THE KOREAN ECONOMIC REVIEW
Volume 25, Number 2, Winter 2009

ECONOMIC DEVELOPMENT AND TRADE OF QUALITY DIFFERENTIATED GOODS*

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In this paper, we investigate the impact of economic development on international trade from a theoretical perspective, which considers quality-differentiated products and preference diversity for quality. We model two trading partners, and evaluate the effects of per capita income and technology on trade volume and intra-industry trade. We show that the volume of trade and the share of intra-industry trade increases as per capita income increases. Furthermore, we also demonstrate that the volume of trade is proportional to the level of technology, and that the intra-industry trade share increases as the technology in the two countries become similar. This theoretical model may explain why trade volume and intra-industry trade tend to be higher in rich countries than in poor countries.

JEL Classification: F12, L13
Keywords: Product Quality, Volume of Trade, Intra-industry Trade, Economic Development, Technology

I. INTRODUCTION

Trade has become more concentrated among rich countries, and trade among these countries is principally intra-industry trade (IIT). For

Received for publication: July 8, 2009. Revision accepted: Nov. 26, 2009.
* We wish to thank two anonymous referees for helpful comments and suggestions. Any remaining error, if any, is of course ours.
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example, trade within the OECD countries has increased at a much faster rate than has OECD trade with the rest of the world (RW). The ratio of OECD-OECD trade to OECD-RW trade increased from 0.84 in 1961 to 1.58 in 1990 (Bergoeing and Kehoe, 2003).

Why is the volume of trade higher among rich countries? Does the growth of income create more trade volume? Why is intra-industry trade share higher in trade among rich countries? In an effort to answer these questions, we constructed a theoretical model that considers quality-differentiated products and preference diversity for quality. We assume that each country produces different types of quality-differentiated products in the same industry, and exchanges its product with the other country. However, this type of intra-industry trade differs from what is developed by Krugman (1979) and Lancaster (1980), who consider horizontally differentiated products, but this paper considers quality differentiated products that can be ranked vertically. Moreover, Krugman (1979) emphasizes supply side factors like economies of scale, but this paper focuses on demand factors like preferences for quality.

Trade of products that can be ranked according to differences in quality in the same industry are called vertical intra-industry trade. Greenaway *et al*. (1994) and Fontagné *et al*. (2006) showed the empirical methodology to decompose total intra-industry trade (IIT) by SITC code into vertical IIT and horizontal IIT, and showed that the increase in total IIT is principally the result of increases in vertical IIT. Since this paper considers two-way trade of quality-differentiated products, it is different from the horizontal intra-industry trade suggested by Krugman (1979).

This paper's approach is associated with Linder (1961), and Flam and Helpman (1987), as it emphasizes the role of demand on the international trade of goods. Flam and Helpman (1987) previously developed a theoretical model in which the North exports high-quality products and the South exports low-quality products, and then assessed the effects on trade from technical progress, income distribution, and population growth. Some empirical studies have demonstrated that the exporter's GDP per capita exerts positive effects on the average unit value (Schott, 2004; Hummels and Klenow, 2005; Hallak, 2006). These studies showed that quality differentiation exerts an effect on trade.

In this paper, we elucidate the impact of increases in income on the volume of trade, as well as vertical intra-industry trade using a theoretical model. As presented by Gabszewics and Thisse (1979), as income increases, preferences for and the willingness to pay for quality increases. This paper also demonstrates that the increase in income per capita increase the volume of trade and vertical IIT.

The paper is organized as follows. Section II provides the basic model with a summary of the price competition section of Wauthy's study (1996). Section III derives the determinants of trade volume and vertical IIT share and their empirical implications. Our conclusions are provided in the final section.

II. THE MODEL

1. Basic Model

We model an industry with quality-differentiated products, in which two firms produce products, of different quality. These products are sold to consumers who have different preferences for quality. Each consumer may purchase a good from one of the firms, or none at all. We assume the quality costs to be zero.

A representative consumer's indirect utility function is described as follows.

$$U(J_i) = J_i q - p \tag{1}$$

in which q is the unit of quality of the good, and p is its price. The population of consumers is differentiated by the parameter J_i, which is uniformly distributed between 1 and b. Where b represents the maximum willingness to pay for a given level of quality, a consumer should buy a good only if it generates non-negative utility. If goods from both firms generate non-negative utility, consumers should purchase the good that generates the higher non-negative utility.

One of the firms is modeled to produce high-quality products, and the other produces low-quality products. We utilize the subscripts H and L for the high and low quality firms, respectively, and denote the corresponding quality level and price by q_H, q_L and p_H, p_L. J_{LH} denotes the marginal consumer who is indifferent with regard to the consumption of either of the two products. That is, the consumer, J_{LH}, satisfies $U(q_H, p_H) = U(q_L, p_L)$. In equation (1), the marginal consumer, J_{LH}, is defined as

$$J_{LH} = \frac{(p_H - p_L)}{(q_H - q_L)} \tag{2}$$

Some consumers do not wish to buy any goods at prevailing prices. We denote by J_L the consumer who is indifferent with regard to the purchase of a low-quality product or refraining from buying. In equation (1), this type of marginal consumer is defined as

$$J_L = \frac{p_L}{q_L} \tag{3}$$

All consumers having $J_i > J_{LH}$ will purchase high-quality goods, all consumers having $J_L < J_i < J_{LH}$ will purchase low-quality goods, and all consumers having $J_i < J_L$ will not purchase any goods.[1]

2. Market Configuration

Wauthy (1996) considered the model presented in section 1, and found the optimal prices of quality-differentiated products. The findings of Wauthy's study (1996) can be summarized briefly, as follows.[2]

Market configurations are derived from the willingness to pay for the different products, represented by the parameter J_i, which is uniformly distributed between 1 and b, and that some consumers who are below J_L do not purchase either of the two products. Therefore, when J_L is lower than 1, all consumers enjoy non-negative utility from the consumption of

[1] See Wauthy (1996) and Beloqui & Usategui (2005).

[2] Wauthy (1996) allowed firms to select quality in the second stage. However, we assumed that quality is exogenously determined by technology. For this reason, we only use the findings of Wauthy (1996) in the first stage of price competition.

the low-quality product and thus, the market is covered in this case. However, when J_L is higher than 1, some consumers may not purchase either of the two products.

Wauthy (1996) derived the Nash equilibrium prices in two steps. First, he computed the equilibrium prices corresponding to $L_L = 1$ and $L_L > 1$. Second, he determined the parameter constellations for which prices effectively generate the corresponding market outcome. In this fashion, he identified four intervals for the value of b whose bounds depend on the degree of product differentiation (q_L, q_H).

For each interval of b, the optimal prices of the low-quality good and the high-quality good (p_L^*, p_H^*) are as follows:[3]

A) If $b \geq \frac{4q_H - q_L}{q_H - q_L}$, the optimal prices are:

$$p_L^* = b(q_H - q_L)\frac{q_L}{4q_H - q_L} \quad \text{and} \quad p_H^* = b(q_H - q_L)\frac{2q_H}{4q_H - q_L} \tag{4}$$

B) If $\frac{2q_H + q_L}{q_H - q_L} \leq b < \frac{4q_H - q_L}{q_H - q_L}$, the optimal prices are:

$$p_L^* = q_L \quad \text{and} \quad p_H^* = \frac{b(q_H - q_L) + q_L}{2} \tag{5}$$

C) If $2 < b < \frac{2q_H + q_L}{q_H - q_L}$, the optimal prices are:

$$p_L^* = \frac{b-2}{3}(q_H - q_L) \quad \text{and} \quad p_H^* = \frac{2b-1}{3}(q_H - q_L) \tag{6}$$

D) If $1 \leq b \leq 2$, the optimal prices are:

$$p_L^* = 0 \quad \text{and} \quad p_H^* = q_H - q_L \tag{7}$$

The domain defined in A is derived from the condition of $1 < J_L$, and the domains in B and C are derived from the condition $J_L \leq 1$. The domain in D implies that the low-quality firm cannot exist because consumers would not buy its product, and that the high-quality firm monopolizes the

[3] See Wauthy (1996) and Beloqui & Usategui (2005) for detailed proofs.

market.

3. Two-Country World Model

We modified the model of Wauthy (1996) by representing two countries, called Home and Foreign, that trade with each other. Each of these countries produces one type of quality-differentiated product, and exchanges its product with the other country. Like Helpman and Krugman (1985), we assume that there are not any trade barriers and that transportation costs are zero.

We assume that both countries are initially at the same level of economic development, so that their per-capita incomes are identical, but that their technologies differ from industry to industry. For example, Home enjoys higher-level technology in some industries, but Foreign has higher-level technology in other industries. As the two countries trade with each other, Home exports higher-quality goods in some industries, but also exports lower-quality goods in other industries.

However, we focus on the trade of goods that differ in quality in a single industry. Moreover, we assume that each country has one in this industry which produces one type of quality-differentiated good, and then exchanges its product with the other country. Thus, trade is defined as intra-industry trade.

Without any loss of generality, we assume that the foreign firm produces high-quality goods and that the home firm produces low-quality goods in the considered industry; this implies that the technology of foreign firm is higher than that of the home firm in this industry.[4] In Grossman and Helpman's (1991) rising product quality model, quality of each product is determined endogenously with R&D investments, but this paper assumes the quality of goods is determined exogenously.[5] Although we consider these qualities to be exogenous, they are dependent on a given technology level. The number of consumers in Home and Foreign are assumed to be same and normalized to 100% and the distribution of

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[4] This assumption is the same as Flam and Helpman (1987).

[5] Grossman and Helpman (1991) considered product quality changes over time due to R&D investments, but our model deals with the situation where quality is given, and considers comparative statics with one time change of product quality.

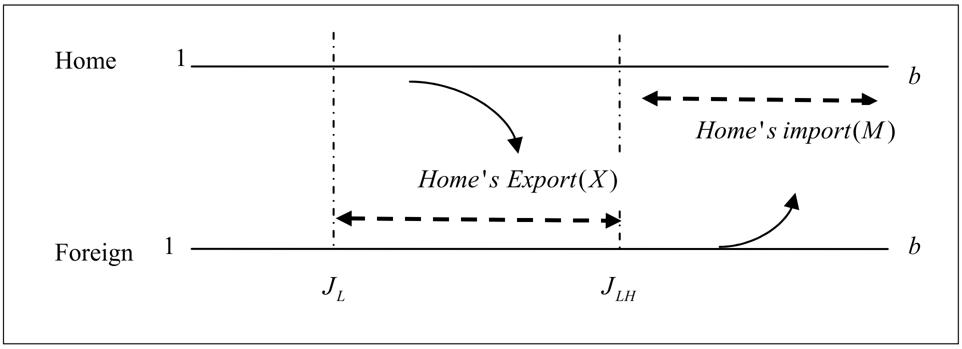
consumers in both countries is also assumed to be the same, uniformly between 1 and b.

The assumption of zero transportation costs helps us to consider the two-country world as a single market with two firms (high-quality and low-quality firms), as elucidated in by Wauthy (1996). Thus, we can utilize the market configurations derived by Wauthy (1996) in investigating trade between the two countries.[6]

4. Trade between the Two Countries

Because each firm can sell its goods in both Home and Foreign markets, consumers will select one good from the Home firm or Foreign firm, depending on their tastes for quality. Because we assume that the Home firm produces low-quality goods and that the Foreign firm produces high-quality goods, Home consumers with a taste for high quality buy high-quality products from Foreign, and Foreign consumers with less of a taste for quality buy low-quality products from Home. Thus, Foreign exports high-quality products to Home, whereas Home exports its low-quality products to Foreign. Foreign's consumption of low-quality products becomes the export value of Home, and Home's consumption of high-quality products becomes the import value of Home. Trade between the two countries can be assessed by viewing Figure 1.

[Figure 1] Trade between the two countries



Note: $J_L = \frac{p_L}{q_L}$, and $J_{LH} = \frac{(p_H - p_L)}{(q_H - q_L)}$.

[6] Note that only results in the price competition of Wauthy (1996) are employed in our paper.

Because we assumed that each consumer buys only one unit of a go od, we can derive the following import value (M) of Home from Figure 1.[7]

$$M(p_L, p_H) = \frac{1}{b-1}[b - \frac{p_H - p_L}{q_H - q_L}]p_H \tag{8}$$

Similarly, because the consumption of low-quality goods in Foreign equals the export value of Home, the export value (X) of Home is represented as:

$$X(p_L, p_H) = \frac{1}{b-1}[\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}]p_L \tag{9}$$

It is worth noting that if $2 < b \leq \frac{4q_H - q_L}{q_H - q_L}$ (domain B and C in section 2), the value of J_L equals 1. Thus, the export value in this case will be:

$$X(p_L, p_H) = \frac{1}{b-1}[\frac{p_H - p_L}{q_H - q_L} - 1]p_L \tag{10}$$

III. TRADE VOLUME AND INTRA-INDUSTRY TRADE SHARE

1. Volume of trade and IIT index

Value of export (X) as well as value of import (M) in Home can be completely defined after price competition by the three exogenous parameters, b, q_H, and q_L. As discussed in Section 2, four market configurations exist in accordance with these parameters. For each of these market configurations, optimal prices as well as the values of exports and imports differ from each other. We allow $q_H = Q$ and $q_L = aQ$, where Q represents the highest technology available in the

[7] Note that the density of consumers both in Home and in Foreign is $1/(b-1)$.

region, and a presents the relative technology level between Home and Foreign. Total trade volume is defined as ($X+M$), and the Grubel-Lloyd intra-industry trade index (*IIT*) is defined as

$$IIT = 1 - \frac{|X-M|}{X+M}$$

The volume of trade and *IIT* index for each market configuration are summarized in Table 1, and mathematical proofs are provided in the Appendix. From these results, we can derive some propositions.

Recall that *b* reflects a consumer's (maximum) willingness to pay for quality. If a consumer has a higher income, he will be willing to pay more for high quality goods. Thus, per-capita income can be employed as a proxy for parameter *b*. Recall also that, if a firm possesses higher technology, it can generate a higher-quality product. Thus, technology can be utilized as a proxy for product quality. Using these proxies, we can derive the following propositions.

[Table 1] Volume of trade and IIT index in each market configuration

Market	Value of *b*	Volume of Trade ($V = X+M$)	*IIT* Index (*IIT*)
A	$\frac{4-a}{1-a} \le b$	$\frac{b^2(1-a)(4+a)}{(b-1)(4-a)^2}Q$	$\frac{2a}{4+a}$
B	$\frac{2+a}{1-a} \le b < \frac{4-a}{1-a}$	$\frac{b^2(1-a)^2+4b(1-a)a-4a+3a^2}{4(b-1)(1-a)}Q$	$\frac{4a[b(1-a)-2+a]}{b^2(1-a)^2+4b(1-a)a-4a+3a^2}$
C	$2 < b < \frac{2+a}{1-a}$	$\frac{(5b^2-8b+5)(1-a)}{9(b-1)}Q$	$\frac{2(b-2)^2}{(b-2)^2+(2b-1)^2}$
D	$1 \le b \le 2$	$(1-a)Q$	0

Note: $a \equiv q_L / q_H$

Proposition 1: *Trade volume increases as the region's income per capita insreases.*

Proof: When $b > 2$, $\frac{\partial V}{\partial b} > 0$ (See Appendix for mathematical derivation). In words the volume of trade generally increases, as *b* increases. Thus, we can assert that trade volume among the high-income regions is higher than that in the low-income regions.

Proposition 2: *The vertical IIT index increases as the region's per capita income increases, until per capita income reaches a critical 1 value, after which there is no relationship.*

Proof: $\frac{\partial IIT}{\partial b} > 0$ in market configuration B and C, but IIT is independent from b in market configuration A (see Appendix for mathematical derivation). In words even though *IIT* increases as the region's per capita income increases, it will eventually stop increasing when the per capita income reaches at a certain level.

Proposition 3: *The volume of trade increases as the available technology in the region increases, whereas the IIT index increases as the technology levels between countries become similar.*

Proof: $\frac{\partial V}{\partial Q} > 0$ in markets A, B, and C, and $\frac{\partial IIT}{\partial a} > 0$ in markets A and B (See Appendix for explanation). Note that Q is the proxy for the highest technology in the region, and parameter a implies the relative technology level of Home to Foreign.

Proposition 4: *If per capita income in the region is very low, the trade will be purely characterized by inter-industry trade.*

Proof: When $1 \leq b \leq 2$, only the Foreign firm produces, and thus Home imports only foreign goods. The firm in Home sells its product because the willingness to pay for low quality products is too low.

2. Empirical Implications

We demonstrated that the volume of trade and IIT index are determined by parameters b, a, and Q; and derived some propositions describing these relationships. The empirical implications of the propositions can be explained as follows.

First, the maximum willingness to pay variable b implies the difference in the willingness to pay for quality among consumers. As presented by

Gabszewics and Thisse (1979), a willingness to pay for quality is dependent on consumers' income; the more income one has, the more quality one will generally pay for. Additionally, Sutton (1986) also derived a similar conclusion, and held that willingness to pay for quality increases as income rises. For this reason, per-capita income can be considered a proxy for b. In other words, b is a proxy for economic development.[8] Therefore, propositions 1 and 2 demonstrate that increases in the per-capita income for all countries will result in increases in the volume of trade and *IIT*.[9] These implications conform to the empirical facts that the volume of trade and *IIT* are higher among rich countries than among poor countries (Bergoeing and Kehoe, 2003; Schott, 2004; Hummels and Klenow, 2005; Hallak, 2006).

Second, the value of Q is determined by the technology available. The level of technology depend on R&D investments. More than 80% of R&D investments are conducted in high-income OECD countries. Proposition 3 explains why trade volumes among high-income countries are larger than those among low-income countries.

Third, the value a, the ratio of the amount of low quality to the amount of high quality, indicates the ratio of technology levels between countries. When the value a approaches 1, the two countries become more similar in terms of technology levels. Proposition 3 shows that *IIT* increases as the value a increases, which is consistent with the empirical fact that *IIT* is higher among countries with similar production technologies. This also implies that technology spillover among countries increases *IIT*. That is, the *IIT* index increases when a low technology country attempts acquire high technology from another country.

Fourth, proposition 4 shows that intra-industry trade does not occur among countries whose income levels are very low. In other words, if the dispersion of the willingness to pay for quality is very low, the

[8] The dispersion of the willingness to pay is caused by differences in income. Additionally, we assume that consumers are uniformly distributed in [1 b] with regard to their willingness to pay. Because per capita income is the average income, it is directly proportional to the average of consumer's willingness to pay (b-1)/2. For simplification, we consider b to be a proxy for per capita income or the level of economic development.

[9] As per-capita income increases, consumers prefer more diversity of products and higher quality products on average. Therefore, intra-industry trade and the total volume of trade increase as per-capita income increases.

opportunity for inter-industry trade will be eliminated. Because one firm's product will monopolize the market while the other firm's product will be eliminated based on a narrow range of preferences. This conclusion explains why vertical *IIT* is unlikely between poor countries.

IV. CONCLUSION

In this paper, we evaluated the impact of economic development on the trade of quality-differentiated goods between two countries. The principal findings are as follows:

First, our model shows that economic development or the growth of per capita income, increases the volume of trade and intra-industry trade, which is consistent with a high *IIT* index and trade volume among rich countries. Second, the volume of trade is related positively increases in technology, whereas the *IIT* index is related positively to similar technology levels between countries. Third, intra-industry trade is unlikely to be observed in regions with very low levels of per capita income. This finding implies that trade among low-income countries is inter-industry trade, rather than intra-industry trade of goods with different quality. The findings in this paper are generally supported by the previous studies conducted by Bergoeing and Kehoe (2003), Schott, (2004), Hummels and Klenow (2005), Hallak (2006).

Economic development can be defined in terms of income per capita and the level of available technology level. This paper demonstrates that the economic development in both senses exerts a positive impact on the volume of trade and *IIT*. It shows that growth of income per capita and growth in technology, through R&D investment and spillover from trade, increase the volume of trade and intra industry trade.

Although we ignore production costs and transportation costs, we expect that our results would be similar even if these costs were significant.

APPENDEX

1. Volume of Trade and Intra-Industry Trade

We let $q_H = Q$ and $q_L = aQ$, and substitute them into conditions of b and optimal prices in each market configuration of A to D in section II of this paper. Then the domain of b for each market configuration and optimal prices can be summarized as in Table A1.

[Table A1] Optimal prices under price competition

Market Configuration	Value of b	Optimal price of high-quality goods	Optimal price of low-quality goods
A	$\frac{4-a}{1-a} \le b$	$p_H^* = \frac{2b(1-a)}{(4-a)}Q$	$p_L^* = \frac{ba(1-a)}{(4-a)}Q$
B	$\frac{2+a}{1-a} \le b < \frac{4-a}{1-a}$	$p_H^* = \frac{b(1-a)+a}{2}Q$	$p_L^* = aQ$
C	$2 < b < \frac{2+a}{1-a}$	$p_H^* = \frac{(2b-1)(1-a)}{3}Q$	$p_L^* = \frac{(b-2)(1-a)}{3}Q$
D	$1 \le b \le 2$	$p_H^* = (1-a)Q$	$p_L^* = 0$

We can get the export value (*X*) and import value (*M*) of Home, by putting the optimal prices in Table A1 into equations (8) and (9) or (10) in part 4 of the section II, and then we obtain volume of trade (*V*) and intra-industry trade index (*IIT*) for each market configuration.

A) For the first domain when $\frac{4-a}{1-a} \le b$

$$M = \frac{1}{b-1}\left(b - \frac{p_H^* - p_L^*}{q_H - q_L}\right)p_H^* = \frac{4b^2(1-a)}{(b-1)(4-a)^2}Q \tag{A1}$$

$$X = \frac{1}{b-1}\left(\frac{p_H^* - p_L^*}{q_H - q_L} - \frac{p_L^*}{q_L}\right)p_L^* = \frac{b^2 a(1-a)}{(b-1)(4-a)^2}Q \tag{A2}$$

From (A1) and (A2), we can derive volume of trade and intra-industry share as follows:

$$IIT = \frac{2a}{4+a} \tag{A3}$$

$$V = \frac{b^2(1-a)(4+a)}{(b-1)(4-a)^2}Q \tag{A4}$$

B) For the second domain when $\frac{2+a}{1-a} \le b < \frac{4-a}{1-a}$

$$M = \frac{1}{b-1}(b - \frac{p_H^* - p_L^*}{q_H - q_L})p_H^c = \frac{[b(1-a)+a]^2}{4(b-1)(1-a)}Q \tag{A5}$$

$$X = \frac{1}{b-1}(\frac{p_H^* - p_L^*}{q_H - q_L} - 1)p_L^c = \frac{[b(1-a)-2+a]a}{2(b-1)(1-a)}Q \tag{A6}$$

From (A5) and (A6), we can derive volume of trade and intra-in dustry share as follows:

$$IIT = \frac{4a[b(1-a)-2+a]}{b^2(1-a)^2+4b(1-a)a-4a+3a^2} \tag{A7}$$

$$V = \frac{b^2(1-a)^2+4b(1-a)a-4a+3a^2}{4(b-1)(1-a)}Q \tag{A8}$$

C) For the third domain when $2 < b < \frac{2+a}{1-a}$

$$M = \frac{1}{b-1}(b - \frac{p_H^* - p_L^*}{q_H - q_L})p_H^* = \frac{(2b-1)^2(1-a)}{9(b-1)}Q \tag{A9}$$

$$X = \frac{1}{b-1}(\frac{p_H^* - p_L^*}{q_H - q_L} - 1)p_L^* = \frac{(b-2)^2(1-a)}{9(b-1)}Q \tag{A10}$$

From (A9) and (A10), we can derive the volume of trade and intra-industry share as follows:

$$IIT = \frac{2(b-2)^2}{(b-2)^2+(2b-1)^2} \tag{A11}$$

$$V = \frac{(5b^2-8b+5)(1-a)}{9(b-1)}Q \tag{A12}$$

D) For the fourth domain when $1 \le b \le 2$

Only Foreign sells goods to Home, and thus there is no intra-industry trade. However, the volume of trade can be calculated by multiplying the optimal price set by the Foreign firm with the quantity of goods bought in Home.

$$VT = (1-a)Q \tag{A13}$$

$$IIT = 0 \tag{A14}$$

2. Proofs of Propositions 1 and 2

It is worth noting that both the volume of trade and the *IIT* index are continuous in *b* provided that $b > 2$. For this reason, we need only to prove that the volume of trade and IIT increase (or remain unchanged) when *b* increases in domains from A to C, as shown in Table 1.

A) When $\frac{4-a}{1-a} \le b$

Differentiating (A4) with regard to *b* and noting that $b > 2$ and $0 \le a < 1$, we can prove

$$\frac{\partial V}{\partial b} = \frac{(1-a)(4+a)(b^2-2b)}{(4-a)^2(b-1)^2} > 0 \tag{A15}$$

This means that the volume of trade increases when *b* increases in this domain.

From the *IIT* formula given in (A3), we can show that parameter *b* does not affect *IIT* in this domain.

B) When $\frac{2+a}{1-a} \le b < \frac{4-a}{1-a}$

Differentiating (A8) with respect to *b*, we get

$$\frac{\partial V}{\partial b} = \frac{b^2(1-a)^2 - 2b(1-a)^2 + a^2}{4(1-a)(b-1)^2} > 0 \tag{A16}$$

Thus, an increase in b leads to a higher level of trade volume in this domain.

Differentiating *IIT* formula given in (A7), we obtain

$$\frac{\partial IIT}{\partial b}=\frac{4a(1-a)[-(1-a)^2b^2+2(1-a)(2-a)b+4a-a^2]}{[b^2(1-a)^2+4b(1-a)a-4a+3a^2]^2} \tag{A17}$$

It is worth noting that whenever $\frac{2+a}{1-a}\le b<\frac{4-a}{1-a}$

$$f(b)=-(1-a)^2b^2+2(1-a)(2-a)b+4a-a^2>0$$

Since $0<a<1$ and $f(b)>0$, we have $\frac{\partial IIT}{\partial b}>0$.

Thus, *IIT* increases in b in this domain.

C) When $2<b<\frac{2+a}{1-a}$

Differentiating (A12) with regard to b and noting that $b>2$ and $0<a<1$, we can prove

$$\frac{\partial V}{\partial b}=\frac{(1-a)(5b^2-10b+3)}{9(b-1)^2}>0 \tag{A18}$$

Thus, the volume of trade increases in b in this domain.

Differentiating the *IIT* formula provided in (A11), we obtain

$$\frac{\partial IIT}{\partial b}=\frac{12(b-2)(2b-1)}{[(b-2)^2+(2b-1)^2]^2}>0 \tag{A19}$$

Thus, *IIT* increases in b in this domain.

From the results derived in A, B, and C, propositions 1 & 2 are proven.

3. Proofs of Propositions 3

From the volume of trade formulas in A, B, and C, we can directly conclude that the volume of trade increases in Q. However, we must also assess the effects of parameter a on *IIT* index in each domain provided in Table 1.[10]

A) When $\frac{4-a}{1-a} \le b$ (or $a \le \frac{b-4}{b-1}$)

Differentiating *IIT* in (A3) with respect to a, we get

$$\frac{\partial IIT}{\partial a} = \frac{8}{(4+a)^2} > 0 \qquad \text{(A20)}$$

Thus, *IIT* increases in a in this domain.

B) When $\frac{2+a}{1-a} \le b < \frac{4-a}{1-a}$ (or $\frac{b-4}{b-1} < a \le \frac{b-2}{b+1}$)

Differentiating *IIT* in (A7) with respect to *a*, we obtain

$$\frac{\partial IIT}{\partial a} = \frac{4[(b-1)^2(b+2)a^2 - 2(b-1)b^2a + (b-2)b^2]}{[(b^2-4b+3)a^2 - 2(b^2-2b+2)a + b^2]^2} \qquad \text{(A21)}$$

It is noteworthy that when $\frac{b-4}{b-1} < a \le \frac{b-2}{b+1}$

$$g(a) = (b-1)^2(b+2)a^2 - 2(b-1)b^2a + (b-2)b^2 > 0$$

Because $g(a) > 0$ in this domain, we conclude that $\frac{\partial IIT}{\partial a} > 0$

Thus, when parameter a increases, *IIT* will increase in this domain.

C) When $2 < b < \frac{2+a}{1-a}$ (or $a > \frac{b-2}{b+1}$)

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[10] Note that *IIT* index is continuous in a given that $0 < a < 1$.

From the *IIT* formula in (A11), we observe that a does not affect *IIT*. From A, B, and C, proposition 3 is proven.

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