed case is true. In the setting of agents' subjective rate of time preference increasing function of utility, agents increase their savings and current account with respect to the worsened terms of trade to maximize their lifetime utility. Svensson and Razin (1983) clarify the role of rate of time preference in the determination of savings and current account in the setting of intertemporal optimization. For a permanent worsening of the terms of trade, saving and current account fall with decreasing rate of time preference, while rise with increasing rate of time preference. In our model, we do not give a role to the

[2] See Arrow (1964). If the "spanning condition" that the number of available financial claims must be greater than or equal to the number of products is satisfied, Pareto-efficient consumption allocations could result.

[3] Complete financial markets mean that financial transaction methods to help households to maximize their lifetime utility by hedging their consumption risk are available in the financial markets, while perfect financial markets just imply that there are no regulations and no transaction costs hampering financial transactions in the financial markets. In our model, financial markets are assumed to be perfect but not complete. For financial market incompleteness, see Davis, Nalewaik, and Willen (2000) and Liu (2002).

rate of time preference by equalizing it to the real interest rate. Instead, we investigate the role of the degree of risk aversion and the share of home-produced goods in the consumption basket in the determination of current account. When the consumption basket is home-biased, the increase in relative prices of home-produced goods resulted from country-specific shocks raises relative cost of living (real appreciation) and reduces relative consumption of home agents. Intertemporal utility maximizing agents with the degree of relative risk aversion greater than unity would like to smooth relative consumption by increasing imports and reducing exports, which leads to trade deficits.

The following is the organization of this paper. Section Ⅱ describes the structure of the model. Section Ⅲ examines the relation between relative consumption and real exchange rate. In Section Ⅳ, trade balance is derived as a function of the real exchange rate. Section Ⅴ solves for the real exchange rate as a function of country-specific shocks. Section Ⅵ concludes.

## Ⅱ. THE MODEL

### Preferences

In the world economy, there are two countries of the same economic size, Home and Foreign. In each country, there is a continuum of identical households, $v$, each of who specializes in a single differentiated product indexed by $v$, $0 \leq v \leq 1$ and $1 \leq v \leq 2$ respectively. The representative household $v$ in the home country is assumed to maximize his lifetime utility given by

$$U_t = E_t\left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{C_s^{1-\rho}}{1-\rho} + \frac{\chi}{1-\varepsilon}\left(\frac{m_s}{P_s}\right)^{1-\varepsilon} - \eta Y_s(v) \right] \right\}, \quad 0 \leq \beta \leq 1,$$
$$0 \leq \rho < \infty, \quad 0 < \varepsilon < \infty, \qquad (1)$$

where $Y(v)$ is the amount of each product. $\beta$ shows the time discount rate, and $\rho$ the degree of relative risk aversion of CES utility function. $C$ is the index of per capita consumption that is a geometric average of the consumption of home and foreign produced goods, $C_H$ and $C_F$. Real money holdings, $M/P$, provide liquidity services of reducing transaction costs of goods and assets, and enter the utility function. The inverse of the elasticity of money demand with respect to consumption is $\varepsilon$, and $\chi$ is some constant. $Y(v)$ is equal to $L(v)$

that denotes the amount of labor supplied by the representative household $v$. Technology shows constant returns to scale. $\eta$ is considered as an expected adverse output shock in the home country that reduces households' utility. Variables in the foreign country are denoted with an asterisk. Households in the foreign country have exactly the same intertemporal utility maximization problem.

Households across countries are assumed to have heterogeneous tastes over home and foreign produced goods. The indexes of per capita consumption of home and foreign countries are as follows:

$$C \equiv \frac{C_H^{\gamma} C_F^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}; \quad C^* \equiv \frac{(C_H^*)^{1-\gamma}(C_F^*)^{\gamma}}{\gamma^{1-\gamma}(1-\gamma)^{\gamma}}, \quad 0 \leq \gamma \leq 1, \tag{2}$$

where $C_H$ and $C_F$ are consumption of home and foreign produced goods by the representative household in home country respectively, and $C_H^*$ and $C_F^*$ are consumption of home and foreign produced goods by the representative household in foreign country respectively. Households' preferences over home and foreign produced goods are the same within the same country with the same weights, $\gamma$, and $(1-\gamma)$. However, preferences over home and foreign produced goods across countries are identically asymmetric with the same weights on the domestic and imported goods.[4]

The sub-indexes of per capita consumption of home and foreign goods in home and foreign countries are respectively,

$$C_H = \left[ \int_0^1 C_H(v)^{\frac{\theta-1}{\theta}} dv \right]^{\frac{\theta}{\theta-1}}; \quad C_F = \left[ \int_1^2 C_F(v)^{\frac{\theta-1}{\theta}} dv \right]^{\frac{\theta}{\theta-1}} \tag{3}$$

$$C_H^* = \left[ \int_0^1 C_H^*(v)^{\frac{\theta-1}{\theta}} dv \right]^{\frac{\theta}{\theta-1}}; \quad C_F^* = \left[ \int_1^2 C_F^*(v)^{\frac{\theta-1}{\theta}} dv \right]^{\frac{\theta}{\theta-1}} \tag{4}$$

where $C_H(v)$ and $C_F(v)$ are respectively the representative home household's consumption of home and foreign produced goods, and $C_H^*(v)$, and $C_F^*(v)$ are the representative foreign household's consumption of home and foreign produced

[4] The assumption of identically asymmetric preference was taken for the convenience of the computations of endogenous variables such as relative consumption and trade balance. Reducing one parameter makes the computation a lot easier. In reality, countries have different combinations of domestic and imported goods in their consumption baskets.

goods respectively.

All agents across countries have the same degree of relative risk aversion, $\rho$, which is assumed to be greater than zero. The elasticity of substitution between goods produced within the same country is $\theta>1$, that is assumed to be greater than one, while the elasticity of substitution between goods produced across countries, Home and Foreign, is assumed to be one.[5]

### Cost of Living of the Representative Agents in Home and Foreign Countries

The overall consumption-based price indexes of home and foreign countries are as follows.

$$P\equiv(P_H)^{\gamma}(P_F)^{1-\gamma};\quad P^*\equiv(P_H^*)^{1-\gamma}(P_F^*)^{\gamma} \tag{5}$$

where $P_H$ and $P_F$ are home country's price indexes for the goods produced in home and foreign countries, and $P_H^*$ and $P_F^*$ are foreign country's price indexes for the goods produced in home and foreign countries.

The sub-price indexes for home and foreign goods are respectively,

$$P_H=\left[\int_0^1 P_H(v)^{1-\theta}dv\right]^{\frac{1}{1-\theta}};\quad P_F=\left[\int_1^2 P_F(v)^{1-\theta}dv\right]^{\frac{1}{1-\theta}} \tag{6}$$

$$P_H^*=\left[\int_0^1 P_H^*(v)^{1-\theta}dv\right]^{\frac{1}{1-\theta}};\quad P_F^*=\left[\int_1^2 P_F^*(v)^{1-\theta}dv\right]^{\frac{1}{1-\theta}} \tag{7}$$

where $P_H(v)$ and $P_F(v)$ are the prices of the representative goods produced in home and foreign countries in home country, while $P_H^*(v)$ and $P_F^*(v)$ are the prices of the representative goods produced in home and foreign countries in foreign country, respectively. The law of one price is assumed to hold for each individual good so that $P(v)=SP^*(v)$, $\forall v\in[0,2]$, where $S$ is the spot exchange rate of home currency to foreign currency. For the sub-price indexes such as $P_H$, and $P_F$, consumption-based purchasing power parity holds so that

[5] Backus, Kehoe, and Kydland (1995) used 1.5 as the benchmark parameter value for the elasticity of substitution between home and foreign produced goods. Generally, it's assumed that the goods produced in the same country are more substitutable for each other than for goods produced in different countries. See Armington (1969) and Tille (1999).

$P_H = SP^*_H$, and $P_F = SP^*_F$. Because home and foreign households do not have an identical preference on home and foreign-produced goods, consumption-based purchasing parity for overall consumer price indexes, $P = SP^*$, does not hold.

**Goods Market Equilibrium**

Under sub-demand functions, (3) and (4), optimal intra-temporal consumption choices are

$$C_H(v) = \left[\frac{P_H(v)}{P_H}\right]^{-\theta} C_H;\quad C_F(v) = \left[\frac{P_F(v)}{P_F}\right]^{-\theta} C_F \tag{8}$$

$$C^*_H(v) = \left[\frac{P^*_H(v)}{P^*_H}\right]^{-\theta} C^*_H;\quad C^*_F(v) = \left[\frac{P^*_F(v)}{P^*_F}\right]^{-\theta} C^*_F \tag{9}$$

where $C_H(v)$ and $C_F(v)$ are the demand for the representative home and foreign goods of the home representative household, while $C^*_H(v)$ and $C^*_F(v)$ are the demand for the representative home and foreign goods of the foreign representative household.

The Cobb-Douglas overall consumption indexes imply that demands for home and foreign goods, $C_H$ and $C_F$, are given by

$$C_H = \gamma\left(\frac{P_H}{P}\right)^{-1} C;\quad C_F = (1-\gamma)\left(\frac{P_F}{P}\right)^{-1} C \tag{10}$$

$$C^*_H = (1-\gamma)\left(\frac{P^*_H}{P}\right)^{-1} C^*;\quad C^*_F = \gamma\left(\frac{P^*_F}{P^*}\right)^{-1} C^*. \tag{11}$$

Combining (8) and (10), and (9) and (11) gives

$$C_H(v) = \gamma\left[\frac{P_H(v)}{P_H}\right]^{-\theta}\left(\frac{P_H}{P}\right)^{-1} C;\ C_F(v) = (1-\gamma)\left[\frac{P_F(v)}{P_F}\right]^{-\theta}\left(\frac{P_F}{P}\right)^{-1} C \tag{12}$$

$$C^*_H(v) = (1-\gamma)\left[\frac{P^*_H(v)}{P^*_H}\right]^{-\theta}\left(\frac{P^*_H}{P^*}\right)^{-1} C^*;\ C^*_F(v) = \gamma\left[\frac{P^*_F(v)}{P^*_F}\right]^{-\theta}\left(\frac{P^*_F}{P^*}\right)^{-1} C^* \tag{13}$$

The world consumption for each individual good produced in home and foreign countries is defined as follows:

$$C_H^W(v) = C_H(v) + C_H^*(v); \quad C_F^W(v) = C_F(v) + C_F^*(v). \tag{14}$$

Substituting equations (12) and (13) into (14) gives $C_H^W(v)$ and $C_F^W(v)$ that are per capita as well as total world consumption for each individual good produced in Home and Foreign countries:

$$C_H^W(v) = \gamma\left[\frac{P_H(v)}{P_H}\right]^{-\theta}\left(\frac{P_H}{P}\right)^{-1}C + (1-\gamma)\left[\frac{P_H^*(v)}{P_H^*}\right]^{-\theta}\left(\frac{P_H^*}{P^*}\right)^{-1}C^* \tag{15}$$

$$C_F^W(v) = (1-\gamma)\left[\frac{P_F(v)}{P_F}\right]^{-\theta}\left(\frac{P_F}{P}\right)^{-1}C + \gamma\left[\frac{P_F^*(v)}{P_F^*}\right]^{-\theta}\left(\frac{P_F^*}{P^*}\right)^{-1}C^*. \tag{16}$$

The goods market for each individual good produced in home and foreign countries clears when the demand equals the supply. Taking account of the population in two countries and evaluating at the symmetric equilibrium, where $P_H(v) = P_H$, and $P_F(v) = P_F$, we attain the world market clearing condition for each individual good produced in home and foreign countries as follows:

$$C_H^W(v) = \left\{\gamma\left(\frac{P_H}{P}\right)^{-1}C + (1-\gamma)\left(\frac{P_H^*}{P^*}\right)^{-1}C^*\right\} = Y(v) \tag{17}$$

$$C_F^W(v) = \left\{(1-\gamma)\left(\frac{P_F}{P}\right)^{-1}C + \gamma\left(\frac{P_F^*}{P^*}\right)^{-1}C^*\right\} = Y^*(v) \tag{18}$$

**Asset Market Equilibrium**

Asset markets are not complete either because Arrow-Debreu state-contingent futures contracts do not exist, or because the number of risky assets to span national output shocks is limited. Instead, in each period, agents can purchase futures contracts in money that promise one unit of money next period. We exclude risky stocks in our model because complete financial markets require enough number of stocks spanning shocks, which are not plausible in reality. Furthermore, their existence in the model only complicates mathematical computations. By these reasons, we assume that only nominal bonds are available in financial markets. Only two nominal bonds denominated in home and foreign currencies are available to agents across countries. Bonds denominated in home currency pay one unit of the home currency, while foreign currency-denominated bonds pay one unit of foreign currency next period. The current price for the futures contract in home currency, $Q_t$, is the discounted present value of one unit of home currency by the current domestic nominal interest rate. When a home agent buys one-year futures contract in money that returns one unit of home currency next period, the price for the position is $1/(1+i)$. The current price of futures contract in foreign currency, $S_t Q_t^*$ that ensures one unit of foreign currency next period for home agents, is $S/(1+i^*)$, the current spot exchange rate times the discounted present value of one unit of foreign currency by the current foreign nominal interest rate. Thus, the price of the bond denominated in foreign currency is subject to changes in the level of the current spot exchange rate as well as in the level of the current foreign nominal interest rate. Because we focus on the expenditure-switching effect of monetary policy, we exclude the liquidity effect of monetary policy by assuming that the interest rate is not changed.

The market clearing conditions in world asset markets are respectively,

$$B_{H,t}+B_{H,t}^{*}=0,\ B_{F,t}+B_{F,t}^{*}=0 \quad \forall t \tag{19}$$

where $B_{H,t}$ and $B_{F,t}$ are respectively home and foreign currency denominated nominal bonds.

## Lifetime Budget Constraint

Given intra-temporal consumption choices, the budget constraint of the representative agent in home country is given as follows:

$$P_tC_t+M_t+Q_tB_{H,t}+S_tQ_t^*B_{F,t}=M_{t-1}+P_{H,t}(v)Y_t(v)+B_{H,t-1} \\ +S_tB_{F,t-1}+T_t, \tag{20}$$

where $Q_t$ and $Q_t^*$ are respectively prices of home and foreign currency denominated bonds, and $T_t$ is monetary transfers from the government to each home private agent. The government budget constraint is given as follows. The change in the money supply by the government is transferred directly to private agents. There are no government expenditures over time:

$$M_t = M_{t-1} + T_t . \tag{21}$$

**First Order Conditions for the Representative Agents in Home and Foreign Countries**

The problem of the representative agent in each country is to choose rules for nominal money balances, $M$, home-currency denominated one-year nominal bonds, $B_H$, and foreign-currency denominated one-year nominal bonds, $B_F$, and the pricing rule for his production to maximize his lifetime expected utility (1) subject to the sequence of inter-temporal budget constraints (20). The initial values of $M$, $B_H$, and $B_F$ are given.

For the representative home agent, inter-temporal Euler equations for $B_{H,t}$ and $B_{F,t}$ are given as follows:

$$Q_t = \beta \frac{E_t(P_{t+1}^{-1} C_{t+1}^{-\rho})}{P_t^{-1} C_t^{-\rho}} \tag{22}$$

$$Q_t^* = \beta \frac{E_t(S_{t+1} P_{t+1}^{-1} C_{t+1}^{-\rho})}{S_t P_t^{-1} C_t^{-\rho}} \tag{23}$$

where $S_t = 1$. These first order conditions for nominal bonds state that the optimal demand for home and foreign currency denominated nominal bonds requires that the marginal rate of substitution between consumption purchased with one unit of each currency at different periods equal the price of each nominal bond.

The inter-temporal Euler equation for money is

$$1 - \frac{\chi P_t^{\varepsilon} C_t^{\rho}}{M_t^{\varepsilon}} = \beta \frac{E_t(P_{t+1}^{-1} C_{t+1}^{-\rho})}{P_t^{-1} C_t^{-\rho}} . \tag{24}$$

Combining (22) and (24) gives the money demand equation for the representative home agent as follows:

$$\left(\frac{M_t}{P_t}\right)^{\varepsilon}=\frac{\chi C_t^{\rho}}{1-Q_t} \tag{25}$$

First order conditions for the representative foreign agent are as follows:

$$Q_t=\beta\frac{E_t(S_{t+1}^{-1}P_{t+1}^{*-1}C_{t+1}^{*-\rho})}{S_t^{-1}P_t^{*-1}C_t^{*-\rho}} \tag{26}$$

$$Q_t^*=\beta\frac{E_t(P_{t+1}^{*-1}C_{t+1}^{*-\rho})}{P_t^{*-1}C_t^{*-\rho}} \tag{27}$$

$$\left(\frac{M_t^*}{P_t^*}\right)^{\varepsilon}=\frac{\chi C_t^{*\rho}}{1-Q_t^*} \tag{28}$$

## Short Run Inflexible Prices in Goods Markets

In our model, each agent in home and foreign countries is a price setter in the sense that, in the current period, they choose their future price to maximize their future profit, based on their expectation of the demand and supply of their products in the future markets. The monopoly pricing decision of the representative producer in home country is given as follows. $P_{H,t-1}(v)$ is set to maximize his lifetime expected utility function (1) given the information at time $t-1$ and inter-temporal budget constraint (20). Pricing rules for $P_{H,t-1}(v)$ and $P_{F,t-1}^*(v)$ in the initial symmetric equilibrium where $P_{H,t-1}(v)=P_{H,t-1}$ and $P_{F,t-1}(v)=P_{F,t-1}$ will be

$$P_{H,t-1}(v)=P_{H,t-1}=\left(\frac{\theta}{\theta-1}\right)\frac{E\{\eta Y_t(v)\}}{E\left\{\frac{Y_t(v)}{P_tC_t^{\rho}}\right\}} \tag{29}$$

$$P_{F,t-1}^*(v)=P_{F,t-1}^*=\left(\frac{\theta}{\theta-1}\right)\frac{E\{\eta^* Y_t^*(v)\}}{E\left\{\frac{Y_t^*(v)}{P_t^*C_t^{*\rho}}\right\}}. \tag{30}$$

The higher the degree of elasticity of substitution between goods produced in the same countries, $\theta$, the smaller the difference between preset prices and marginal costs, the markup, is. The increase in the expectations of the adverse output shocks, $\eta$, and $\eta^*$, raises the preset price of goods produced in home and foreign countries because the output from the same amount of labor in the next period is expected to decrease. Producers increase their preset price if the demand for their products or the general price level is expected to be high in the next period. Producers will be reluctant to change their preset prices with respect to unexpected shocks in the current period, if preset prices in the previous period are high enough to compensate the loss from the changes in the marginal values and the marginal costs. Instead, monopolistic producers react to unexpected current shocks by changing output, or the labor supply, while leaving price unchanged.[6]

## Ⅲ. RELATIVE CONSUMPTION AND REAL EXCHANGE RATES

The result of Cole and Obstfeld [1991]'s implies that if the preferences of agents across countries are identical and isoelastic, agents' international consumption risk sharing would be complete even without financial transactions because country specific shocks change agents' cost of living across countries by the same extent so that agents' relative cost of living and relative consumption across countries would not change. If agents have different compositions of goods in their consumption basket, however, idiosyncratic shocks differentiate the cost of living of agents across countries when the initial cost of living across countries is assumed to be symmetric. If financial markets are incomplete with only nominal bonds, consumption across agents would be ensured until the marginal utility of consumption per one unit of each currency is the same across countries. Agents will have to change their consumption level in accordance with the changes in the cost of living so that complete consumption risk sharing across countries may not be possible.

From the first order conditions for bond holdings, (22), (23), (26), and (27), equating two equations (22) and (26), and (23) and (27) respectively since the price of one-period nominal bonds denominated in each currency is the same across countries, gives the following relationships.[7]

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[6] The theory tells that, under constant or increasing returns to scale, and iso-elastic demand, unexpected multiplicative shifts in productivity, $\eta$ and $\eta^*$, do not change profit-maximizing monopoly prices.

[7] In the model of intertemporal utility maximization, agents' optimal intertemporal consumption

$$Q_t = \beta\frac{E_t(P_{t+1}^{-1}C_{t+1}^{-\rho})}{P_t^{-1}C_t^{-\rho}} = \beta\frac{E_t(S_{t+1}^{-1}P_{t+1}^{*-1}C_{t+1}^{*-\rho})}{S_t^{-1}P_t^{*-1}C_t^{*-\rho}}; \tag{31}$$

$$Q_t^* = \beta\frac{E_t(S_{t+1}P_{t+1}^{-1}C_{t+1}^{-\rho})}{S_tP_t^{-1}C_t^{-\rho}} = \beta\frac{E_t(P_{t+1}^{*-1}C_{t+1}^{*-\rho})}{P_t^{*-1}C_t^{*-\rho}} \tag{32}$$

These equations tell us that risk-averse agents across countries demand each nominal bond until the marginal rate of substitution between consumption purchased with one unit of each currency at different periods across home and foreign countries are the same.

Assuming that both countries are initially in the symmetric situation where $P_t^{-1}C_t^{-\rho} = (P_t^*)^{-\rho}(C_t^*)^{-\rho}$ gives the following expression.

$$\frac{C_t^{-\rho}}{P_t} = \frac{(C_t^*)^{-\rho}}{S_tP_t^*} \qquad \forall t \tag{33}$$

This optimality condition says that, in each period, consumption across countries would be ensured until the marginal utility of consumption per one unit of each currency is the same across countries.

To see the effect of real exchange rate changes on relative consumption, rearrange the equation (33) as follows:

$$\left(\frac{C_t}{C_t^*}\right)^{\rho} = \frac{S_tP_t^*}{P_t} \qquad \forall t \tag{34}$$

The presence of asset markets for nominal one-year bonds in different currencies ensures that the ratio of the marginal utility of consumption in home and foreign countries equals the real exchange rate, implying that relative consumption between countries varies with respect to the changes in the real exchange rate. This condition is called 'consumption risk sharing condition'.

The real exchange rate can be defined as relative cost of living across

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allocations of home and foreign produced goods, and the demand for futures contracts in different currencies are determined by the structural parameters such as the degree of risk aversion ( $\rho$), the shares of home and foreign produced goods ( $\gamma$), the elasticity of substitution among goods produced in the same country ( $\theta$), the elasticity of substitution between home and foreign produced goods in the consumption basket ( $\sigma = 1$), the economic size ( $n$), and the elasticity of money demand with respect to consumption ( $\varepsilon$).

countries as follows. Substituting home and foreign price indexes (5) into the expression of the real exchange rate gives

$$\frac{S_t P_t^*}{P_t} = \frac{S_t[(P_{H,t}/S_t)^{1-\gamma}(P_{F,t}^*)^{\gamma}]}{[(P_{H,t})^{\gamma}(S_t P_{F,t}^*)^{1-\gamma}]} = \left(\frac{S_t P_{F,t}^*}{P_{H,t}}\right)^{2\gamma-1}. \tag{35}$$

In equation (35), the real exchange rate depends on the share of home and foreign produced goods in the consumption basket and the terms of trade (consisting of the spot exchange rate and the prices of goods produced in home and foreign countries). Since, in our model, it's assumed that agents across countries have identically asymmetric preferences on home and foreign produced goods for computational convenience, the real exchange rate is expressed as the terms of trade to the $(2\gamma-1)$ power. If the share of domestically produced goods in the consumption index is different across countries, the power will be $(\gamma+\gamma^*-1)$. If all agents across countries have an identical preference, the purchasing power parity always holds, or the real exchange rate between countries does not change with respect to the changes in the terms of trade. If the elasticity of substitution between home and foreign produced goods ( $\sigma$) is not assumed to be one, the larger is the cross-country elasticity of substitution, the smaller the changes in the real exchange rate are. We also found that the greater is the share of home-produced goods in the consumption basket, the larger the changes in the real exchange rate are. Our finding is consistent with Hau (2000)'s.

Next, to see the change in relative consumption, combining (34) with (35) gives

$$\frac{C_t}{C_t^*} = \left(\frac{S_t P_{Ft}^*}{P_{Ht}}\right)^{\frac{2\gamma-1}{\rho}} \quad \forall t. \tag{36}$$

Relative consumption is affected not only by the change in the terms of trade but also by the parameters of the preference structure such as the degree of openness ( $\gamma$), the degree of risk aversion ( $\rho$) and the cross-country elasticity of substitution ( $\sigma$), which is assumed one. The change in the terms of trade affects consumption by two channels; substitution and income effects. Suppose in the current period the prices of home goods increase due to an expected adverse output shock. The current increase in the prices of home produced goods induces home agents to switch to the relatively cheap foreign goods. On the other hand, the increase in the prices of home goods reduces home agents' real income by increasing their cost of living. If the consumption basket is home-biased ( $\gamma > \frac{1}{2}$ ), the positive substitution effect of the increase in the relative price of home goods is smaller than the negative income effect so that the cost of living of home agents rises more than the cost of living of foreign agents. This increase in relative cost of living reduces relative consumption of a home agent.

The unitary relative risk aversion or a linear quadratic problem produces the certainty equivalent solution that would be obtained if there was no uncertainty, or if agents held the expectations of income with subjective certainty. Grossman and Shiller (1981) show that agents with the larger coefficient of relative risk aversion than unity try to reduce the variability of consumption relative to the variability of asset returns. If agents' degree of relative risk aversion is greater than one, agents respond to permanent shocks by less than one for one by incorporating risk premium for the uncertainty of the lifetime borrowing constraint. This proposition is called "Precautionary Savings" or "Excess Smoothness" of consumption and developed by Leland (1968), Turnovsky (1971), Kimball (1990), and Deaton (1991).[8]

As mentioned earlier, the elasticity of substitution between home and foreign produced goods, $\sigma$, is assumed to be lower than the elasticity of substitution between products produced in the same country, $\theta$ in general. In our model, the elasticity of substitution between goods produced in different countries is assumed to be one, $\sigma = 1$, while the elasticity of substitution between products produced in the same country is assumed to be greater than one, $\theta > 1$. If goods produced in home and foreign countries are more substitutable, the decrease in home relative consumption would be smaller with respect to the increase in the price of home produced goods because the increase in the price of home produced goods would make home agents switch to relatively cheap foreign produced goods easily to reduce the increase in the cost of living and the decrease in consumption.[9]

[8] Liu (2002) shows the presence of aggregate uncertainty can bring 'precautionary savings' for any concave utility function of agents.

[9] For the effect of the cross-country elasticity of substitution, see Svensson and van Wijnbergen (1989), van der Ploeg (1993), Backus, Kehoe, and Kydland (1995), Tille (1999), and Corsetti and Pesenti (2001).

## Ⅳ. TRADE BALANCE AND REAL EXCHANGE RATES

The effect of terms-of-trade shocks on saving and current account was first studied by Harberger (1950) and Laursen and Metzler (1950), using single-good, Keynesian open-economy models without international capital mobility barriers. They argue that a fall in real income resulted from an adverse terms-of-trade shock would lower savings and current account. Intertemporal utility optimization approach to the determination of savings and current account was taken by Obstfeld (1982) and Svensson and Razin (1983). Obstfeld (1982) finds that intertemporal utility maximizing agents increase their savings and current account if the rate of time preference is increasing in utility, which is the reversed Harberger, Laursen and Metzler effect. Svensson and Razin [1983] show that the result of Obstfeld (1982)'s is one special case when the rate of time preference is increasing to the wealth and that the effect of a terms of trade deterioration on current account would have any sign if no restrictions on the rate of time preference are imposed. Backus, Kehoe, and Kydland (1995) researched on the effect of the relation between the intertemporal and intra-temporal elasticity of substitution on current account balance in an intertemporal optimization model without investment. They show that relative consumption of home agents falls by less with respect to an adverse terms-of-trade shock by running current account deficit if intra-temporal elasticity of substitution is greater than intertemporal elasticity of substitution.[10]

In our model, we focus on the role of the degree of risk aversion and the share of home-produced goods in the consumption basket in the households' CES utility function in determining the current account. When agents' relative risk aversion is not equal to unity and they have heterogeneous consumption baskets in a world of incomplete financial markets,[11] trade balance may worsen or improve depending on the values of the degree of risk aversion and the degree of openness measured by the share of imports in the consumption basket. If agents' consumption baskets are home-biased, the increase in the prices of

[10] The inter-temporal elasticity of substitution is interpreted as the inverse of the coefficient of risk aversion. They also documented that the home-biased preference has the effect of dampening fluctuations in trade balance.

[11] In complete financial markets, people hold assets sharing all the consumption risk so that every individual in the world has the same marginal rate of substitution of present for future consumption across all states of nature. The fact that consumption risk is completely insured implies that trade account is always balanced.

home-produced goods reduces home agents' relative consumption. Agents with the degree of relative risk aversion greater than unity would like to smooth relative consumption over time by reducing exports and increasing imports, worsening trade balance.

From our model economy, trade balance can be expressed as follows:

$$Trade\ balance = Exports - Imports = S_t P^*_{Ht} C^*_{Ht} - P_{Ft} C_{Ft} \tag{37}$$

From the demand equations for representative home and foreign products, (12) and (13),

$$Exports = S_t P^*_{Ht} C^*_{Ht} = (1-\gamma) S_t P^*_t C^*_t, \quad Imports = P_{Ft} C_{Ft} = (1-\gamma) P_t C_t.$$

Substituting these expressions into (37) gives

$$Trade\ Balance = S_t P^*_{Ht} C^*_{Ht} - P_{Ft} C_{Ft} = (1-\gamma)[S_t P^*_t C^*_t - P_t C_t]. \tag{38}$$

According to equation (38), $(1-\gamma)$ portion of foreign demand is exported while $(1-\gamma)$ portion of home demand is imported. Thus, trade balance is $(1-\gamma)$ portion of the difference between home and foreign demand. Using the demand functions for representative home and foreign products, $C^*_{Ht}$ and $C^*_{Ft}$, evaluating them at the initial symmetric equilibrium where $P_{Ht}(v) = P_{Ht}$ and $P_{Ft}(v) = P_{Ft}$, and combining the consumption risk sharing condition produces the following expression for trade balance as a fraction of GDP:

$$\frac{Trade\ Balance}{GDP} = 1 - \frac{1}{\left[\gamma + (1-\gamma)\left(\frac{SP^*}{P}\right)^{\left(\frac{\rho-1}{\rho}\right)}\right]} \tag{39}$$

where $P = (P_H)^{\gamma}(SP^*_F)^{1-\gamma} = Home\ Consumption\ Price\ Index$

$P^* = (P_H/S)^{1-\gamma}(P^*_F)^{\gamma} = Foreign\ Consumption\ Price\ Index$

Trade balance as a fraction of GDP in equation (39) is a function of the real exchange rate.[12] If the degree of relative risk aversion of agents is greater than

[12] The real exchange rate is the relative cost of living between countries.

unity, $\rho>1$, the increase in the relative cost of living of home agents worsens trade balance to smooth relative consumption over time. The greater is the degree of relative risk aversion, the more do agents try to smooth relative consumption. As a result, the fluctuations in trade balance also become greater. If agents have a log utility function, that is, $\rho=1$, agents do not care about the changes in their cost of living and relative consumption. In this case, trade would be always balanced because agents would adjust their consumption in order to balance the trade account.

Combining (39) with (36) gives the following expression for trade balance as a function of the terms of trade.[13] The increase in the price of exports worsens the terms of trade while the decrease in the price of exports improves it:

$$\frac{Trade\ Balance}{GDP}=1-\frac{1}{\left[\gamma+(1-\gamma)\left(\frac{SP_F^*}{P_H}\right)^{(2\gamma-1)(\frac{\rho-1}{\rho})}\right]} \tag{40}$$

where $SP_F^*/P_H= Terms\ of\ Trade$

In our model, we assume agents' preferences across countries are identically asymmetrical for computational convenience: the share of imported goods in the consumption basket is the same across countries. Therefore, when the share of imported goods in the consumption basket is half, that is, $\gamma=\frac{1}{2}$, the consumption baskets across countries become identical. When agents across countries have an identical preference, the change in the terms of trade does not have an influence on the relative cost of living and relative consumption. In this case, trade will be always balanced as well.

According to equation (40), trade balance will be zero in the case of $\gamma=\frac{1}{2}$ or $\rho=1$. If $\rho>1$ and $\gamma>\frac{1}{2}$, the worsening of the terms of trade appreciates the real exchange rate and deteriorates trade balance while the improvement of the terms of trade depreciates the real exchange rate and improves trade balance.[14]

[13] The terms of trade is defined as the relative price of imports to exports.

[14] See Obstfeld [1986] for this absorption approach.

## V. REAL EXCHANGE RATES AND COUNTRY-SPECIFIC OUTPUT AND MONETARY SHOCKS

The real exchange rate is relative cost of living across countries. To see the effects of expected output and monetary shocks on the real exchange rate, log-linearize the real exchange rate as a function of the terms of trade:[15]

$$s+p^{*}-p=(2\gamma-1)(s+p_{F}^{*}-p_{H}) \tag{41}$$

To see the determination of the spot exchange rate in the foreign exchange market, suppose that a monetary shock($\mu_t$) or revisions in the current period goods' prices due to the expected output shock($\xi_t$) occurred only in the current period and that the growth of output and money supply in both home and foreign countries follow a random walk:

$$\log\eta_t=\log\eta_{t-1}+\xi_t;\ \log\eta_t^{*}=\log\eta_{t-1}^{*}+\xi_t^{*};\ m_t=m_{t-1}+\mu_t;\ m_t^{*}=m_{t-1}^{*}+\mu_t^{*}$$

where $m_t=\log M_t$, $m_t^{*}=\log M_t^{*}$, and $\xi_t$, $\xi_t^{*}\sim N(0,\sigma_\xi^2)$, and $\mu_t$, $\mu_t^{*}\sim N(0,\sigma_\mu^2)$ for every date $t$.[16] Using the money demand equations for the home and foreign countries, (25), and (28), log-linearlized at a non-stochastic steady state where $i=i^{*}=\bar{i}$,[17] consumption price indexes of home and foreign countries, and the consumption risk sharing condition, the equilibrium spot exchange rate can be solved for as follows:

$$s_t=\left(\frac{\varepsilon}{(2\gamma-1)+2\varepsilon(1-\gamma)}\right)\{\mu_t-\mu_t^{*}\}-\left(\frac{(\varepsilon-1)(2\gamma-1)}{\varepsilon}\right)(\xi_t-\xi_t^{*}) \tag{42}$$

Now, the terms of trade as a function of country-specific shocks can be derived by combining (29), (30) and (42):

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[15] By solving for the real exchange rate as a function of exogenous money and expected output shocks, the endogeneity problem raised by Krugman [1978] and Obstfeld (1995) can be avoided.

[16] Since the distributions from which output and money shocks redrawn are assumed to be time-invariant and lognormal, all of the variances and co-variances in the model will be constant over time.

[17] Or equivalently $Q=Q^{*}$ where $Q$ and $Q^{*}$ are respectively the prices of nominal bonds denominated in home and foreign currencies.

$$s_t + p_F^* - p_H = \left(\frac{\varepsilon}{(2\gamma-1)+2\varepsilon(1-\gamma)}\right)\{\mu_t - \mu_t^*\} - (\xi_t - \xi_t^*) \tag{43}$$

Combining (42) and (43) gives the following expression for the real exchange rate:

$$s + p^* - p = (2\gamma-1)\left\{\left(\frac{\varepsilon}{(2\gamma-1)+2\varepsilon(1-\gamma)}\right)\{\mu_t - \mu_t^*\} - (\xi_t - \xi_t^*)\right\} \tag{44}$$

According to equation (44), the increase in relative home money supply depreciates the real exchange rate while the adverse home output shock appreciates it. As Stockman (1983, 1990), Stockman and Dellas (1989), and Stockman and Tesar (1995) argue, as trading economies become more integrated in terms of preferences, the deviations from purchasing power parity decrease.

## Ⅵ. CONCLUDING REMARKS

In our paper, we construct an open economy macroeconomic model that shows that the changes in the relative prices of home-produced goods resulted from country-specific shocks influence relative consumption and trade balance through the changes in real exchange rate in the setting of intertemporal utility maximization. If the consumption baskets across countries are home-biased, an increase in the relative prices of home produced goods would raise home country's cost of living more than foreign country's, and then reduce home relative consumption. Intertemporal utility maximizing agents with the degree of relative risk aversion greater than unity would like to smooth their relative consumption over time. By reducing exports and increasing imports, home agents are able to increase their consumption, which leads to trade deficit in the current period. The magnitude of real exchange rate fluctuations, the degree of consumption synchronization and the magnitude of trade balance deviations critically depend on the parameters of households' CES utility function such as the degree of relative risk aversion and the share of home-produced goods in the households' consumption baskets.

In particular, we would like to emphasize that country specific shocks are channeled to other trading countries through the changes in the cost of living. We find that the main determinant of transmission effects of the idiosyncratic shock is the degree of openness of trading countries that is measured by the similarity of their consumption baskets. This implies that the similar consumption baskets across trading countries may result in quite the same degree of changes in trading economies' cost of living due to the idiosyncratic shocks, which leads to more synchronized consumption baskets. These outcomes are consistent with those of Stulz (1987), Stockman and Dellas (1989), Backus and Smith [1993], Tesar (1993), Stockman and Tesar (1995) and Hau (2000) who argue that the presence of non-traded goods or home biased preference may account for large and persistent deviations from the purchasing power parity and for smaller correlations of consumption fluctuations and large deviations of trade balance across countries.

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