

Investment Option Game for Controlling Global Stock Pollution under Uncertainty

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Stock pollution involves a tradeoff between irreversible environmental impacts and investment costs. Such a tradeoff creates opposing incentives between early- and late-policy adoptions to reduce stock pollution emissions. Earlier studies consider the effects of the uncertainty of environmental impacts on investment timing. This paper extends these studies by considering interactions between two players in order to abate stock pollution. When the uncertainty of environmental damage exists, the absence of cooperative policy may result in further environmental damage because the irreversibility of investments dominates that of environmental damages.

JEL Classification: Q50

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I. Introduction

Stock pollution problems, such as the emission of greenhouse gases (GHGs), typically involve a tradeoff between environmental damages and investment costs that are both irreversible. Environmental damages caused by stock pollutants tend to be irreversible because it is almost impossible to restore the environmental system once it is disturbed. In fact, according to a recent study of the National Oceanic and Atmospheric Administration, climate change is largely irreversible for the next 1,000 years in terms of temperature, rainfall, and sea level (Solomon et al., 2009). Such irreversible environmental damages call for immediate actions so as to curb GHG emission before the irreversibility becomes prevalent. On the other hand, environmental investment is irreversible due to the property of sunk cost required at

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an initial stage of the investment. Those investments made to reduce CO₂, such as the Integrated Gasification Combined Cycle (IGCC) or Carbon Capture and Storage (CCS), require substantial initial costs. Option values emerge from the investment decision when there are uncertainties regarding the costs of environmental damage that can be partially reduced through the investment (Dixit and Pindyck, 1994). By considering these issues altogether, discussing the problems in controlling global stock pollutants becomes more complicated in terms of identifying the optimal time at which to undertake irreversible investments to reduce the irreversible effects of climate change.

In response to this question, Pindyck (2000, 2002) provides models to determine the optimal investment timing for controlling global stock pollutants. Using real option models to simultaneously take into account irreversibility and uncertainty, he examines the case in which there is uncertainty over GHG concentrations and there are sunk costs of policy adoption to invest in reducing GHG emission. He finds that uncertainty leads to hysteresis of policy adoption and less stringent emission reduction. The rationale behind this result is that policy adoption reduces future emissions over the entire trajectory at a large sunk cost incurred initially, whereas inaction to cut back GHG emissions does not give rise to substantial increase of GHG stock. Hence, Pindyck (2000, 2002) shows that investment in reducing GHGs may be optimally delayed in the presence of the uncertainty of future benefits of GHG emission reduction. When uncertainty is sufficiently resolved, undertaking investment immediately is an ideal option.

Similarly, earlier studies of Kolstad (1996) examine how information about the future economic impact of GHGs would affect investment decision to curb emissions. His finding reveals that irreversibility associated with abatement investment is more significant than the irreversibility of environmental damage. This is because emissions with slow stock effect could always be slightly reduced in the future. This calls for a conservationist approach operating under the principle that it is not yet too late to initiate a policy to abate GHG emissions once uncertainty is sufficiently resolved.

The presumption of the abovementioned literature is that immediate action to abate emission is possible at any given time through the existence of a kind of supranational organization. However, literature has largely ignored the fact that most international environmental problems, such as global warming and ozone depletion, involve several countries without supranational authority that can effectively adopt and enforce policies to reduce global pollutants. Such lack or absence of effective international coordination complicates the possibility of reaching an agreement to take immediate abatement actions, as presumed in the above literature. Thus, the motivation of the present paper is to extend Pindyck's model by incorporating interactions between countries to control global pollutants. A model is analyzed, in which two players strategically determine the optimal

investment timing to reduce stock pollutants. The model basically follows Pindyck (2000) in analyzing the stock pollutant control problem and the environmental damage caused by the stock pollutant subject to uncertainty. Interactions between countries are captured by employing an option game approach as developed by Weeds (2002). Developing a two-player option model is helpful in understanding conditions, under which environmental cooperation is sustained, in order to reduce global stock pollutants. It is shown that the optimal abatement time in a two-player game is much more delayed compared with Pindyck's case, wherein international cooperation is presumed.

The remainder of the paper is organized as follows. Section 2 presents a game-theoretic real option model to analyze control game for stock pollutants. Brief numerical illustrations are provided in order to compare the results of our model with those of Pindyck's results. Section 3 concludes the paper.

II. Model

In our model, cooperative and non-cooperative games with two players are analyzed when both players are affected by a global stock pollutant. The game is presented as an investment option game to reduce pollution damage. This setting allows one to investigate the effects of uncertainty and irreversibility on the investment timing of each player. The players specified in the models are identical. Their investment timings are determined either by cooperation or non-cooperation. In addition, they optimally undertake environmental investment either sequentially or simultaneously. By observing the relative outcomes from different schemes, each player strategically decides the optimal time to invest. The resulting equilibrium is a Markov Perfect equilibrium, in which each player's strategy consists of current state variables.

Players are indexed by $i \in \{1, 2\}$. The emission rate for each player is E^N before undertaking environmental investment. Investment is lumpy by which E^N is reduced to E^A , denoting emission level after the investment ($E^N > E^A$). The superscripts N and A stand for "no action" and "action" for the investment, respectively. Prior to abatement activity, it is assumed that global stock pollutant $S(t)$ evolves according to:

$$\frac{dS(t)}{dt} = 2E^N(t) - \delta S(t), \quad (1)$$

where δ is a natural decay rate. In a case when one takes "action" while the other takes "no action," Equation (1) is transformed into $dS(t)/dt = E^N(t) + E^A(t)$

$-\delta S(t)$. The environmental investment cost, K_i , is completely sunk, and $K_i = K$ due to identical property. Following Pindyck (2002), the damage function of each player is explicitly represented using a convex function, $D(S(t); \theta(t)) = \theta(t)S(t)^2$, where $\theta(t)$ is subject to a stochastic process given by:

$$d\theta(t) = \alpha\theta(t)dt + \sigma\theta(t)dz(t), \quad \theta(t) > 0. \quad (2)$$

In Equation (2), α is a drift parameter and σ is a variance parameter. The initial state $\theta(0)$ is known to players. Given that $dz(t)$ is a Wiener increment with respect to $(\Omega, \mathcal{F}, \mathbb{P})$, the payoff-relevant information is represented by the continuous filtration, $\mathcal{F}(t)|_{t \geq 0}$, generated by $\theta(t)|_{t \geq 0}$ and $S(t)|_{t \geq 0}$. The problem is to find an optimal rule for environmental investment, which maximizes the net present value of players subject to (1) and (2), while considering the other player's move. As in Weeds (2002), the problem becomes an investment option game, in which various investment equilibria may arise. The game proceeds as described below.

A strategy space for player at time, t , is denoted by $\Phi(\phi_1(t), \phi_2(t))$. Here, $\phi_i(t) = \{N, A\}$ implies that Player i does not invest ($\phi_i(t) = N$) or invests ($\phi_i(t) = A$) in pollution reduction. If $\phi_i(t) = N$, its action set is $\phi_i(t) = \{N, A\}$. On the other hand, if $\phi_i(\tau) = A$ at some point, τ , then for $t > \tau$, $\phi_i(t)$ is just null because the investment option is already exercised and the investment is irreversible. Each player determines when to choose investment given the underlying stochastic process, $\theta(t)$, that evolves according to (2). Markov Perfect equilibrium is derived by assuming that each player exercises a Markov strategy based on the other player's best response, which follows a Markov strategy as well. Hereafter, time notation, t , is suppressed for notational convenience.

1. Cooperative Game

Consider a case, in which two players choose their investments cooperatively. A cooperative investment schedule may have two schemes: either both players invest at the same threshold or they sequentially invest at distinct points. A simultaneous cooperative game is first analyzed by specifying the optimal joint investment rule, after which sequential investment is considered. Two outcomes under different strategies are compared in order to characterize the ordering of investment schedule.

Let V_c denote the value function where the leader and follower make the commitment to invest simultaneously at time $\tau_c = \inf\{t \mid \theta(t) \geq \theta_c\}$. Here, θ_c denotes the associated investment threshold above which investment option is exercised. The decision problem is equivalent to a single player's problem with $V_c(\theta) = -\varepsilon_0 \int_0^\infty \theta S^2 e^{-rt} dt - \varepsilon_0 K e^{r\tau_c}$, where r is the annual discount rate, and ε_0 is the expectation operator at $t=0$ because players are identical. Therefore, the

policy objective is to maximize the net present value function specified above. Given that the problem is solved using dynamic programming, this leads to the following Bellman equation:

$$V_c(\theta, S; \phi_1, \phi_2) = -\theta S^2 + V_c(\theta, S; A, A) - \theta S^2 + (1+r)^{-1} \mathbb{E}[V_c(\theta + \Delta\theta, S + \Delta S, N; N)]. \quad (3)$$

The first expression in the RHS of (3), $-\theta S^2 + V_c(\theta, S; A, A)$, represents the return at which the player invests in the current period t . The second expression in the RHS of (3) corresponds to the continuation region, i.e., the “no action” region. This refers to the value of delaying the investment for another period by keeping the emission level at E^N . Using Ito’s lemma, the differential equation, V_c^N , in the “no action” region is obtained as:

$$rV_c^N = -\theta S^2 + (2E^N - \delta S) \frac{\partial V_c^N}{\partial S} + \alpha \theta \frac{\partial V_c^N}{\partial \theta} + \frac{1}{2} \sigma^2 \theta^2 \frac{\partial^2 V_c^N}{\partial \theta^2}. \quad (4)$$

The relevant boundary conditions are $\lim_{\theta \rightarrow 0} V_c^N(\theta, S) = 0$ and $\lim_{S \rightarrow \infty} V_c^N(\theta, S) = -\infty$. Likewise, the associated Bellman equation in the “action” region is given by:

$$rV_c^A = -\theta S^2 + (2E^A - \delta S) \frac{\partial V_c^A}{\partial S} + \alpha \theta \frac{\partial V_c^A}{\partial \theta} + \frac{1}{2} \sigma^2 \theta^2 \frac{\partial^2 V_c^A}{\partial \theta^2}, \quad (5)$$

where both players’ emission rates are reduced from E^N to E^A at the same time. By solving (4) and (5), the optimal timing problem is completely characterized as:

$$V_c = \begin{cases} -\theta_v - \theta v_{00} + A_c \theta^\beta & \text{if } \theta < \theta_c \\ -\theta_v - \theta v_{11} - K & \text{if } \theta \geq \theta_c \end{cases}, \quad (6)$$

where $v = \frac{S^2}{\gamma + 2\delta}$, $v_{00} = \frac{4E^N(2E^N + S\gamma)}{\gamma(\gamma + \delta)(\gamma + 2\delta)}$, $v_{11} = \frac{4E^A(2E^A + S\gamma)}{\gamma(\gamma + \delta)(\gamma + 2\delta)}$, and

$\gamma = r - \alpha$. Note that V_c for $\theta \geq \theta_c$ does not retain the option value term since the investment option is already exercised in the “action” region. In addition, $\theta_v + \theta_{v_{00}}$ denotes the expected present value of damage cost when neither player invests, $\theta_v + \theta_{v_{11}}$ is the expected present value when both players invest, and $A_c \theta^\beta$ is an option value for the investment whereas the characteristic equation yields β as shown below:

$$\beta = \left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right) + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} > 1. \quad (7)$$

The constant term, A_c , and the optimal threshold, θ_c , are determined using the standard value-matching and smooth-pasting conditions as indicated in Dixit and Pindyck (1994). The results are as follows:

$$\theta_c = \frac{K}{v_{00} - v_{11}} \left(\frac{\beta}{\beta - 1} \right), \quad (8)$$

$$A_c = \frac{(\theta_c)^{1-\beta} (v_{00} - v_{11})}{\beta}. \quad (9)$$

Brief comparative statics confirms our intuition about investment threshold, that is, the investment threshold (8) is increasing in K and σ , indicating that the investment timing is delayed with increasing cost and uncertainty. The optimal decision rule yields a Markov strategy, θ_c , as a function of the current state variable, S .

It is also interesting to see how the optimal investment threshold changes in the absence of uncertainty. In the real option literature, deterministic threshold is called a Marshallian threshold. Given that $\beta \rightarrow \infty$ as $\sigma \rightarrow 0$, $\lim_{\beta \rightarrow \infty} \beta / (\beta - 1) = 1$. Hence, in a deterministic setting, the multiplier $\beta / (\beta - 1)$ simply disappears and the Marshallian investment threshold is given as:

$$\theta_c^m = \frac{K}{v_{00} - v_{11}}. \quad (10)$$

The superscript m is used to denote the Marshallian investment threshold. Due to $\beta / (\beta - 1) > 1$, it is clear that $\theta_c^m < \theta_c$, implying that uncertainty raises the investment threshold at which players simultaneously invest to mitigate stock pollution.

Next, we considered a cooperative sequential investment involving two cases: Player 1 invests first and Player 2 invests as a follower, or vice versa. Given that every player is identical, the resulting solution is the same. Under the rule, one player invests first at θ_{cL} and the other player makes an investment immediately after at θ_{cF} . The joint value function associated with sequential investment, V_{cs} , is described as:

$$V_{cs} = \begin{cases} -2\theta v - 2\theta v_{00} + A_{cL}\theta^\beta & \text{if } \theta < \theta_{cL} \\ -2\theta v - 2\theta v_{10} + A_{cF}\theta^\beta - K & \text{if } \theta_{cL} \leq \theta < \theta_{cF} \\ -2\theta v - 2\theta v_{11} + 2K & \text{if } \theta \geq \theta_{cF} \end{cases} \quad (11)$$

where $v_{10} = \frac{2(E^A + E^N)(E^A + E^N + S\gamma)}{\gamma(\gamma + \delta)(\gamma + 2\delta)}$. The first line of the equation represents

the value function before the leader implements the investment, whereas the second line corresponds to a case when the leader invests but the follower is still waiting. Once θ crosses over to θ_{cF} , the value function is obtained, as shown in the third line of the equation. Note that A_{cL} and A_{cF} are the respective values of option terms attached to both players since (11) is the joint value function. The “equal sharing principle” is implicitly assumed in a cooperative game. Therefore, the shares of A_{cL} and A_{cF} attached to each player are the same, i.e., the value for each of is simply divided by 2. The optimal thresholds of θ_{cL} and θ_{cF} are determined by imposing the value-matching and smooth-pasting conditions at the relevant ranges. By solving backwards, the following equations are obtained:

$$\theta_{cF} = \frac{K}{2(v_{10} - v_{11})} \left(\frac{\beta}{\beta - 1} \right), \quad (12)$$

$$A_{cF} = \frac{2\theta_{cF}^{1-\beta}(v_{10} - v_{11})}{\beta}, \quad (13)$$

$$\theta_{cL} = \frac{K}{2(v_{00} - v_{10})} \left(\frac{\beta}{\beta - 1} \right), \quad (14)$$

$$A_{cL} = \frac{2\theta_{cL}^{1-\beta}(v_{00} - v_{10})}{\beta}. \quad (15)$$

From the visual inspection of (12) and (14), it is easy to verify $\theta_{cL} \leq \theta_c \leq \theta_{cF}$, thereby confirming that the leader invests first, followed by the follower. The chosen strategy, whether simultaneous or sequential, is determined by comparing $2V_c$ and V_{cs} . A simultaneous cooperative agreement is said to be *internally sustainable* when neither player prefers a sequential cooperative agreement to a simultaneous investment. More precisely, when θ crosses over to θ_{cL} , if $(1/2)V_{cs} > V_c$ at $\theta = \theta_{cL}$, the assigned leader does not have an incentive to make a simultaneous investment agreement and would rather prefer to make a sequential move. Conversely, if V_c exceeds V_{cs} at θ_{cL} , the assigned leader has an incentive to engage in simultaneous investment and, therefore, postpone the investment until θ reaches θ_c . As a consequence, it is sufficient to compare the size of value

functions over the range of $\theta < \theta_{cL}$. This leads to the following lemma:

Lemma 1. *The sufficient condition for the simultaneous cooperative investment to be sustained is:*

$$\left(\frac{v_{00} - v_{11}}{v_{10} - v_{11}} \right)^\beta > 2^{\beta-1}. \quad (16)$$

Proof: Sustainable simultaneous cooperative investment requires $V_c > (1/2)V_{cs}$ at θ_{cL} , indicating that from (6) to (11), $\theta_{cL}(v_{10} - v_{00}) + (A_c - 0.5A_{cF})\theta_{cL}^\beta + K > 0$. Given that $\theta_{cL} > 0$, the sufficient condition for sustainable simultaneous cooperative investment only requires $A_c - 0.5A_{cF} > 0$. After rearranging the terms using (9) and (13), (16) is finally obtained.

The lemma implies that when the payoff from the simultaneous investment is sufficiently larger than the payoff from the sequential cooperative investment, the former could be sustained definitely.

2. Non-Cooperative Game

Without loss of generality, one player is a leader and the other is a follower due to fact that the players are identical. In this case, as the option game is solved backwards, the follower's problem is evaluated first. From the perspective of a follower, optimal investment timing, θ_F , can be analyzed using a single agent real option model, because the leader has already exercised her investment option. Thus, the optimal timing problem for the follower can be completely characterized as:

$$V_F = \begin{cases} -\theta v - \theta v_{10} + A_F \theta^\beta & \text{if } \theta < \theta_F \\ -\theta v - \theta v_{11} - K & \text{if } \theta \geq \theta_F \end{cases}. \quad (17)$$

Using boundary conditions, the following is obtained:

$$\theta_F = \frac{K}{v_{10} - v_{11}} \left(\frac{\beta}{\beta - 1} \right), \quad (18)$$

$$A_F = \frac{(\theta_F)^{1-\beta} (v_{10} - v_{11})}{\beta}. \quad (19)$$

One important feature in (18) is that θ_F increases as E^A decreases,

representing an emission leakage phenomenon, which is well known in stock pollution control literature. It is caused by the free rider incentive of the follower when emissions are significantly reduced through the efforts of the leader.

The next task is to derive the value of being a leader. This is somewhat different from the follower's problem because in this case, the leader's payoff is affected by the follower's strategy, θ_F . There are three components in the value function of the leader over different ranges of θ : (region i) neither the leader nor the follower is yet to invest; (region ii) only the leader invests; and (region iii) both invest. Then, V_L must satisfy:

$$V_L = \begin{cases} -\theta v - \theta v_{00} + A_L \theta^\beta & \text{if } \theta < \theta_L \\ -\theta v - \theta v_{10} + B_L \theta^\beta - K & \text{if } \theta_L \leq \theta < \theta_F \\ -\theta v - \theta v_{11} - K & \text{if } \theta \geq \theta_F \end{cases} \quad (20)$$

The optimal threshold is denoted by θ_L , which signifies that it is better for Player 1 to invest unilaterally. For $\theta \geq \theta_L$, (region ii) holds when only the leader invests. Immediately after the investment, the emission level of the leader is reduced to E^A from E^N . Weeds (2002) calls $B_L \theta^\beta$ an option-like term, which is not actually the value of the investment option to be exercised by the leader, but the value to the leader of the future adoption by the follower. Given that this value is not controllable by the leader, it does not require a smooth-pasting condition; thus, the value of constant term, B_L , is solved using only the value-matching condition evaluated at θ_F . By solving backwards and given θ_F from (18) without a smooth-pasting condition, we obtain:

$$B_L = (\theta_F)^{1-\beta} (v_{10} - v_{11}), \quad (21)$$

which is a function of θ_F . The investment threshold, θ_L , for the leader cannot be determined optimally, such as in the case of the follower's threshold. In other words, θ_L is not obtained by solving value-matching and smooth-pasting conditions. Instead, the first player to invest does so at the point at which he or she prefers to be the leader rather than be the follower; this point is not necessarily the point at which the benefits from being the leader are the largest. The threshold point, θ_L , is given by indifference $V_L(\theta_L) = V_F(\theta_L)$. From (20), the indifference relation gives $\theta_L^\beta B_L - K = A_F \theta_L^\beta$. By utilizing (19) and (21), the following result is derived:

$$\theta_L = \frac{K}{v_{10} - v_{11}} \left(\frac{\beta}{\beta - 1} \right) = \theta_F. \quad (22)$$

Non-cooperative investment threshold is denoted by $\theta_{nc} = \theta_L = \theta_F$. It is clear that $\theta_c < \theta_{nc}$, because $v_{00} - v_{11} > v_{10} - v_{11}$. Then,

Lemma 2. *The incentive to a free ride induces players to delay investment until the payoff for the leader is equal to that for the follower, thus yielding simultaneous non-cooperative investment such that $\theta_{nc} > \theta_{cF} > \theta_c > \theta_{cL}$.*

Obviously, Lemma 2 implies that the non-cooperative game aggravates environmental deterioration. A brief comparison of Pindyck's threshold, θ_c , and non-cooperative threshold, θ_{nc} , provides significant changes, resulting in more hysteresis for environmental investment in the case of the non-cooperative game. As opposed to a Pindyck's conjecture for global optimal investment timing to mitigate GHGs, non-cooperative game results in the delay of investment timing. Given that $\theta_{cF} < \theta_{nc}$, the emission leakage effect is evidently more prevalent in the non-cooperative strategy. This provides a somewhat striking result because in a non-cooperative game, the sequential strategy yields a situation, in which both players behave as if they both want to be followers, thereby delaying the investment timing until θ reaches θ_{nc} . This is mainly due to the fact that both players have incentives to free ride while waiting for the other player's investment. Note that:

$$A_L = \theta_{nc}^{-\beta} K \left[\frac{\beta}{\beta-1} \left(\frac{v_{00} - v_{11}}{v_{10} - v_{11}} \right) - 1 \right]. \quad (23)$$

$$\text{By rearranging (23), } A_L = \theta_{nc}^{1-\beta} \left[(v_{00} - v_{11}) - (v_{10} - v_{11}) \frac{(\beta-1)}{\beta} \right] > \theta_{nc}^{1-\beta} (v_{10} - v_{11}) /$$

$\beta = A_F$ is obtained, implying that the option value term for a potential leader has a greater value than the option value terms for a potential follower. Therefore, a potential leader reveals greater hysteresis than a potential follower and wants to delay investment as long as possible.

One can characterize equilibria depending on the relative outcome of value functions. The condition is examined under which cooperative investment is sustained in equilibrium as opposed to simultaneous non-cooperative outcomes. Note that in the above analysis of the cooperative game, there are two possible equilibria depending on whether or not internal sustainability holds. It is interesting to investigate the possibility of transition from a cooperative to a non-cooperative regime. Suppose that initially, two players agree on the cooperative simultaneous investment. For future reference, let $\Delta = (v_{00} - v_{11})^\beta - 2^{\beta-1} (v_{10} - v_{11})^\beta$, denoting the gains from simultaneous move. The following proposition is then made (the proof is in the Appendix):

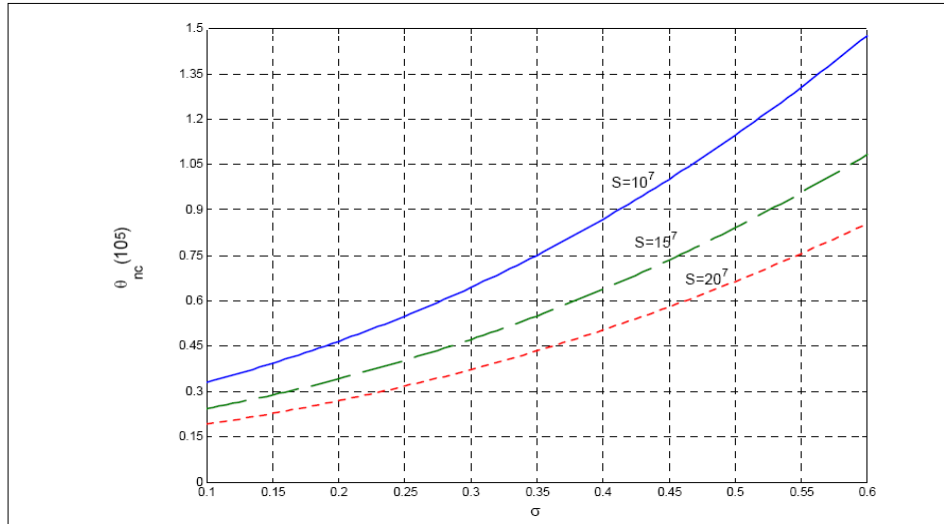
Proposition 1. *If $\Delta < 0$ and $V_{nc} > V_{cL}$ at $\theta = \theta_{cL}$, there exists an incentive to deviate from the cooperative regime to the non-cooperative sequential regime. On the other hand, if $\Delta > 0$ and $V_{nc} > V_c$ at $\theta = \theta_c$, there exists an incentive to deviate from the cooperative to the non-cooperative simultaneous regime.*

The first statement implies that simultaneous cooperative investment, which is a dominant strategy, is in equilibrium if the payoff from the simultaneous cooperation is greater than or equal to the payoff from being a follower in the non-cooperative game, less the investment cost adjusted by the threshold at that point. The second statement is analogous, but both the assigned leader and follower must have greater payoff when they are involved in a sequential cooperative investment as opposed to the case when they deviate away from the simultaneous non-cooperative game. More precisely, this implies that $2\theta_{cL}^{1-\beta}(v_{00} - v_{10})/\beta \geq \theta_F^{1-\beta}(v_{00} - v_{11}) - K\theta_F^{-\beta}$ for the leader and $2\theta_{cF}^{1-\beta}(v_{10} - v_{11})/\beta \geq \theta_F^{1-\beta}(v_{00} - v_{11}) - K\theta_F^{-\beta}$ for the follower. However, since $\theta_{cF}^{1-\beta} < \theta_{cL}^{1-\beta}$ and $v_{00} - v_{10} > v_{10} - v_{11}$, if the latter is satisfied, the former always holds. The final case is considered wherein uncertainty is sufficiently large.

Corollary 1. *In a limiting case where uncertainty grows infinitely, simultaneous cooperative strategy is only a sustainable equilibrium.*

When uncertainty on environmental damage becomes greater, irreversible environmental effect will dominate the irreversible investment effect, so that the earlier reduction of stock pollutant is optimal.

[Figure 1] Sensitivity of θ_{nc} with respect to σ and S

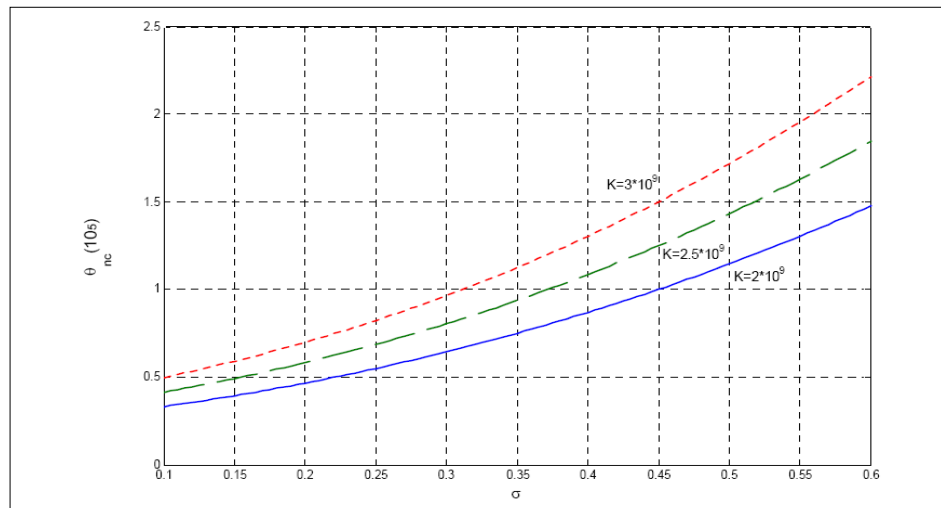


To explore the characteristics of the solutions as well as to facilitate comparison, the same numerical values used in Pindyck (2002) is imported for numerical illustration. The values are given as $r = 0.04$, $\delta = 0.02$, $\sigma = 0.2$, $K = \$2$ billion, and $\alpha = 0$. Pindyck set α to 0, so that the social cost per unit of S is expected to remain constant. The current paper set $S = 10,000,000$, although some range is allowed for S of up to 16 million tons as in Pindyck (2002). The total emission $E = 300,000$ tons per year is divided by $E^N = 150,000$. Following Pindyck (2002), we also let $E^A = 0$.

Using these values, the equilibrium condition is checked as in Proposition 1. To do this, $\theta_{cF} = 2.3273 \times 10^{-6}$ and $\theta_F = 4.6545 \times 10^{-6}$ are calculated. The value of Δ is 3.3081, implying that the only feasible equilibrium is simultaneous non-cooperative investment. From Equations (7) and (22), $\beta = 2$ and $\theta_F = 4.6545 \times 10^{-6}$. The Marshallian threshold, $\theta_c^m = 9.1429 \times 10^{-7}$, which has been computed by (10), is shown to be always less than θ_F . Figure 1 shows θ_{nc} as a function of S with different levels of σ . The figure also shows that the threshold is increasing in σ , confirming the adverse effect of uncertainty on investment.

Figure 2 shows the effect of irreversibility versus uncertainty effect by varying K and σ from 2×10^9 to 3×10^9 and 0.1 to 0.6, respectively. The numerical values adopted in this analysis support the argument of Pindyck (2002) and Kolstad (1996) by showing that θ_{nc} is more sensitive to the irreversible investment cost than the uncertainty.

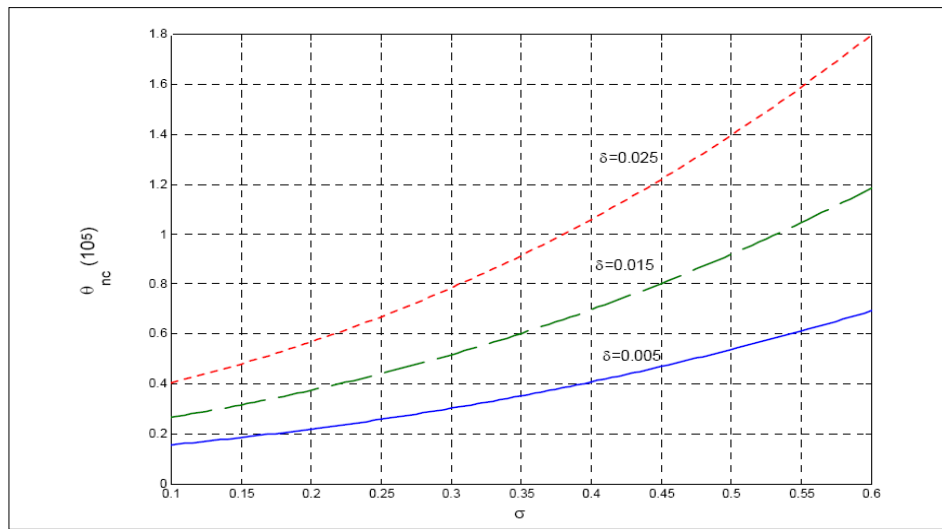
[Figure 2] Sensitivity of θ_{nc} with respect to σ and K



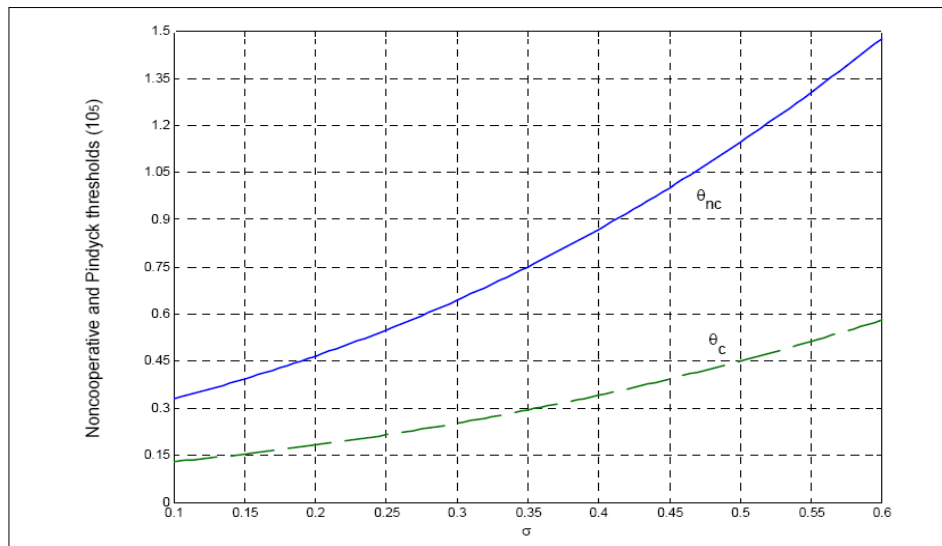
As mentioned in Pindyck (2002, footnote 7), the value of natural decay rate, $\delta = 0.02$, may be too high for GHGs based on the fact that a consensus estimate is close to $\delta = 0.005$. Figure 3 shows that, with fixed $\sigma = 0.2$, θ_{nc} rapidly decreases

when δ decreases. This implies that the longer pollutants persist in the air, thus warranting an earlier investment. According to Nordhaus (1994) and Kolstad (1996), the fact that the existing stock decays very slowly implies that abatement “today” does not create too much difference from abatement “tomorrow.” The result of this paper shows that the decay rate is inversely related to the investment incentives. Moreover, when decay rate is sufficiently small, resolving uncertainty is not as significant an issue as irreversible investment cost effect.

[Figure 3] Sensitivity of θ_{nc} with respect to σ and δ



[Figure 4] Comparison of θ_{nc} and θ_c



For fixed $\sigma = 0.2$, the investment threshold for non-cooperative investment is illustrated in Figure 4. As reference, the figure also shows Pindyck's threshold, which is derived by a single agent model equivalent to θ_c . Evidently, the investment threshold of the non-cooperative game is higher than that of a single player, implying that the environmental outcome from non-cooperative game is more deteriorated than the one from a cooperative (or social planner's) model. Furthermore, the figure shows that the incentive for emission leakage prevails, i.e., because θ_L is delayed to be equal to θ_F . The last figure shows that higher uncertainty reduces the gains from simultaneous investment relative to sequential investment.

III. Conclusion

A real option game model is presented, in which environmental damage is subject to uncertainty. This is done in order to analyze stock pollutant control in a framework of non-cooperative and cooperative models. In the presence of both irreversibilities, there exist some tensions between the delay of investment until further information is revealed, and precautionary investment before irreversible environmental damage takes place. The present analysis shows that in the absence of cooperative policy, environmental condition is further aggravated by the war of attrition feature, in which both players delay to gain free-ride payoffs. A substantial emission leakage is also created. The cooperative strategy is a viable option to overcome such investment delay. However, uncertainty erodes the gains from cooperation, thereby reducing the cooperative investment incentive. Given the uncertainty of knowledge on environmental damages, this may provide a partial explanation for the deficiency of international environmental cooperation.

The model can be extended to several directions. In the current model, only damage uncertainty is allowed, and there is no uncertainty regarding the evolution of pollutant stock. However, the prediction of GHGs is subject to a wider range of variances, although scientific evidence has led to a consensus that the GHGs can increase in a long-term trend. One way to analyze ecological uncertainty is to introduce disturbance term into the evolution equation of stock, as demonstrated by Pindyck (2002) and Saphores (2004). However, this may further delay the investment timing, thus exacerbating the uncertainty effect of environmental damage.

Another significant way to extend the current model is to allow gradual emission reduction. The present model only deals with a one-time emission reduction case, in which emissions are reduced from a certain level to a lower level by a one-time lumpy investment. Furthermore, there is no consideration on time-to-reduce effect,

i.e., actual investment instantaneously results to emission. However, it is more conceivable to allow some time-to-reduce because the emission reduction is more likely to entail significant lags between the investment time and the effective reduction time. A useful real option model is that proposed by Bar-Ilan (1996) who analyzed real estate development plan with construction lags.

Appendix

Proof of Proposition 1. In the first case, since $V_{nc} - V_{cL} = \theta_{cL}(v_{10} - v_{00}) + (A_L - 0.5A_{cF})\theta_{cL}^\beta + K > 0$ at $\theta = \theta_{cL}$, the sufficient condition for $V_{nc} > V_{cL}$ is $A_L > 0.5A_{cF}$. The proof proceeds as follows:

$$\begin{aligned}
 V_{nc} - V_{cL} &= \theta_{cL}(v_{10} - v_{00}) + (A_L - 0.5A_{cF})\theta_{cL}^\beta + K \\
 &= -\frac{K}{2} \frac{\beta}{\beta-1} + K \left(\frac{\beta}{\beta-1} \frac{v_{00} - v_{11}}{v_{10} - v_{11}} - 1 \right) \left(\frac{v_{10} - v_{11}}{v_{00} - v_{10}} \right)^\beta + K \\
 &> -\frac{K}{2} \frac{\beta}{\beta-1} + K \left(\frac{\beta}{\beta-1} \frac{v_{00} - v_{10}}{v_{10} - v_{11}} - 1 \right) \left(\frac{v_{10} - v_{11}}{v_{00} - v_{10}} \right)^\beta + K \quad \text{since } v_{10} > v_{11} \\
 &> -\frac{K}{2} \frac{\beta}{\beta-1} + K \left(\frac{\beta}{\beta-1} \frac{v_{00} - v_{10}}{v_{10} - v_{11}} - 1 \right) \left(\frac{v_{10} - v_{11}}{v_{00} - v_{10}} \right) + K \\
 &= \frac{K\beta}{2(\beta-1)} + K \left(\frac{v_{00} - v_{10} - v_{10} + v_{11}}{v_{00} - v_{10}} \right) = \frac{K\beta}{2(\beta-1)} + K \left(\frac{v_{11} - v_{10}}{v_{00} - v_{10}} \right) > 0.
 \end{aligned}$$

This proves the first argument. In the second case, $V_{nc} - V_c = \theta_c(v_{11} - v_{00}) + A_L\theta_c^\beta + K$ at $\theta = \theta_c$, which can be proven as follows:

$$\begin{aligned}
 V_{nc} - V_c &= \theta_c(v_{11} - v_{00}) + K \left(\frac{\beta}{\beta-1} \frac{v_{00} - v_{11}}{v_{10} - v_{11}} - 1 \right) \left(\frac{\theta_c}{\theta_{nc}} \right)^\beta + K \\
 &> \theta_c(v_{11} - v_{00}) + K \left(\frac{\beta}{\beta-1} \frac{v_{00} - v_{11}}{v_{10} - v_{11}} - 1 \right) \left(\frac{v_{10} - v_{11}}{v_{00} - v_{11}} \right) + K \\
 &= -K \frac{v_{10} - v_{11}}{v_{00} - v_{11}} + K = K \left(\frac{v_{00} - v_{10}}{v_{00} - v_{11}} \right) > 0.
 \end{aligned}$$

This completes the proof for the latter argument.

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