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Perks in Long-term Contracts*

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Perks are a commodity bundle offered by an employer to an employee. We provide two dynamic models. First, we assume non-separable utility function between effort and both of a perk good and money, extending Bennardo, Chiappori and Song (2010). There are two forces affecting the incentive compatibility constraint: higher promised utility makes the incentive compatibility constraint more binding, and if the higher promised utility is too costly then a principal may reduce the implemented effort. When the first effect is stronger than the second, the principal gives more perk good as successful outcomes accumulate. In the second model, an agent can save money privately (i.e. hidden saving), but not a perk good. Increasing monetary payment today makes it more difficult to satisfy the today's hidden saving constraint, but makes it easier to satisfy the yesterday's hidden saving constraint. When the second effect is larger than the first, the principal gives more perk as successful outcomes accumulate.

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I. Introduction

Why do perks exist? Why not just pay an employee in cash and let the employee purchase these products? A quick answer would be that many such perks are products that an employee seems unlikely to purchase, even if he is given the money.

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In other words, perks are usually luxurious. This begs the question: "Why does an employer want her employees to consume these luxuries?" Our focus is especially on the optimal perks in dynamic contracts.

Some economists argue that perks lead to a moral hazard problem: an employer cannot monitor whether an employee abuses them or not. In this view, perks are "non-productive goods." Other economists, following Alchian and Demsetz (1972), consider perks as a consequence of a moral hazard problem. When the members of a profit-sharing firm have to purchase input factors personally, there is an under-investment problem (or, equivalently, a free-rider problem) since each does not fully appropriate the profit from these investments. If the problem is severe, it could be efficient to give the input factor as a perk, in spite of the possible abuse. This second view considers perks as "productive goods."[1]

Both of these views share the idea that an employer cannot observe the use of perks. However, many expensive perks can be monitored easily. For example, it is not difficult to check whether a private jet is used for business or for personal reasons. The cost of monitoring the use of a private jet will be insignificant compared with the cost of flying it. It is even a legal requirement to report such expensive perks to the public.[2] If the use of perks is observable, it is explicitly contractible. Thus, we treat perks as a contingent payment, especially, in a repeated contractual relationship.

We do not assume that perks provide intrinsic motivation (no consumption complementarities between perks and effort), nor do we assume that perks have a productive use, as in most of the literature. Those assumptions automatically justify the existence of perks. However, there are many perks that do not seem to help production or reduce an agent's cost of effort (e.g, luxurious corporate retreats may be useful for "team building," but more inexpensive destinations are available for the same purpose).[3]

We also do not assume that a principal and an agent can save on tax by having perks. Hypothetically, a principal could report perks as a cost of production, get a tax deduction, and thus provide the perks at a lower cost than the agent would pay privately. However, the tax advantage explanation fails to explain why we do not often see perks in lower paid jobs. For example, there is typically no commuting

[1] Yermack (2006) uses this term for the consumption of non-productive goods and services. Jensen and Meckling (1976) and Rajan and Wulf (2006) distinguish productive and non-productive perks. Marino and Zábojník (2006) mainly consider perks as the consumption of productive goods. Oyer (2007) considers perks that have complementarities with effort and production.

[2] For example, new SEC rules since 2006 require public companies to list all perks over $10,000. For top rankers in receiving perks, see

http://www.bayareanewsgroup.com/multimedia/mn/biz/specialreport/wtbm ceosorts.pdf

http://www.bayareanewsgroup.com/multimedia/mn/biz/specialreport/wtbm underceosorts.pdf

[3] Other examples include fancy company cars, a "training program" on a Mediterranean island, a car service home in a Lincoln town car, and a lavish corporate holiday party.

subsidy for general office workers, while executives often receive corporate cars with a chauffeur. Furthermore, many perks are now fully subject to tax.[4]

Bennardo, Chiappori and Song (2010) assume asymmetry in utility function where the cross derivatives between effort and monetary income and between effort and a perk good are different. They find that the principal gives more perks (in the sense that the wedge between the marginal rate of substitution and the price ratio is larger) when the incentive problem is more severe. However, it is not possible to ask questions on how dynamically optimal perks evolve depending on the past history in their framework, since they lack timing in the model. For example, the media and the general public will be interested if we can justify the perks that high profile CEOs receive, i.e., does successful past history justify luxurious perks? To answer the question, we propose two dynamic models, in which we use the method of accumulated multipliers (following Marcet, 2008; Chien and Lustig, 2009; Mele, 2009) to analyze the models. We can summarize our technical contribution as the application of the method to the dynamic problems having the hidden saving and moral hazard problems.

In our first model extending Bennard, Chiappori and Song (2010) to a dynamic setting, we show that a binding incentive compatibility constraint requires the agent's marginal rate of substitution to differ from the price ratio (in the presence of the aforementioned asymmetry in utility function). More specifically, we show that the higher shadow value of the incentive compatibility constraint requires a larger discrepancy between the marginal rate of substitution and the price ratio.

Then we characterize two effects on how strongly the incentive compatibility constraint binds, i.e., how much of the perk good the principal provides. In the first place, the principal has to promise higher and higher utility to the agent as success outcomes accumulate. This, in turn, makes the next period's incentive compatibility constraint harder to satisfy. This first effect will make the incentive compatibility constraint more binding as successful outcomes accumulate. However, if a more binding incentive compatibility constraint imposes too much (shadow) cost, the principal might lower the implemented effort in ongoing periods. This second effect will make the incentive compatibility constraint less binding.

We provide two simulations to illustrate these two effects. We show that if the second effect is larger than the first, the perks decreases as successful outcomes accumulate. We also show that the first effect can dominate the second effect if the agent's task is dependent on how costly the moral hazard problem was (measured by the shadow value of the incentive compatibility constraint) and how crucial the agent's effort for the outcome was (measured by the marginal percentage increase of

[4] Since many perks are listed to the public, they could be taxed. For example, Meg Whitman (eBay) was invited to use corporate planes for up to 200 hours of personal travel annually. That added up to more than $773,000, plus nearly $231,000 more to cover her tax bills for the perk.

outcome probability in effort). Note that we do not characterize the conditions when the first effect dominates the second. We only show the possibility of ever increasing perks by our simulations.

In our second model extending Chien and Song (2013) to a dynamic setting (but without the double deviation of effort and saving), we do not assume the asymmetry of utility function. Instead, we assume asymmetry in storage technology for the perk good and money. The agent can save money for the purpose of consumption smoothing against uncertainty in the future, but he cannot save perks. For example, a CEO owning the right to use a private jet for personal use cannot make a saving account for the use of the private jet, while the CEO can make a saving account for wage income. An agent's ability of the hidden saving is known to decrease efficiency in many contexts (Abraham and Panovi, 2008, 2009; Kocherlakota, 2004[5]). As the monetary payment increases due to success, marginal utility in money decreases. Thus the principal's (shadow) cost for the agent's hidden saving becomes more expensive as successful outcomes accumulate. Since the saving of perks is not allowed, there is no shadow cost for the hidden saving of the perk good. Therefore, providing the perk good could circumvent efficiency loss due to the hidden saving. However, the decrease of the today's monetary payment (compared to the increase of the perks) could make the yesterday's hidden saving constraint more binding. These two effects on the today's and yesterday's hidden saving constraints determine whether the principal gives perks and how much perks she gives. When the first effect is larger than the second, the principal gives more perks as successful outcomes accumulate. Our second model shows that perks can arise due to different saving technologies for different commodities, even with a symmetric utility function.

We provide simulations to illustrate these findings. These simulations also show that perks can glean the efficiency loss due to the hidden saving. Again note that we do not characterize the conditions when the first effect dominates the second. We only show the possibility of ever increasing perks in the presence of the hidden saving problem with symmetric utility function.

To our best knowledge, our paper is the first one to apply the method of accumulated multipliers to dynamic problems having a moral hazard problem (and the hidden saving problem).

Scholars have studied high profile CEO compensation empirically (e.g., Yermack, 1995; Kole, 1997; Jensen and Murphy, 1990; Murphy, 1999), in a dynamic framework (e.g., Wang, 1997; Hopenhayn and Jarque, 2007), and in Matching

[5] He considers the problem of optimal unemployment insurance where the unemployed agent's job-search effort is unobservable and his level of saving is unobservable. He finds that the agent's consumption is constant while he is unemployed, and jumps up to a higher constant and history-independent level of consumption when he finds a job. Our model is not the same to his in the specification of the moral hazard problem, and the efficiency loss is not as extreme as Kocherlakota's.

framework (Gabaix and Landier, 2008; Edmans, Gabaix, and Landier, 2007). The main interest was why they are paid so much. However, there are only limited studies on perks (Marino and Zabojnik, 2006; Oyer, 2007; Rajan and Wulf, 2006; Yermack, 2006; Bennardo, Chiappori and Song, 2010). Moreover, there was no study on perks in a dynamic setting as far as we know.

We presents the first model in Section 2 and the second model in Section 3. We conclude in Section 4.

II. First Model

There are one principal and one agent. Their contractual relationship lasts for T periods. T could be infinity. There are two kinds of commodities: a perk good in period t denoted by $c_t \in \mathbb{R}$, and a *numeraire* good in period t denoted by $m_t \in \mathbb{R}$ (say *money*). The price of the perk good is p in every period, and the price of money is normalized to be unity. The agent makes effort $e_t \in \varepsilon \subset \mathbb{R}$ in each period t. This effort determines the distribution function of the next period's output, $\pi(s_{t+1} \mid e_t)$, where $s_t \in S \subset \mathbb{R}$ is the outcome with the exception that the support of s_1 is singleton. We denote the history of outcomes by $s^t = (s_2, s_3, \ldots, s_t)$. There is unverifiability of effort $e_t \in \varepsilon$ with the exception that the support of e_T is singleton; thus, there is moral hazard problem in each period t. Output at each history s^t is denoted by $y(s^t)$, which is potentially dependent upon the past outcomes. Consumption of the perk good and money in period t is a function of outcome history s^t, denoted by $c_t(s^t)$ and $m_t(s^t)$, or simply $c(s^t)$ and $m(s^t)$. The implemented effort is also a function of s^t, $e_t(s^t)$. The history of effort is denoted by $e^t(s^t) = (e_1(s^1), e_2(s^2), \ldots, e_t(s^t))$. The agent's temporal utility in period t is $u(c(s^t), e_t) + v(m(s^t), e_t)$. Note that we are assuming separability between c_t and m_t to have a more clear-cut illustration. Given the consumption schedule $(c(s^t), m(s^t))_{t\geq 1}$ and effort schedule $(e_t(s^t))_{t\geq 1}$, the expected utility is:

$$\sum_{t=1}^{T}\sum_{s^t}\beta^{t-1}[u(c(s^t),e_t(s^t))+v(m(s^t),e_t(s^t))]\,\Pi(s^t \mid e_{t-1}(s^{t-1})),$$

where

$$\Pi(s^t \mid e_{t-1}(s^{t-1})) = \prod_{\tau=1}^{t}\pi(s_\tau \mid e_{\tau-1}(s^{\tau-1})).$$

We assume that the agent cannot opt out of a contract once the contract starts. Thus there is only one individual rationality constraint, which is

$$\sum_{t=1}^{T-t}\sum_{s^t}\beta^{t-1}[u(c(s^t),e_t(s^t))+v(m(s^t),e_t(s^t))]\,\Pi(s^t \mid e_{t-1}(s^{t-1})) \geq U_0. \tag{1}$$

We denote the multiplier of the individual rationality constraint by ρ.

We adopt first-order approach, so the incentive compatibility constraint in period t is

$$-u_e(c(s^t),e_t(s^t))-v_e(m(s^t),e_t(s^t))$$
$$=\sum_{j=1}^{T-t}\beta^j\sum_{s^{t+j}}\frac{\pi_e(s_{t+1}\mid e_t(s^t))}{\pi(s_{t+1}\mid e_t(s^t))}\begin{bmatrix}u(c(s^{t+j}),e_{t+j}(s^{t+j}))\\+v(m(s^{t+j}),e_{t+j}(s^{t+j}))\end{bmatrix}\Pi(s^{t+j}\mid s^t,e_{t+j-1}(s^{t+j-1}))\quad(2)$$

We denote the multiplier of each incentive compatibility constraint by $\gamma(s^t)$.

The principal's problem is:

$$\max_{e_t(\bullet),c(\bullet),m(\bullet)}\sum_{t=1}^{T}\sum_{s^t}\left(\frac{1}{1+r}\right)^{t-1}[y(s^t)-pc(s^t)-m(s^t)]\Pi(s^t\mid e_{t-1}(s^{t-1}))$$

s.t. (1) and (2). (3)

Lagrangean is:

$$\begin{aligned}L=&\sum_{t=1}^{T}\sum_{s^t}\left(\frac{1}{1+r}\right)^{t-1}[y(s^t)-pc(s^t)-m(s^t)]\,\Pi(s^t\mid e_{t-1}(s^{t-1}))\\&+\rho\left[\sum_{t=1}^{T}\sum_{s^t}\beta^{t-1}[u(c(s^t),e_t(s^t))+v(m(s^t),e_t(s^t))]\,\Pi(s^t\mid e_{t-1}(s^{t-1}))-U_0\right]\\&+\sum_{t=1}^{T}\sum_{s^t}\beta^{t-1}\gamma(s^t)\begin{bmatrix}u_e(c(s^t),e_t(s^t))+v_a(m(s^t),e_t(s^t))\\+\sum_{j=1}^{T-t}\beta^j\sum_{s^{t+j}}\frac{\pi_e(s_{t+1}|e_t(s^t))}{\pi(s_{t+1}|e_t(s^t))}\\\times\begin{pmatrix}u(c(s^{t+j}),e_{t+j}(s^{t+j}))\\+v(m(s^{t+j}),e_{t+j}(s^{t+j}))\end{pmatrix}\\\times\Pi(s^{t+j}\mid s^t,e_{t+j-1}(s^{t+j-1}))\end{bmatrix}\Pi(s^t\mid e_{t-1}(s^{t-1})).\end{aligned}\quad(4)$$

The first order conditions with respect to $c(s^t)$ and $m(s^t)$ are:

$$\left(\frac{1}{(1+r)\beta}\right)^{t-1}\frac{p}{u_c(c(s^t),e_t(s^t))}$$
$$=\rho+\gamma(s^{t-1})\frac{\pi_e(s_t\mid e_{t-1}(s^{t-1}))}{\pi(s_t\mid e_{t-1}(s^{t-1}))}+\gamma(s^t)\frac{u_{ec}(c(s^t),e_t(s^t))}{u_c(c(s^t),e_t(s^t))},$$

$$\left(\frac{1}{(1+r)\beta}\right)^{t-1}\frac{1}{v_m(m(s^t),e_t(s^t))}$$

$$= \rho + \gamma(s^{t-1}) \frac{\pi_e(s^t \mid e_{t-1}(s^{t-1}))}{\pi(s^t \mid e_{t-1}(s^{t-1}))} + \gamma(s^t) \frac{v_{em}(m(s^t), e_t(s^t))}{v_m(m(s^t), e_t(s^t))}.$$

The difference of the two equalities is

$$\left(\frac{1}{(1+r)\beta}\right)^{t-1} \left[\frac{p}{u_c(c(s^t), e_t(s^t))} - \frac{1}{v_m(m(s^t), e_t(s^t))}\right]$$
$$= \gamma(s^t) \left[\frac{u_{ec}(c(s^t), e_t(s^t))}{u_c(c(s^t), e_t(s^t))} + \frac{v_{em}(m(s^t), e_t(s^t))}{v_m(m(s^t), e_t(s^t))}\right].$$

If there were no moral hazard problem, $\gamma(s^t) \equiv 0$. Then $\frac{v_m(m(s^t), e_t(s^t))}{u_c(c(s^t), e_t(s^t))} = \frac{1}{p}$, i.e., the marginal rate of substitution is equivalent to price ratio. However, we derive the following with non-zero $\gamma(s^t)$:

$$MRS_{mc} := \frac{v_m(m(s^t), e_t(s^t))}{u_c(c(s^t), e_t(s^t))} = \frac{1}{p} \times \frac{\left(\frac{1}{(1+r)\beta}\right)^{t-1} - \gamma(s^t) v_{em}(m(s^t), e_t(s^t))}{\left(\frac{1}{(1+r)\beta}\right)^{t-1} - \gamma(s^t) \frac{u_{ec}(c(s^t), e_t(s^t))}{p}} \tag{5}$$

Under the interpretation that effort is reciprocal of leisure, empirical evidence demonstrates the increasing marginal disutility of effort in money. See Grossman and Hart (1983), Blundell, Browning and Meghir (1991), and Bennardo and Chiappori (2003) for detailed discussion. We assume the following.

Assumption 1 *As effort e increases, per-dollar marginal utility gain in consumption m decreases faster than the one in consumption c, i.e.,*

$$v_{em}(m(s^t), e_t(s^t)) < \frac{u_{ec}(c(s^t), e_t(s^t)}{p} \leq 0 .$$

Under this assumption, we derive our first result directly from equation.

Proposition 1 *Under Assumption 1, the marginal rate of substitution with respect to money and perk good is larger than the price ratio* $\frac{1}{p}$*, i.e.,*

$$MRS_{mc} > \frac{1}{p}$$

Note that Assumption 1 is not so restrictive as it seems. It merely states that any two commodities have different effects on the marginal disutility of effort. Under Assumption 1, Proposition 1 implies that the principal wants the agent to consume more of the commodity that has less (per-dollar) effect on the marginal disutility of effort than the agent would freely choose to consume (i.e., the equivalence of marginal rate of substitution and price ratio). In other words, Assumption 1 holds for any two commodities, and Proposition 1 indicates which commodity should be supplied as a an excessive perk. We say the perk good is *luxurious* if $MRS_{mc} > 1/p$ from now on.

Further to see the departure of the marginal rate of substitution from the price ratio, assume that the cross derivatives of money and effort and the perk good and effort are constant.

Assumption 2 $v_{me} \equiv -\alpha < u_{ce} \equiv -\delta \leq 0$.

Assumption 2 is apparently a simplification of Assumption 1. We do assume this to see the first order effect only. For example, this is equivalent to considering the first terms in the taylor expansions of functions $u(\bullet)$ and $v(\bullet)$.

Under Assumption 2, we get the following from equation (5).

$$MRS_{mc} := \frac{v_m(m(s^t), e_t(s^t))}{u_c(c(s^t), e_t(s^t))} = \frac{1}{p} \times \frac{\left(\frac{1}{(1+r)\beta}\right)^{t-1} + \alpha\gamma(s^t)}{\left(\frac{1}{(1+r)\beta}\right)^{t-1} + \delta\gamma(s^t)} \tag{6}$$

Since $\alpha > \delta$, an immediate result from the above is that the difference between the marginal rate of substitution MRS_{mc} and price ratio $\frac{1}{p}$ becomes larger as $\gamma(s^t)$ increases and vice versa.

Proposition 2 *Assumption 2 implies that* $[MRS_{mc} - \frac{1}{p}]$ *increases* (*decreases*) *as* $\gamma(s^t)$ *increases* (*decreases*).

Proposition 2 (and Assumption 2) makes it possible to directly connect the shadow value of the incentive compatibility constraint to the departure of MRS from the price ratio. In other words, once we understand how the shadow value evolves as t increases and the uncertainty on state s^t is unfolded, we would understand how the marginal rate of substitution evolves. For tractable computational analysis, we further assume the following.

Assumption 3 $u_{ce} \equiv \delta = 0$ *and* $v_{me} \equiv -\alpha$.

Bennardo, Chiappori and Song (2010) study a static version of our model, and they report the same proposition. They also show that if $MRS_{mc}(am, ac) -$

$MRS_{mc}(m,c) \geq -k$ with positive and sufficiently small k for all $a > 1$ with Assumption 3, the larger gap between MRS_{mc} and $1/p$ implies the larger $c(s^t)/m(s^t)$. We adopt their assumption on MRS_{mc} throughout the paper, and focus on the divergence between MRS_{mc} and price ratio $1/p$. They also do a comparative statics on an agent's outside option, and conclude that the consumption of the perk good will increase as the outside option increases, assuming that the perk good is a small portion of the entire compensation. However, since they lack dynamic setting, the explanation on how the implemented effort level and the perk good interact is not illustrated. We provide an illustration in the following subsection.

2.1 Simulation

For simplicity, we assume $\frac{1}{1+r} = \beta$. We use recursive Lagrangean approach. Lagrangean (4) changes into:

$$L = \sum_{t=1}^{T} \sum_{s^t} \beta^{t-1} \begin{bmatrix} y_t(s^t) - m_t(s^t) - pc_t(s^t) \\ +\phi(s^t)[u(c(s^t), e_t(s^t)) + v(m(s^t), e_t(s^t))] \\ +\gamma(s^t)[u_e(c(s^t), e_t(s^t)) + v_e(m(s^t), e_t(s^t))] \end{bmatrix}$$

$$\Pi(s^t \mid e_{t-1}(s^{t-1})) - \rho U_0 . \tag{7}$$

where

$$\phi(s^{t+1}) = \rho + \sum_{i=0}^{t} \gamma(s^i) \frac{\pi_e(s_{i+1} \mid e_i(s^i))}{\pi(s_{i+1} \mid e_i(s^i))} = \phi(s^t) + \gamma(s^t) \frac{\pi_e(s_{t+1} \mid e_t(s^t))}{\pi(s_{t+1} \mid e_t(s^t))} \tag{8}$$

As was noted in Marcet (2008), Chien and Lustig (2009), and Mele (2009), $\phi(s^t)$ can be interpreted as *Pareto weight*. The first line in the bracket is the principal's profit, the second line is the agent's Pareto-weighted expected utility, and the third line is the incentive cost that the principal has to pay because of the moral hazard problem. Lastly, ρU_0 is principal's another cost due to the individual rationality constraint.

Under Assumption 3, the first order conditions are:

$$c(s^t): 1 = \phi(s^t) u_c(c(s^t), e_t(s^t)) \tag{9}$$

$$m(s^t): 1 = \phi(s^t) v_m(m(s^t), e_t(s^t)) + \gamma(s^t) v_{em}(m(s^t), e_t(s^t)) \tag{10}$$

$$e_t(s^t): 0 = \gamma(s^t) v_{ee}(m(s^t), e_t(s^t)) + \phi(s^t) v_e(m(s^t), e_t(s^t))$$

$$+\sum_{j=1}^{\infty} \beta^j \sum_{s^{t+j}} \frac{\pi_e(s_{t+1} \mid e_t(s^t))}{\pi(s_{t+1} \mid e_t(s^t))} \begin{bmatrix} y(s^{t+j}) - pc(s^{t+j}) - m(s^{t+j}) \\ +\phi(s^{t+j})[u(c(s^{t+j})) + v(m(s^{t+j}), e_{t+j}(s^{t+j}))] \\ +\gamma(s^{t+j}) v_e(m(s^{t+j}), e_{t+j}(s^{t+j})) \end{bmatrix}$$

$$\times\Pi(s^{t+j} \mid s^t, e_{t+j-1}(s^{t+j-1}))$$

$$+\beta\gamma(s^t)\sum_{s_{t+1}} \frac{\partial\left(\frac{\pi_e(s_{t+1}|e_t(s^t))}{\pi(s_{t+1}|e_t(s^t))}\right)}{\partial e_t}[u(c(s^{t+1}))+v(m(s^{t+1}),e_{t+1}(s^{t+1}))]\pi(s_{t+1} \mid e_t(s^t)) \quad (11)$$

where $\Pi(s^{t+j} \mid s^t, e_{t+j-1}(s^{t+j-1}))$ is the conditional probability of s^{t+j} for given history s^t.

The recursive formulation has two state variables (s,ϕ). We define

$$K(s,\phi) = [y - m - pc + \phi(u(c,e)+v(m,e)) + \gamma v_e(m,e)] \\ +\beta\sum_{s'} K(s',\phi')\pi(s' \mid e). \quad (12)$$

Then we can write the first order condition with respect to e into a recursive form:

$$0 = \gamma v_{ee} + \phi v_e + \beta\sum_{s'} \frac{\pi_e(s' \mid e)}{\pi(s' \mid e)} K(s',\phi')\pi(s' \mid e) \\ +\beta\sum_{s'} \frac{\partial\left(\frac{\pi_e(s'|e)}{\pi(s'|e)}\right)}{\partial e}[u(c(s'),e(s'))+v(m(s'),e(s'))]\pi(s' \mid e).$$

We need to solve the following five functions:

$$c(\phi), m(\phi), e(\phi), \gamma(\phi), K(s,\phi).$$

We have three FOCs, one incentive compatibility constraint, and equation (12). Thus we can solve the five functions with the law of motion for state variable ϕ, given in equation (8).

For simulation, we simplify the environment even further. There are two possible outcomes: we denote success by 1 and failure by 0, so $S = \{0,1\}$. For given history s^t, $s^{t+1} = (s^t, s_{t+1})$ denotes success in period $t+1$ if $s_{t+1} = 1$, and failure if $s_{t+1} = 0$. In these notations, $\gamma(s^t,1) > \gamma(s^t)$ means that the multiplier of the incentive compatibility constraint increases in success, and $\gamma(s^t,0) < \gamma(s^t)$ means that the multiplier decreases in failure.

For a given history s^t, suppose $s_{t+1} = 1$. Then $\gamma(s^t)\frac{\pi_e(s_{t+1}|e_t(s^t))}{\pi(s_{t+1}|e_t(s^t))}$ is positive, since $\pi_e(s_{t+1} \mid e_t(s^t))$ is positive for $s_{t+1} = 1$. Therefore, we conclude $\phi_{t+1}(s^{t+1}) > \phi(s^t)$ from equation (8). On the other hand, if $s_{t+1} = 0$, we conclude $\phi_{t+1}(s^{t+1}) < \phi(s^t)$ because of negative $\pi_e(s_{t+1} \mid e_t(s^t))$. Thus we can say that $\phi(s^t)$ *increases in success, and decreases in failure*. Thus if we show that $\phi(s^t)$ and $\gamma(s^t)$ moves to the same direction, we conclude that a success increases $\gamma(s^t)$; hence, more perk good from

Proposition 2. Our next two simulations investigate the cases when $\phi(s^t)$ and $\gamma(s^t)$ move together to the same direction, and when not.

An economic interpretation of $\phi(s^t)$ is that it measures the promised utility at the event of s_t for given history s^{t-1}. The inverse of Pareto weight in a model with information asymmetry is known to be equivalent to marginal utility in income. Since the marginal utility decreases as the utility level increases, the Pareto weight $\phi(s^t)$ and utility level will have monotonic relationship if there is no information asymmetry. In fact, an observation of the first FOC, equation (9), reveals that there is one-to-one mapping from $\phi(s^t)$ to consumption $c(s^t)$. One-to-one mapping between $\phi(s^t)$ and $m(s^t)$ is not clear from equation (10) because of $\gamma(s^t)$. By total-differentiating equation (10) with respect to $\phi(s^t)$, $m(s^t)$ and $\gamma(s^t)$, we derive

$$\left(\frac{1+\gamma(s^t)\alpha}{\phi(s^t)^2}\right)d\phi(s^t)=(-v_{mm})dm(s^t)+\frac{\alpha}{\phi(s^t)}d\gamma(s^t)\,.$$

Thus, small k guarantees the positive relationship between $\phi(s^t)$ and $m(s^t)$. Moreover, if $\gamma(s^t)$ and $\phi(s^t)$ moves to the same direction, larger $\phi(s^t)$ always implies larger $m(s^t)$. From (10), we conclude that $\phi(s^t)$ is an approximate measure of promised utility if $\gamma(s^t)$ and/or $v_{me}=\alpha$ is small enough.

We assume the following functional forms in the next three simulation:

$$u(c,e)=\frac{c^{1-\sigma}}{1-\sigma},\quad v(m,e)=\frac{m^{1-\sigma}}{1-\sigma}-\alpha me\,,\quad \pi(s_t=1\,|\,e)=1-\exp(-e^{\mu})\,,\quad e\geq 0\,.$$

This makes our model to satisfy the sufficient conditions for the valid first order condition approach in Jewitt (1988) as long as $\mu\leq 1$.

2.1.1 Simulation: decreasing implemented effort

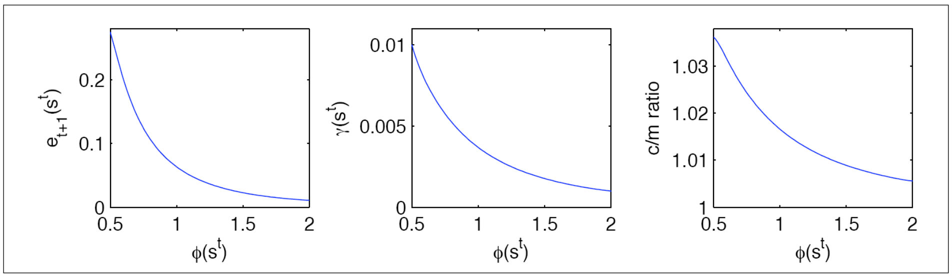
Output is determined only by the current state, so $y_t(s^t)=y_t(s_t)$. Parameters are

$$\alpha=0.5\,,\ \ \mu=0.5\,,\ \ \sigma=2\,,\ \ y(s_t=1)=1\,,\ \ y(s_t=0)=0\,,\ \ \beta=0.90\,,\ \ T=50\,.$$

Since $\beta^T=0.0052$, it is almost an infinite horizon problem in the beginning of the periods. Thus the first period is an approximate solution to the infinite horizon problem ($T=\infty$), a steady state we pick the first period to draw Figure 1.

The first diagram shows that the implemented effort decreases as more successful outcomes accumulate (i.e., as $\phi(s^t)$ becomes larger). The second diagram shows that $\phi(s^t)$ and $\gamma(s^t)$ move to the opposite direction. Thus the perk/consumption ratio will decrease when successful outcomes accumulate (see the third diagram), as stated in equation (6).

[Figure 1] the 1st period (steady state), $y(s^{t-1}, s_t = 1) = \phi(s^t)$



In the first place, the principal has to promise higher utility to the agent as successful outcomes accumulate. This, in turn, makes the next period's incentive compatibility constraint harder to satisfy. Thus, the principal might lower the implemented effort to make the next period's incentive compatibility constraint easier to satisfy. In the second diagram above, the second effect of lowered implemented effort is stronger than the effect of promising more utility; thus the graph goes downwards.

2.1.2 Simulation: output as a function of history s^t

We assume the same parameters except the output functions. We assume

$$y(s^{t-1}, s_t = 1) = \phi(s^t), \quad y(s^{t-1}, s_t = 0) = 0$$

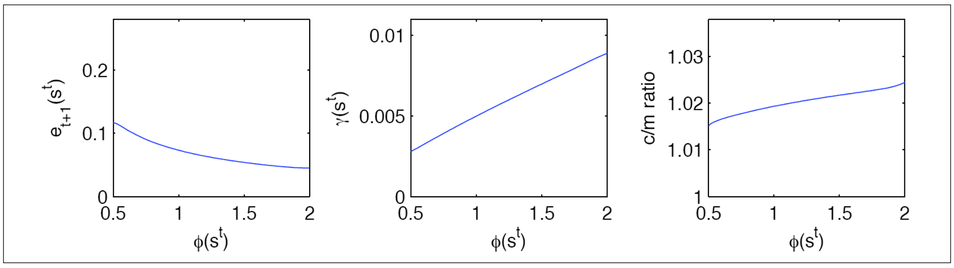
Notice that $\gamma(s^t)\frac{\pi_e(s_{t+1}|e_t(s^t))}{\pi(s_{t+1}|e_t(s^t))}$ in equation (8) measures the effect of how important the task was in two ways: (i) $\gamma(s^t)$ is the shadow cost for the incentive compatibility constraint, and (ii) $\gamma(s^t)\frac{\pi_e(s_{t+1}|e_t(s^t))}{\pi(s_{t+1}|e_t(s^t))}$ is the marginal percentage increase/decrease of the outcome probability in effort. We can think of this output function in two ways. Firstly, the output is literally history dependent. For example, R\&D task is heavily dependent on the past success or failure. Secondly, the agent is assigned to a different task according to his past history. For example, more successful employees are assigned to more important task (e.g., Gabaix and Landier, 2008).

Note that when the principal solves problem (3), she does not take it into account that her choice of $\gamma(s^t)$ and effort $e_t(s^t)$ influence the aforementioned output, i.e., FOCs do not take the $\gamma(s^t)$ and $e_t(s^t)$ contained in the output technology $y(s^t)$ into consideration.

Again, we pick the first period to draw Figure 2.

The implemented effort still decreases with the given output function; however, the decrease is slower than that of the simulation in section 2.1.1. Since the task becomes more important as successful outcomes accumulate, the principal does not want to decrease the implemented effort as much as she would do in the simulation of section 2.1.1. This slower decrease makes it possible that $\phi(s^t)$ and $\gamma(s^t)$ move to the same direction. Thus the perks/consumption ratio increases when successful outcomes accumulate (see the third diagram), as stated in equation (6).

[Figure 2] the 1st period (steady state), $y(s^{t-1}, s_t = 1) = \phi(s^t)$



III. Second Model

Our second model has two differences from the first model. The first difference is that the disutility from effort is additively separable from the consumption of c and m, so the temporal utility is $u(c)+v(m)-w(e)$ where $w(e)$ is the cost of effort. The second difference is that the agent can save money privately (i.e., *hidden saving* of money), but not the perk good. For example, a CEO cannot make a saving account for his personal use of a corporate private jet. On the other hand, the CEO can save his wage income to a saving account without being monitored by the board. Thus an additional constraint for money is required. One unit of money tomorrow can be purchased by paying q unit of money today. The agent's discount factor is β. Then, the constraint for money is

$$qv'(m(s^t)) \geq \beta \sum_{s_{t+1} \in S} v'(m(s^t, s_{t+1}))\pi(s_{t+1} \mid e_t(s^t)). \tag{13}$$

The interest rate an economic agent pays to borrow money is typically larger than the one he gets for saving. The inequality (13) is a simplification that the agent cannot borrow at all. We refer this constraint by *hidden saving constraint*. Let the multiplier of constraint (13) to be $\eta(s^t)$.

We first assume that the principal can perfectly control the consumption of the perk good, i.e., the agent is not allowed to sell or buy the perk good. This is of course an extreme assumption. As mentioned in Introduction, it is more likely that the principal cannot prevent the agent to buy more perk good, although she can typically prohibit the agent to sell the perk good. This asymmetry in controlling the agent's consumption (referred as *spot market constraint*) is represented by

$$v'(m(s^t)) \geq \frac{u'(c(s^t))}{p}. \tag{14}$$

However, we keep the assumption that the principal can perfectly control the agent's consumption for the time being because of two reasons: it will illustrate the intuition behind simulations clearer,[6] and it will make an interesting theoretical comparison with Kocherlakota (2004). In appendix A.1, we put constraint (14) back into the model, and we confirm that the qualitative outcome with the spot market constraint is similar to that of the model without it.

The incentive compatibility and individual rationality constraints are similarly defined.

$$\sum_{t=1}^{T}\sum_{s^t}\beta^{t-1}[u(c(s^t))+v(m(s^t)-w(e_t(s^t))]\,\Pi(s^t \mid e_{t-1}(s^{t-1}))\geq U_0 \tag{15}$$

$$\begin{aligned} w'(e_t(s^t)) &= \sum_{j=1}^{T-t}\beta^j\sum_{s^{t+j}}\frac{\pi_e(s_{t+1}\mid e_t(s^t))}{\pi(s_{t+1}\mid e_t(s^t))}[u(c(s^{t+j}))+v(m(s^{t+j})\\ &\quad -w(e_{t+j}(s^{t+j}))]\,\Pi(s^{t+j}\mid s^t, e_{t+j-1}(s^{t+j-1})) \end{aligned} \tag{16}$$

The principal's problem is:

$$\begin{aligned} &\max_{e_t(\bullet),c(\bullet),m(\bullet)}\sum_{t=1}^{T}\sum_{s^t}\left(\frac{1}{1+r}\right)^{t-1}[y_t-pc(s^t)-m(s^t)]\Pi(s^t\mid e_{t-1}(s^{t-1}))\\ &\text{s.t. (13), (15) and (16).} \end{aligned} \tag{17}$$

The Lagrangean for the principal's problem is:

$$\begin{aligned} L &= \sum_{t=1}^{T}\sum_{s^t}\left(\frac{1}{1+r}\right)^{t-1}[y_t(s^t)-m(s^t)-pc(s^t)]\,\Pi(s^t\mid e_{t-1}(s^{t-1}))\\ &\quad +\rho\left[\sum_{t=1}^{T}\sum_{s^t}\beta^{t-1}[u(c(s^t))+v(m(s^t))-w(e_t(s^t))]\,\Pi(s^t\mid e_{t-1}(s^{t-1}))-U_0\right] \end{aligned}$$

[6] Without spot market constraint (14), we track down only two shadow values of constraints: one for the moral hazard problem and the other for the hidden saving constraint. Tracking down two shadow values (instead of three) is not only simpler, but also easier to visualize.

$$-\sum_{t=1}^{T}\sum_{s^t}\beta^{t-1}\gamma(s^t)\left\{\begin{array}{l} w'(e_t(s^t)) \\ -\sum_{j=1}^{T-t}\beta^j\sum_{s^{t+j}}\frac{\pi_e(s^{t+1}|e_t(s^t))}{\pi(s^{t+1}|e_t(s^t))}\left[\begin{array}{c} u(c(s^{t+j})) \\ +v(m(s^{t+j})) \\ -w(e_{t+j}(s^{t+j})) \end{array}\right] \\ \times\Pi(s^{t+j}\mid s^t, e_{t+j-1}(s^{t+j-1})) \end{array}\right\}\Pi(s^t\mid e_{t-1}(s^{t-1}))$$

$$+\sum_{t=1}^{T}\sum_{s^t}\beta^{t-1}\eta(s^t)\left\{v'(m(s^t))-\frac{\beta}{q}\sum_{s_{t+1}}v'(m(s^{t+1}))\pi(s_{t+1}\mid e_t(s^t))\right\}$$
$$\Pi(s^t\mid e_{t-1}(s^{t-1})) \tag{18}$$

The first order conditions with respect to c_1, m_1, $c(s^t)$, $m(s^t)$, $c(s^T)$, and $m(s^T)$ where $1<t<T$ are:

$$c_1:\frac{p}{u'(c_1)}=\rho \tag{19}$$

$$m_1:\frac{1}{v'(m_1)}=\rho+\eta_1\frac{v''(m_1)}{v'(m_1)} \tag{20}$$

$$c(s^t):\frac{p}{[\beta(1+r)]^{t-1}u'(c(s^t)}=\rho+\gamma(s^{t-1})\frac{\pi_e(s^t\mid e_{t-1}(s^{t-1}))}{\pi(s^t\mid e_{t-1}(s^{t-1}))} \tag{21}$$

$$m(s^t):\frac{1}{[\beta(1+r)]^{t-1}v'(m(s^t)}=\rho+\gamma(s^{t-1})\frac{\pi_e(s^t\mid e_{t-1}(s^{t-1}))}{\pi(s^t\mid e_{t-1}(s^{t-1}))}$$
$$+\left(\eta(s^t)-\frac{\eta(s^{t-1})}{q}\right)\frac{v''(m(s^t))}{v'(m(s^t))} \tag{22}$$

$$c(s^T):\frac{p}{[\beta(1+r)]^{T-1}u'(c(s^T)}=\rho+\gamma(s^{T-1})\frac{\pi_e(s^T\mid e_{T-1}(s^{T-1}))}{\pi(s^T\mid e_{T-1}(s^{T-1}))} \tag{23}$$

$$m(s^T):\frac{1}{[\beta(1+r)]^{T-1}v'(m(s^T)}=\rho+\gamma(s^{T-1})\frac{\pi_e(s^T\mid e_{T-1}(s^{T-1}))}{\pi(s^T\mid e_{T-1}(s^{T-1}))}$$
$$-\frac{\eta(s^{T-1})}{q}\frac{v''(m(s^T))}{v'(m(s^T))} \tag{24}$$

From (19) and (20), (21) and (22), and (23) and (24), we get

$$\frac{1}{v'(m_1)}-\frac{p}{u'(c_1)}=\eta_1\frac{v''(m_1)}{v'(m_1)} \tag{25}$$

$$\frac{1}{[\beta(1+r)]^{t-1}}\left(\frac{1}{v'(m(s^t))}-\frac{p}{u'(c(s^t))}\right)=\left(\eta(s^t)-\frac{\eta(s^{t-1})}{q}\right)\frac{v''(m(s^t))}{v'(m(s^t))} \tag{26}$$

$$\frac{1}{[\beta(1+r)]^{T-1}}\left(\frac{1}{v'(m(s^T))}-\frac{p}{u'(c(s^T))}\right)=-\frac{\eta(s^{T-1})}{q}\frac{v''(m(s^T))}{v'(m(s^T))} \tag{27}$$

From these equations, we derive the following proposition.

Proposition 3

1. *In period 1, the perk good is given more than the agent would purchase in the spot market.*
2. *In period* $t\in\{2,\dots,T-1\}$*, the perk good is given more than the agent would want to purchase in the spot market when* $\eta(s^t)>\eta(s^{t-1})/q$.
3. *In the last period, the perk good is given less than the agent would want to purchase.*

Proposition 3.1, or equation (25), implies that the perk good is given in period 1 more than the agent would purchase in the spot market (if he were able to access it) since $MRS_{mc}>1/p$. The wedge between the marginal rate of substitution and price ratio is a function of the multiplier for the hidden saving constraint (13). This means that the wedge comes from the ability to save m privately, not only from the moral hazard. Proposition 3.3, or equation (27), implies that the perk good is given less than the agent would want to purchase in the last period.

Proposition 3.2, or equation (26), implies that the perk good is given more than the agent would want to purchase in the spot market when $\eta(s^t)>\eta(s^{t-1})/q$ in period $t\in\{2,\dots,T-1\}$, and the perk good is given less otherwise.

Note that increasing $m(s^t)$ makes the hidden saving constraint in period $t-1$ less binding as term $m(s^t)$ appears on the right-hand side of the hidden saving constraint (14). Thus, the (shadow) benefit of increasing one unit of money at state s^{t-1} in period $t-1$ will be $\frac{\eta(s^{t-1})}{q}$ when it is measured in period t (note that one unit of money tomorrow is equivalent to q unit of money today). On the other hand, increasing $m(s^t)$ makes the hidden saving constraint in period t more binding as term $m(s^t)$ appears on the left-hand side of the hidden saving constraint (14). Thus, the (shadow) cost of increasing one unit of money at state s^t is $\eta(s^t)$. In summary, the net cost of increasing $m(s^t)$ becomes $[\eta(s^t)-\frac{\eta(s^{t-1})}{q}]$. Also note that $\frac{v''(m(s^t))}{v'(m(s^t))}$ measures the absolute risk aversion of utility function $v(\bullet)$. Thus, the right hand side of equation (26) measures the net benefit of providing one more unit of $m(s^t)$.

On the other hand, the left-hand side of equation (26) measures the efficiency loss from the discrepancy of the marginal rate of substitution and the price ratio. To be more precise, $\frac{p}{u'(m(s^t))}$ is the inverse of the per-dollar marginal utility, i.e., per-utility expenditure on good c; thus, the difference of the per-utility expenditure on good c and good m measures the efficiency loss. In summary, equation (26) describes the trade-off between the marginal benefit and marginal cost of perturbing

$m(s^t)$.

In short, starting from allocation such that $MRS_{mc} = \frac{1}{p}$, if the benefit of relaxing the hidden saving constraint in period $t-1$ is larger than the cost of making the period t hidden saving constraint more binding, we derive $MRS_{mc} = \frac{1}{p}$, i.e., the principal provides luxurious perk good.

Suppose there are only two states, i.e, $S = \{0,1\}$\$ where $s = 0$ is *failure*, and $s = 1$ is *success*. Suppose that the project that the agent put effort on in period $t-1$ succeeded, then $s^t = (s_t = 1, s^{t-1})$. The wage increases as the project succeeded. *If* the principal implements the same effort level in period t, then the incentive compatibility constraint at $s^t = (s_t = 1, s^{t-1})$ becomes harder to satisfy due to the increased wage, i.e., the shadow value of the incentive compatibility constraint increases. Thus, we can say that $q\eta(s^t) > \eta(s^{t-1})$. (Note that the shadow value has to be measured with proper discount, that is, with q since one unit of money tomorrow is equivalent to q unit of money today.) However, if lower effort is implemented, it is not clear whether the inequality holds or not. In summary, there are two forces even when the agent's project succeeds: one of which lowers the shadow value of the incentive compatibility constraint, and the other increases the shadow value. We will investigate when $\eta(s^t) > \eta(s^{t-1})/q$ or not by providing simulations in section 3.1.

In the first model, the principal awards the agent luxurious perk good due to $v_{me} < \frac{u_{ce}}{p}$. In the current model, both of v_{me} and u_{ce} are nil. The wedge between the marginal rate of substitution and price ratio exists in the current model due to the asymmetry in storage technology for money and the perk good: the agent can save money for the purpose of consumption smoothing, but cannot save the perk good. Even though the wedge exists in both of the models, whether more perk good or less is awarded is not clear in the second model: the determination requires whether $\eta(s^t)$ is larger or smaller than $\eta(s^{t-1})/q$.

3.1 Simulation

We assume $\frac{1}{1+r} = \beta$ for simplicity. We use Recursive Lagrangean approach again. The Lagrangean (18) changes into the following.

$$
L = \sum_{t=1}^{T} \sum_{s^t} \beta^{t-1} \begin{bmatrix} y(s^t) - c(s^t) - pm(s^t) \\ +\phi(s^t)\left[u(c(s^t)) + v(m(s^t) - w(e_t(s^t))\right] \\ -\gamma(s^t) w'(e_t(s^t)) \\ +[\eta(s^t) - \eta(s^{t-1})/q] v'(m(s^t)) \end{bmatrix}
$$

$$
\Pi(s^t \mid s^t, e_{t-1}(s^{t-1})) - \gamma U_0 \tag{28}
$$

where

$$\phi(s^{t+1})=\rho+\sum_{i=1}^{t}\gamma(s^i)\frac{\pi_e(s_{i+1}\,|\,e_i(s^i))}{\pi(s_{i+1}\,|\,e_i(s^i))}=\phi(s^t)+\gamma(s^t)\frac{\pi_e(s_{t+1}\,|\,e_t(s^t))}{\pi(s_{t+1}\,|\,e_t(s^t))}. \tag{29}$$

The marginal decrease of $m(s^t)$ in period t makes the hidden saving constraint in period $t-1$ harder to satisfy since $v'(m(s^t))$ increases; hence, the cost of $\frac{\eta(s^{t-1})}{q}v'(m(s^t))$ is incurred since $\eta(s^t)$ is the shadow value of the hidden saving constraint. On the other hand, this decrease makes the hidden saving constraint in period t easier to satisfy; hence, the benefit of $\eta(s^t)v'(m(s^t))$. In total, the last term in the bracket is the net cost due to the hidden saving constraint. Again $\phi(s^t)$ is interpreted as Pareto weight, and the explanation in the first model applies for the other terms in the same manner.

The first order conditions are:

$$m(s^t):1=\phi(s^t)v'(m(s^t))+\left[\eta(s^t)-\frac{\eta(s^{t-1})}{q}\right]v''(m(s^t))$$

$$c(s^t):p=\phi(s^t)u'(c(s^t))$$

$$\begin{aligned}e_t(s^t):0=&-\gamma(s^t)w''(e_t(s^t))+\phi(s^t)w'(e(s^t))\\&+\sum_{j=1}^{T-t}\beta^j\sum_{s^{t+j}}\frac{\pi_e(s_{t+1}\,|\,e_t(s^t))}{\pi(s_{t+1}\,|\,e_t(s^t))}\left[\begin{array}{c}y(s^{t+j})-c(s^{t+j})-pm(s^{t+j})\\+\phi(s^{t+j})[v(m(s^{t+j}))+u(c(s^{t+j})),w(e_{t+j}(s^{t+j}))]\\-\gamma(s^{t+j})w'(e_{t+j}(s^{t+j}))\\+[\eta(s^{t+j}-\frac{\eta(s^{t+j-1})}{q}]v'(m(s^{t+j}))\end{array}\right]\\&\times\Pi(s_{t+j}\,|\,s^t,e_{t+j-1}(s^{t+j-1}))\\&+\beta\gamma(s^t)\sum_{s^{t+1}}\frac{\partial(\frac{\pi_e(s^{t+1}|e_t(s^t))}{\pi(s^{t+1}|e_t(s^t))})}{\partial e_t(s^t)}[u(m(s^{t+1}))+u(c(s^{t+1}))-w(e_{t+1}(s^{t+1}))]\pi(s_{t+1}\,|\,e_t(s^t))\end{aligned}$$

We assume the following functional forms in the following simulations.

$$u(x)=v(x)=\frac{x^{1-\sigma}}{1-\sigma},\quad w(e)=\alpha e^{\varepsilon},\quad \pi(s_t=1\,|\,e)=1-\exp(-e^{\mu}),\quad e\geq 0$$

Note that we are setting the utility function for money and the perk good identical to see the effect of the hidden saving constraint clearer.

The recursive formulation has three state variables (s,ϕ,ζ). We define recursive function $J(s,\phi,\zeta)$,

$$J(s,\phi,\zeta)=\left[\begin{array}{c}y-m-pc+\phi(u(c)+u(m)-w(e))\\-\gamma w'(e)+[\eta-\frac{\zeta}{q}]u'(m)\end{array}\right]+\beta\sum_{s'}J(s',\phi',\zeta')\pi(s'\,|\,e) \tag{30}$$

where ζ represents the multiplier for the previous period hidden saving constraint, $\eta(s^{t-1})$ in period t.

Then we can rewrite the first order condition with respect to e into a recursive form.

$$0=-\gamma w''(e)-\phi w'(e)+\beta\sum_{s'}\frac{\pi_e(s'\mid e)}{\pi(s'\mid e)}J(s',\phi',\zeta')\pi(s'\mid e)$$
$$+\beta\gamma\sum_{s'}\frac{\partial\left(\frac{\pi_e(s'|e)}{\pi(s'|e)}\right)}{\partial e}[u(c(s'))+u(m(s'))-w(e(s'))]\pi(s'\mid e)$$

Note $c(s')$ and $m(s')$ denote the next period's consumption when state s' is realized.

We need to solve the following six functions.

$$c(\phi,\zeta),m(\phi,\zeta),\eta(\phi,\zeta),e(\phi,\zeta),\gamma(\phi,\zeta),J(s,\phi,\zeta)$$

Since we have three first order conditions, one incentive compatibility constraint, one hidden saving constraint, and equation (30), we will be able to solve the six functions with the two laws of motion for state variables, equation (29) and $\zeta_{t+1}(s^t,s_{t+1})=\eta(s^t)$ for all s_{t+1}.

We adopt the following parameters unless mentioned otherwise.

$$\alpha=1,\ \ \varepsilon=1,\ \ \mu=0.5,\ \ \sigma=2,\ \ \beta=0.90,\ \ T=50$$

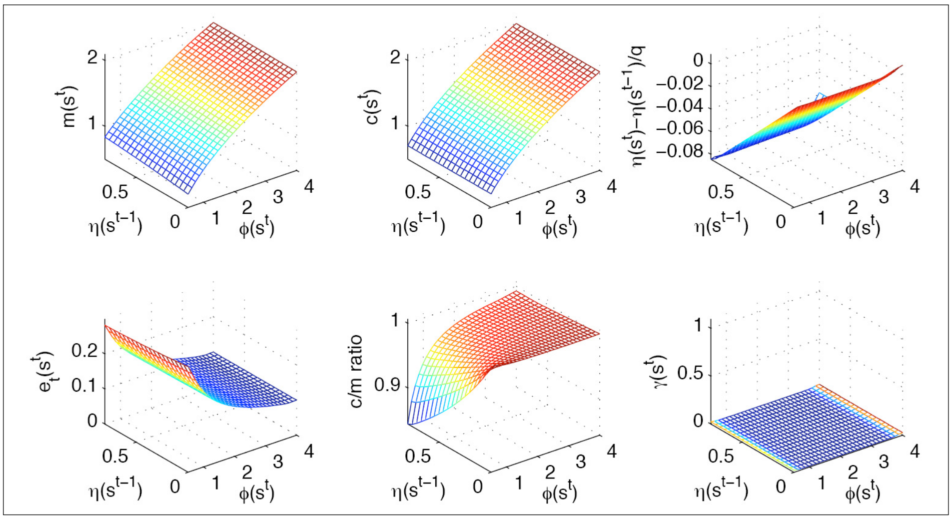
Since $\beta^T=0.0052$, the characterization of the first period is a good approximation of infinite horizon problem.

3.1.1 Simulation: decreasing implemented effort

We assume $y(s_t=0)=0$ and $y(s^{t-1},s_t=1)=1$. We pick period 1 to draw Figure 3.

The fourth diagram shows that the implemented effort $e_t(s^t)$ decreases in $\phi(s^t)$ for given $\eta(s^{t-1})$. An explanation similar to that of the first model applies here: since the higher promised utility makes it more difficult to satisfy the incentive compatibility constraint *and* the hidden saving constraints in the current period, the implemented effort level decreases in $\phi(s^t)$. Note that the shadow value of the incentive compatibility constraint remains very small in the sixth diagram. The value of $\gamma(s^t)$ remains near 0.01 with much stability (the interpolation used in the computation makes it look like increasing and decreasing sharply in the two boundaries of $\phi(s^t)$; however that is not the case. In fact, computation with enlarged boundaries for $\phi(s^t)$ confirms that this is indeed an interpolation issue).

[Figure 3] Period 1 (steady state), hidden saving with perk good, $y(s^{t-1}, s_t = 1) = 1$



Low and stable $\gamma(s^t)$ implies that the (shadow) cost of the moral hazard problem is low and stable. Since equation (13) is automatically satisfied when there is no moral hazard problem, the shadow value $\eta(\bullet)$ of the constraints will be stable. Thus $\eta(\bullet)$ will be almost constant, $\eta(s^{t-1}) \approx \eta(s^t)$. Thus $\eta(s^{t-1}) - \eta(s^t)/q$ will be approximately $\frac{q-1}{q}\eta(s^{t-1})$. This fact describes the third diagram,[7] which in turn describes the fifth diagram as implied in equation (26).

3.1.2 Simulation: output as a function of history s^t

The only difference in parameters is $y(s^{t-1}, s_t = 1) = \phi(s^t)$. Figure 4 is from period 1.

The consumption of money $m(s^t)$ is lower than that of the previous simulation. The hidden saving constraint becomes more binding as $m(s^t)$ becomes large. Thus it becomes more difficult to pay much of $m(s^t)$. This difficulty becomes more severe in increasing rate as $\phi(s^t)$ becomes larger, which makes the first diagram more concave than that of the previous simulation. In the end, the perk good becomes a more efficient way to implement the effort.

The third diagram shows that there is larger area where $\eta(s^t) - \eta(s^{t-1})/q > 0$ than that of the previous simulation. Also $[\eta(s^t) - \eta(s^{t-1})/q]$ is increasing in $\phi(s_t)$ for given $\eta(s^{t-1})$, i.e., there will be more luxurious perk good as successful outcomes accumulate. The fifth diagram agrees with this observation.

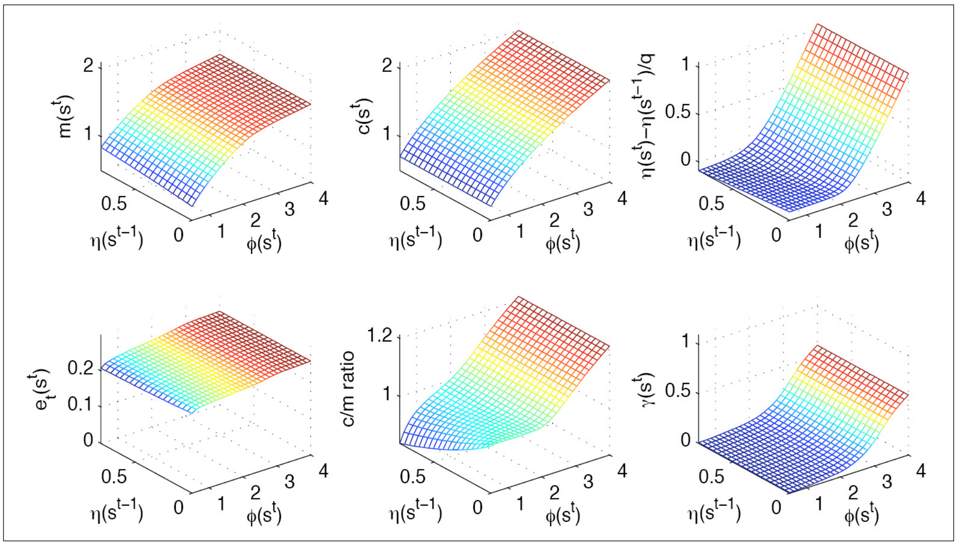
The fourth diagram shows that the effort level increases in $\phi(s^t)$ for given

[7] However, notice that when $\eta(s^{t-1})$ is close to zero, $\eta(s^t)$ will be positive even though it could be very small. Thus $\eta(s^{t-1}) - \eta(s^t)/q$ will be positive when $\eta(s^{t-1})$ is very close to zero. Since this value is so small that it does not show up in the diagram, but an enlarged diagram confirms it.

$\eta(s^{t-1})$. As a consequence, the (shadow) cost of incentive $\gamma(s^t)$ is significantly larger than that of the previous simulation, and also increases in $\phi(s^t)$ for given $\eta(s^{t-1})$. Again, this is because of the increasing output function $y(s^{t-1}, s_t = 1) = \phi(s^t)$.

With the introduction of spot market constraint (14), we expect that the shape of all the diagrams will still remain roughly the same except that the consumption ratio between the perk good and money is not smaller than unity.

[Figure 4] Period 1 (steady state), hidden saving with perk good, $y(s^{t-1}, s_t = 1) = \phi(s^t)$



3.2 Comparison with Kocherlakota (2004)

Kocherlakota (2004) considers the problem of optimal unemployment insurance where the unemployed agent's job-search effort is unobservable and his level of saving is unobservable. He finds that the agent's consumption is constant while he is unemployed, and jumps up to a higher constant and history-independent level of consumption when he finds a job.

His and our models include agents' unobservable efforts with the issue of hidden saving, however the contexts in which agents exert effort are different. Thus our model does not have the history-independent wage as Kocherlakota's model has, although we also show that the hidden saving constraint exacerbates the moral hazard problem. Additionally we suggest that the introduction of the perk good can alleviate the moral hazard problem worsened by the hidden saving problem. More specifically, we provide three simulations with the moral hazard problem: (i) one without hidden saving, (ii) one with hidden saving, and (iii) one with hidden saving and a perk good. All these simulations assume increasing return of output as successful outcomes accumulate, i.e., $y(s^{t-1}, s_t = 1) = \phi(s^t)$. We show that the outcome in (ii) is worse than that of (i), and that the outcome in (iii) is better than (ii).

More specifically, since the saving technologies for money and the perk good are identical in the model (i), the principal will provide consumption $m(s^t)$ and $c(s^t)$ such that $\frac{u'(c(s^t))}{p} = v'(m(s^t))$, which implies $c(s^t) = u'^{-1}(pv'(m(s^t)))$. Under our functional assumption that $u(x) = v(x)$ and $p = 1$, we derive $c(s^t) = m(s^t)$. Thus the principal's problem of model (i) is

$$\max_{e_t(\bullet), m(\bullet)} \sum_{t=1}^{T} \sum_{s^t} \left(\frac{1}{1+r} \right)^{t-1} [y_t - 2m(s^t)] \Pi(s^t \mid e_{t-1}(s^{t-1}))$$

$$s.t. \sum_{t=1}^{T} \sum_{s^t} \beta^{t-1} [2v(m(s^t) - w(e_t(s^t))] \Pi(s^t \mid e_{t-1}(s^{t-1})) \geq U_0 ,$$

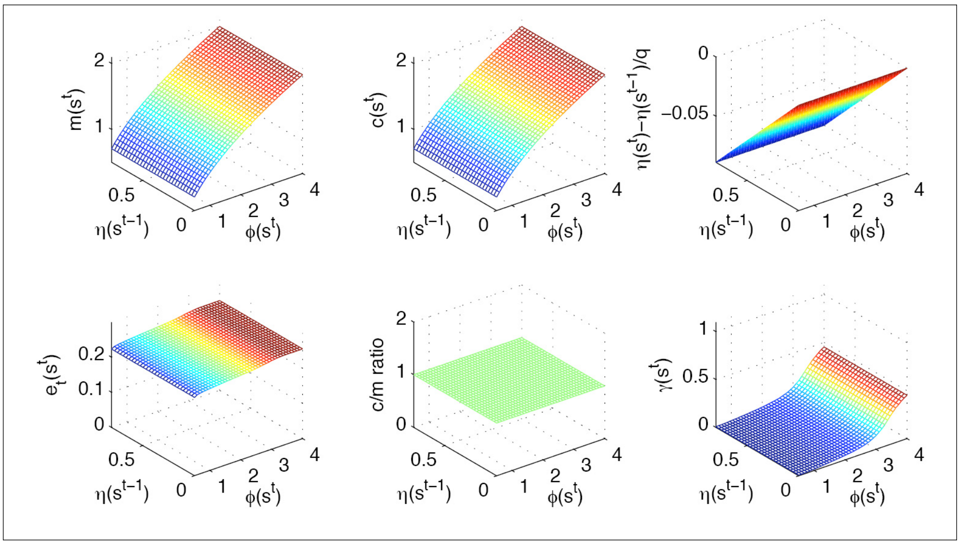
$$w'(e_t(s^t))$$

$$= \sum_{j=1}^{T-t} \beta^j \sum_{s^{t+j}} \frac{\pi_e(s_{t+1} \mid e_t(s^t))}{\pi(s_{t+1} \mid e_t(s^t))} [2v(m(s^{t+j})) + w(e_{t+j}(s^{t+j}))] \Pi(s^{t+j} \mid s^t, e^{t+j-1}(s^{t+j-1})) .$$

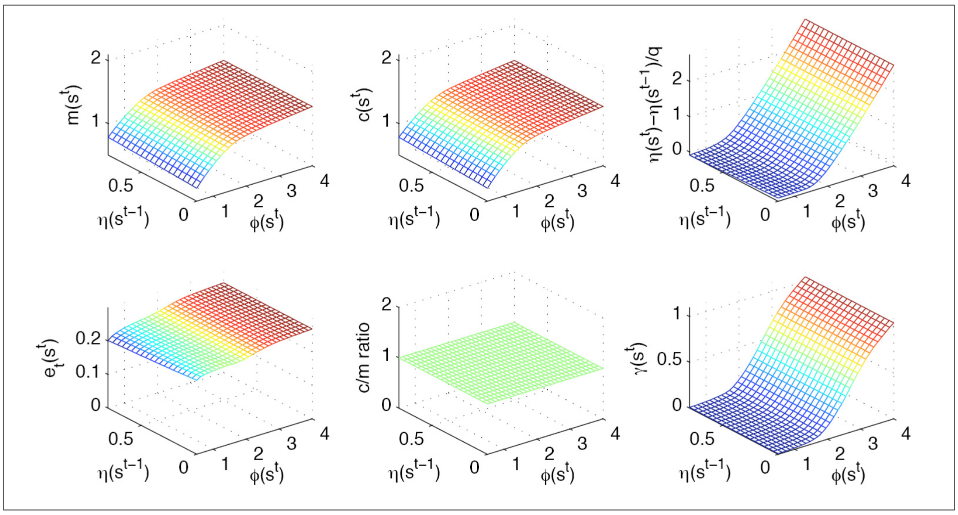
For model (ii), hidden saving constraint (13) is added to the above problem. Model (iii) is already presented in section 3.1.2.

For model (i) without the hidden saving constraint, the diagrams for period 1 is Figure 5. For model (ii) with the hidden saving constraint but without perk good, the diagrams for period 1 is Figure 6.

[Figure 5] Period 1 (steady state), no hidden saving, $y(s^{t-1}, s_t = 1) = \phi(s^t)$



[Figure 6] Period 1 (steady state), hidden saving without perk good, $y(s^{t-1}, s_t = 1) = \phi(s^t)$



Again consumption levels are lower when hidden saving constraint (13) is present than when it is not. The shadow value of the incentive compatibility constraint, $\gamma(s^t)$, is higher than the one without a hidden saving constraint, i.e., the hidden saving exacerbate the moral hazard problem. Although the implemented effort levels look similar, the effective domains are different for model (i) and (ii). $\phi(s^t)$ varies less in the case of model (i) as $\gamma(s^t)$ is lower. Thus the effective domain for the fourth diagram is narrower for model (i). In other words, when there is a hidden saving problem, the variation of the implemented effort is higher. This higher variation gives more uncertainty to the agent; thus, it becomes more costly to satisfy the individual rationality constraint, i.e., more costly implementation.

In summary, the introduction of the perk offsets the adverse effect of a hidden saving problem. Comparing the Figure 6 with Figure 4, we observe that the introduction of the perk good increased the consumption level, lowered the (shadow) cost of the moral hazard problem, narrowed the effective domain for $e_t(\eta(s^{t-1}), \phi(s^t))$, and lowered the (shadow) cost of the incentive compatibility constraint $\gamma(s^t)$. This narrower domain implies that there is less variation in the compensation to the agent; thus, it becomes less costly to satisfy the individual rationality constratint, i.e., less costly implementation.

Note that there is no substitution effect between money and the perk good as the cross-derivative with respect to $m(s^t)$ and $c(s^t)$ is nil. If we were to use different utility function with negative cross-derivative ($m(s^t)$ and $c(s^t)$ are substitutes), then the efficiency gain from the use of the perk good will be even higher. For example, if we use utility function of $u(c+m)-w(e)$ or a CES utility function $[a_c^{\frac{1}{s}} c(s^t)^{\frac{s-1}{s}} + a_m^{\frac{1}{s}} m(s^t)^{\frac{s-1}{s}}]^{\frac{s}{s-1}}$ instead of $u(c)+v(m)-w(e)$, the efficiency gain due to the use of the perk will be even larger.

IV. Conclusion

The empirical studies have shown that there is much perks in the high-profile CEO's compensation package (Rajan and Wulf, 2006; Yermack, 2006). However, there is little attention on the theoretical underpinnings for the existence of perks, especially on dynamically optimal perks. We provide two potential explanation for the existence of perks. The first one is when a perk good and monetary income have asymmetric effect on effort in the utility function. In this case, we show that the amount of the perk good may increase as successful outcomes accumulate. This is consistent with conventional view on perks. The second one is the asymmetric storage technologies for the perk good and money. In this environment, we show that perks can make a contract more efficient, and perks may increase as successful outcomes accumulate.

The idea of our paper can be applied to a border research agenda that attempts to improve the efficiency of a contract in a repeated moral hazard environment. Commodities' different effect on effort and/or different saving technologies can play an important role in designing an efficient contract. There is relatively few attention on expanding literature along this dimension. Our paper attempts an exercise in this direction.

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A. Appendix

A.1 Constraint on spot market trading

We simply report the derivation and the outcome diagrams of the model with spot market constraint (14). The outcome is very similar to the outcome of section 3.1.2 except that the consumption ratio c/m between the perk good and money cannot go below unity due to the spot market constraint.

The Lagrangean for the principal's problem is:

$$
\begin{aligned}
L = & \sum_{t=1}^{T}\sum_{s^t}\left(\frac{1}{1+r}\right)^{t-1}[y_t(s^t)-m(s^t)-pc(s^t)]\,\Pi(s^t \mid e_{t-1}(s^{t-1})) \\
& +\rho\left[\sum_{t=1}^{T}\sum_{s^t}\beta^{t-1}[u(c(s^t))+v(m(s^t))-w(e_t(s^t))]\,\Pi(s^t \mid e_{t-1}(s^{t-1}))-U_0\right] \\
& -\sum_{t=1}^{T}\sum_{s^t}\beta^{t-1}\gamma(s^t)\left\{\begin{matrix} w'(e_t(s^t)) \\ -\sum_{j=1}^{T-t}\beta^j\sum_{s^{t+j}}\frac{\pi_e(s^{t+1}|e_t(s^t))}{\pi(s^{t+1}|e_t(s^t))}\begin{bmatrix} u(c(s^{t+j})) \\ +v(m(s^{t+j})) \\ -w(e_{t+j}(s^{t+j})) \end{bmatrix} \\ \times\Pi(s^{t+j} \mid s^t, e_{t+j-1}(s^{t+j-1})) \end{matrix}\right\}\Pi(s^t \mid e_{t-1}(s^{t-1})) \\
& +\sum_{t=1}^{T}\sum_{s^t}\beta^{t-1}\eta(s^t)\left\{v'(m(s^t))-\frac{\beta}{q}\sum_{s_{t+1}}v'(m(s^{t+1}))\pi(s_{t+1} \mid e_t(s^t))\right\} \\
& \qquad \Pi(s^t \mid e_{t-1}(s^{t-1})) \\
& +\sum_{t=1}^{T}\sum_{s^t}\beta^{t-1}\xi_t(s^t)\left\{\frac{v'(c(s^t))}{p}-v'(m(s^t))\right\}\ \Pi(s^t \mid e_{t-1}(s^{t-1})),
\end{aligned}
$$

which transforms into:

$$
L=\sum_{t=1}^{T}\sum_{s^t}\beta^{t-1}\begin{bmatrix} y(s^t)-c(s^t)-pm(s^t) \\ +\phi(s^t)[u(c(s^t))+v(m(s^t)-w(e_t(s^t))] \\ -\gamma(s^t)w'(e_t(s^t)) \\ +[\eta(s^t)+\xi_t(s^t)-\eta(s^{t-1})/q]v'(m(s^t)) \\ -\xi_t(s^t)\frac{u'(c_t(s^t))}{p} \end{bmatrix}\Pi(s^t \mid e_{t-1}(s^{t-1}))-\gamma U_0
$$

The first order conditions are:

$$
m(s^t):1=\phi(s^t)v'(m(s^t))+\left[\eta(s^t)+\xi_t(s^t)-\frac{\eta(s^{t-1})}{q}\right]v''(m(s^t))
$$

$$c(s^t): p = \phi(s^t)u'(c(s^t)) - \xi_t(s^t)u''(c(s^t))$$

$$e_t(s^t): 0 = -\gamma(s^t)w''(e_t(s^t)) + \phi(s^t)w'(e(s^t))$$

$$+\sum_{j=1}^{T-t}\beta^j \sum_{s^{t+j}} \frac{\pi_e(s_{t+1} \mid e_t(s^t))}{\pi(s_{t+1} \mid e_t(s^t))} \begin{bmatrix} y(s^{t+j}) - c(s^{t+j}) - pm(s^{t+j}) \\ +\phi(s^{t+j})[v(m(s^{t+j})) + u(c(s^{t+j})) - w(e_{t+j}(s^{t+j}))] \\ -\gamma(s^{t+j})w'(e_{t+j}(s^{t+j})) \\ +[\eta(s^{t+j}) + \xi(s^{t+j}) - \frac{\eta(s^{t+j-1})}{q}]v'(m(s^{t+j})) \end{bmatrix}$$

$$\times \Pi(s_{t+j} \mid s^t, e_{t+j-1}(s^{t+j-1}))$$

$$+\beta\gamma(s^t)\sum_{s^{t+1}} \frac{\partial\left(\frac{\pi_e(s^{t+1} \mid e_t(s^t))}{\pi(s^{t+1} \mid e_t(s^t))}\right)}{\partial e_t(s^t)}[v(m(s^{t+1})) + u(c(s^{t+1})) - w(e_{t+1}(s^{t+1}))]\pi(s_{t+1} \mid e_t(s^t))$$

Using the parameters for the previous simulation, we derive the following diagrams.

[Figure 7] Period 1, with hidden saving, perk good and spot market constraint, $y(s^{t-1}, s_t = 1) = \phi(s^t)$