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VALUE-AT-RISK ANALYSIS FOR ASIAN EMERGING MARKETS: ASYMMETRY AND FAT TAILS IN RETURNS INNOVATION*

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This paper examines value-at-risk (VaR) analysis performance in the context of the market volatility of five Asian emerging stock markets. From the performance of VaR analysis, we found that the skewed Student's t APARCH model is the best for incorporating the skewness and excess kurtosis of stock returns, and the appropriate assumption of return distribution can provide more accurate VaR models for Asian stock markets. This means that risk-averse investors or portfolio managers of long and short trading positions in Asian stock markets can build optimal margin levels using the VaR computation based on the skewed Student's t APARCH model.

JEL Classification: C32, C52, G11, G15
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I. INTRODUCTION

Especially after the recent financial turmoil financial regulators and supervisory committees of financial institutions are seeking to avoid

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exposure losses and minimize possible investment risk. Thus, they place great emphasis on the development and use of accurate measures of market risk (Stambaugh, 1996; Duffie and Pan, 1997).

Value-at-risk (VaR) is one of the most popular techniques for measuring market risk and has been widely used in banking and securities sectors. VaR simply calculates the maximum loss (or worst-case scenario) for an investment with a specified significance level over a given time period.[1] That is, VaR defines a quantile of a probability distribution which quantifies the probability of a given loss. Several VaR approaches have been introduced to appraise possible losses that financial institutions can incur (Jo and Lee, 2006; Linsmeier and Pearson, 2000; Stambaugh, 1996). Among these, the RiskMetrics model of the J.P. Morgan Group (1996) is one of the most popular tools for measuring the market volatility risk of asset portfolios under the assumption of normality.

However, the main problem with the popular RiskMetrics model is that the set of possible portfolio gains and losses corresponds to a normal distribution (Alexander, 1996; Pafka and Kondor, 2001). Most empirical studies have observed that significant deviations of returns from conditional normality in the form of a skewed and fat-tail distribution, implying that the assumption of Gaussian error innovation is inappropriate for estimating VaR in financial markets (Bali and Theodossiou, 2007; So and Yu, 2006; Angelidis, Benos and Degiannakis, 2007). Thus, VaR models based on a normal distribution may especially lead to spurious results because they are mainly concerned with the tail properties of the distribution in measuring risk.

To account for the skewed and fat-tailed distribution, empirical studies have developed a GARCH-type framework with different distribution innovations.[2] For instance, Giot and Laurent (2003) and Tang and Shieh

[1] The Bank for International Settlement (BIS) imposes a confidence level of 99% and a time horizon of 10 days to measure the adequacy of bank capital.

[2] Some applications introduced an alternative VaR methodology: the extreme value theory (EVT) that models only the tails of the distribution (Daníelsson and Morimoto, 2000). Although the EVT approach provides robust VaR estimation, it focuses on extreme events in financial markets and thus requires the choice of threshold point, which is vaguely defined in the literature (Sriananthakumar and Silvapulle, 2003). Bao, Lee, and Saltoğlu (2006) empirically compared the VaR performance between the GARCH class models (including asymmetry models) and the EVT model. Their evidence indicates that both VaR models produce a similar forecasting of VaR estimation in Asian emerging stock markets.

(2006) considered a skewed Student's *t*-distribution of Lambert and Laurent (2001) to accommodate the excess kurtosis and observed skewness of stock returns. They found that the skewed Student's *t*-distribution produces better one-step-ahead forecasts than do the normal and Student's *t*-distributions. In addition, Choi and Nam (2008) proposed the S_U-normal distribution in capturing the skewness and excess kurtosis in the returns of exchange rates and Dow Jones shares. They found that the S_U-normal distribution is practically useful for adopting univariate and multivariate GARCH models.[3]

Another problem is that the stock returns often exhibit a leverage effect, i.e., stock return volatility tends to increase more following a large decrease in price (bad news) than following an increase in price (good news) of the same magnitude (Black, 1976; Nelson, 1991; Engle and Ng, 1993; Hentschel, 1995). To circumvent this problem, Ding, Granger, and Engle (1993) proposed a class of asymmetric power ARCH (APARCH) models to capture this asymmetry in the stock return volatility. Subsequent studies have widely used the APARCH model in modeling the VaR measurement using daily stock returns (Giot and Laurent, 2003, 2004; Huang and Lin, 2004; Sriananthakumar and Silvapulle, 2003).

Our primary aim was to investigate the volatility asymmetry for five daily indices of Asian emerging markets: Straits Times (Singapore), KOSPI 200 (Korea), KLSE (Malaysia), JAKCOMP (Indonesia), and SET (Thailand). To further enhance the robustness of the estimation results, we compared the performance of the various VaR models with that of normal distribution innovation models (RiskMetrics and normal APARCH models), as well as with that of non-normal distribution innovation models (symmetric Student's *t*, APARCH, and asymmetric skewed Student's t APARCH models).

The main contribution of this study is twofold. First, this study adopted the back-testing approach of Giot and Laurent (2003) that tests not only in-sample VaR values to evaluate the estimated model's goodness-of-fit, but also out-of-sample VaR values to assess the forecasting quality of the

[3] Choi and Nam (2008, p. 42) suggested that "the S_U-normal distribution not only represents a more general distribution to capture skewed and leptokurtic characteristics, but shares some convenient properties of the normal distribution."

estimated model. In particular, the out-of-sample VaR analysis provides forecasting information about the worst potential risk or loss for traders or portfolio managers.

Second, relatively little work has been done on modeling the VaR of Asian equity indices, which exhibit higher expected returns, as well as higher risks, than other major markets. In addition, it is quite possible that the unconditional distribution often exhibits asymmetry and tail-fatness in data from these markets (Harvey, 1995). For this reason, the study of non-normal VaR models could provide accurate forecasts of risk in Asian emerging markets.

The rest of this paper is organized as follows. In Section 2, we describe the theoretical properties of symmetric and asymmetric VaR models. In Section 3, we provide the statistical characteristics of sample data and the estimation results. The final section contains our concluding remarks.

II. METHODOLOGY

Here, we provide a framework of the RiskMetrics and the APARCH models, following the model framework of Giot and Laurent (2003). The first step is to take into account the serial correlation in the daily percentage returns: $y_t = \ln(P_t / P_{t-1}) \times 100$, with $t = 1, \ldots, T$. The daily stock returns exhibit a strong serial correlation, which might distort a conditional variance equation. To account for the serial correlation, we set an $\mathrm{AR}(2)$ process on the y_t series as follows:[4]

$$y_t = \mu + \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varepsilon_t , \tag{1}$$

where μ, φ_1, and φ_2 are constant parameters. We then consider the APARCH model for the conditional variance of ε_t.

2.1. RiskMetrics Model

The RiskMetrics model specifies the variance of the portfolio returns

[4] In order to determine the lags of the AR process, this study considers the lowest value of the Akaike information criterion (AIC).

under the assumption of a Gaussian error distribution. Generally, the RiskMetrics model is equivalent to a normal Integrated GARCH (IGARCH) specification in which the autoregressive parameter is set at a pre-specified value λ, and the coefficient of ε_{t-1}^2 is equal to $1-\lambda$ (Giot and Laurent, 2003). The basic RiskMetrics model can be defined as:

$$\varepsilon_t = z_t \sigma_t, \quad z_t \sim N(0,1), \tag{2}$$

$$\sigma_t^2 = (1-\lambda)\varepsilon_{t-1}^2 + \lambda \sigma_{t-1}^2, \tag{3}$$

where $0 \le \lambda \le 1$. The RiskMetrics Group (1996) suggests $\lambda = 0.94$ for the best back-testing results. Therefore, the RiskMetrics specification does not require the estimation of unknown parameters in the volatility equation because all parameters are already present at given values.

2.2. APARCH Model

The APARCH model of Ding, Ganger, and Engle (1993) extended the GARCH model of Bollerslev (1986) by transforming the squared power on the lagged errors terms with optimally estimated power term since. The squared power may not necessarily be optimal to capture volatility clustering (Ding, Granger, and Engle, 1993). The general APARCH$(1,1)$ model is given as:

$$\sigma_t^\delta = \omega + \alpha_1(\varepsilon_{t-1} + \gamma_1 \varepsilon_{t-1})^\sigma + \beta_1 \sigma_{t-1}^\delta, \tag{4}$$

where ω, α_1, γ_1, β_1, and δ are parameters. The δ ($\delta > 0$) is a coefficient for the power term, whereas γ_1 ($0 < \gamma_1 < 1$) accounts for asymmetric volatility in which negative shocks cause higher volatility than positive shocks of the same magnitude. Furthermore, when $\alpha_1 E(|z| - \gamma_1 z)^\delta + \beta_1 < 1$, the APARCH process is stationary (Ding, Granger, and Engle, 1993).

2.3. Model Densities

The model parameters can be estimated using non-linear optimization procedures to maximize the logarithm of the Gaussian likelihood function. Under the assumption that the random variable $z_t \sim N(0,1)$, the log-likelihood of the Gaussian or normal distribution (L_{Norm}) can be expressed as:

$$L_{Norm} = -\frac{1}{2}\sum_{t=1}^{T}\left[\ln\left(2\pi\right)+\ln\left(\sigma_t^2\right)+z_t^2\right], \tag{5}$$

where T is the number of observations. However, residuals generally have excess kurtosis and skewness, namely, that their distribution tends to have higher means and fatter tails than a normal distribution. Bollerslev (1987) and Baillie and Bollerslev (1989) argued that the Student's *t*-distribution can more accurately represent a distribution of residuals because it may be used to capture leptokurtosis.

We used the Student's *t*-distribution to capture any excess kurtosis in the distribution of the residuals. If the random variable is $z_t \sim ST(0,1,\upsilon)$, the log-likelihood function of the Student's *t*-distribution (L_{Stud}) is defined as:

$$\begin{aligned} L_{Stud} = T&\left\{\ln\Gamma\left(\frac{\upsilon+1}{2}\right)-\ln\Gamma\left(\frac{\upsilon}{2}\right)-\frac{1}{2}\ln[\pi(\upsilon-2)]\right\} \\ &-\frac{1}{2}\sum_{t=1}^{T}\left[\ln(\sigma_t^2)+(1+\upsilon)\left[\ln\left(1+\frac{z_t^2}{\sigma_t^2(\upsilon-2)}\right)\right]\right], \end{aligned} \tag{6}$$

where $2<\upsilon\leq\infty$ and $\Gamma(\cdot)$ is the gamma function. υ represents the number of degrees of freedom in the student's *t*-distribution, which measures the degree of fat tails of the distribution. Lower values of υ indicate fatter tails.

Despite accounting for tail leptokurtosis, a Student's *t*-distribution innovation cannot represent asymmetry. By the skewed Student's *t*-distribution proposed by Lambert and Laurent (2001) can. If

$z_t \sim SKST(0,1,k,\upsilon)$, the log-likelihood of the skewed Student's *t*-distribution (L_{SkSt}) is:

$$L_{SkSt} = T\left\{\ln\Gamma\left(\frac{\upsilon+1}{2}\right) - \ln\Gamma\left(\frac{\upsilon}{2}\right) - \frac{1}{2}\ln[\pi(\upsilon-2)] + \ln\left(\frac{2}{k+\frac{1}{k}}\right) + \ln(s)\right\}$$
$$-\frac{1}{2}\sum_{t=1}^{T}\left[\ln(\sigma_t^2) + (1+\upsilon)\ln\left[1+\frac{(sz_t+m)^2}{\upsilon-2}k^{-2I_t}\right]\right], \tag{7}$$

where $I_t = 1$ if $z_t \geq -m/s$ or $I_t = -1$ if $z_t < -m/s$, k is an asymmetry parameter, and υ is the degrees of freedom of the distribution. The constants $m = m(k,\upsilon)$ and $s = \sqrt{s^2(k,\upsilon)}$ are the mean and standard deviation of the skewed Student's *t*-distribution:

$$m(k,\upsilon) = \frac{\Gamma\left(\frac{\upsilon-1}{2}\right)\sqrt{\upsilon-2}}{\sqrt{\pi}\,\Gamma\left(\frac{\upsilon}{2}\right)}\left(k-\frac{1}{k}\right),$$
$$s^2(k,\upsilon) = \left(k^2+\frac{1}{k^2}-1\right) - m^2. \tag{8}$$

The natural logarithm of the asymmetry parameter $\ln(k)$ can represent the asymmetry of the residual distribution. For example, if $\ln(k) > 0$ $(\ln(k) < 0)$, the density is right (left)-skewed. When $\ln(k) = 0$ (i.e., $k = 1$), the skewed Student's t-distribution equals the general Student's t-distribution, i.e., $z_t \sim ST(0,1,\upsilon)$ in Equation (6).

2.4. VaR Model and Test

2.4.1. VaR Models

This paper compares the performance of the VaR models estimated using the assumption of three different distributions: the normal, Student's *t*, and skewed Student's *t*. In practice, traders or portfolio managers find that the value of their portfolios can change dramatically from one day to the next, through both long positions short positions. The VaR with a long positions is at the (left tail of the distribution). The VaR with a short positions is at the (right tail of the distribution).

In addition, the one-step-ahead VaR was computed using the results of the estimated models and the given distribution. The VaR of the α quantile for long and short trading positions, respectively, was computed as follows.

Under the assumption of a normal distribution,

$$VaR_{long} = \mu_t - n_\alpha \sigma_t \text{, and}$$
$$VaR_{short} = \mu_t + n_\alpha \sigma_t \text{,} \tag{9}$$

where n_α is the left or right quantile at α % for the normal distribution in Equation (5).[5]

Under the assumption of a Student's *t*-distribution,

$$VaR_{long} = \mu_t - st_{\alpha,\upsilon} \sigma_t \text{, and}$$
$$VaR_{short} = \mu_t + st_{\alpha,\upsilon} \sigma_t \text{,} \tag{10}$$

where $st_{\alpha,\upsilon}$ is the left or right quantile at α % for the Student's *t*-distribution in Equation (6).

Under the assumption of a skewed Student's *t*-distribution,

$$VaR_{long} = \mu_t - skst_{\alpha,\upsilon,k} \sigma_t \text{, and}$$
$$VaR_{short} = \mu_t + skst_{\alpha,\upsilon,k} \sigma_t \text{,} \tag{11}$$

where $skst_{\alpha,\upsilon,k}$ is the left or right quantile at α % for the skewed Student's *t*-distribution in Equation (7). If $k < 1$, the VaR for a long trading position will be larger than the VaR for a short trading position for the same conditional variance. When $k > 1$, the opposite holds true.

[5] When calculating the VaR, μ_t and σ_t are computed with the replacement of the unknown parameters in Equation (1) with their maximum likelihood estimates.

2.4.2. Tests of Accuracy for VaR Estimates

We calculated the pre-specified VaR of the 5% and 1% quantiles and evaluated their performance by calculating the failure rate for both the left and right tails of the distribution of the sample return series $\{y_t\}$. Kupiec (1995) proposes a likelihood-ratio test that can be employed to test whether the sample point estimate is statistically consistent with the VaR model's prescribed confidence level. Testing the accuracy of the model is equivalent to testing the hypothesis $H_0: f = \alpha$ versus $H_1: f \neq \alpha$, where f is the failure rate. If the VaR model is correctly specified, the failure rate should be equal to the pre-specified significance level of α. The failure rate is the probability of a failure on any one of the independent trials, estimated by the empirical failure rate $\hat{f}$. The estimated failure rate is calculated as:

$$\hat{f} = \frac{x}{T}, \tag{12}$$

where x is the number of times when returns (in absolute value) exceed the forecasted VaR in the sample, and T is the sample size. The LR statistic is defined as:

$$LR = -2\ln[(1-\alpha)^{T-x}\alpha^{x}] + 2\ln\left[(1-\hat{f})^{T-x}(\hat{f})^{x}\right] \sim \chi^2(1). \tag{13}$$

Under the null hypothesis, the Kupiec LR statistic has a chi-squared distribution with one degree of freedom.

III. EMPIRICAL RESULTS

3.1. Preliminary Analysis of the Data

We considered the representative stock indices of five Asian emerging markets: Straits Times (Singapore), KOSPI 200 (Korea), KLSE (Malaysia), JAKCOMP (Indonesia), and SET (Thailand).[6] All of these

[6] All sample index data were obtained from the DataStream database.

index series consist of daily observations and cover the sample period from January 1990 to December 2005. Both skewness and kurtosis statistics indicate that the returns distribution does not follow conditional normality (Table 1). The statistics for the Jarque–Bera test (J-B) also confirm the non-normality in the sample return series.

[Table 1] Descriptive Statistics of Five Sample Return Series

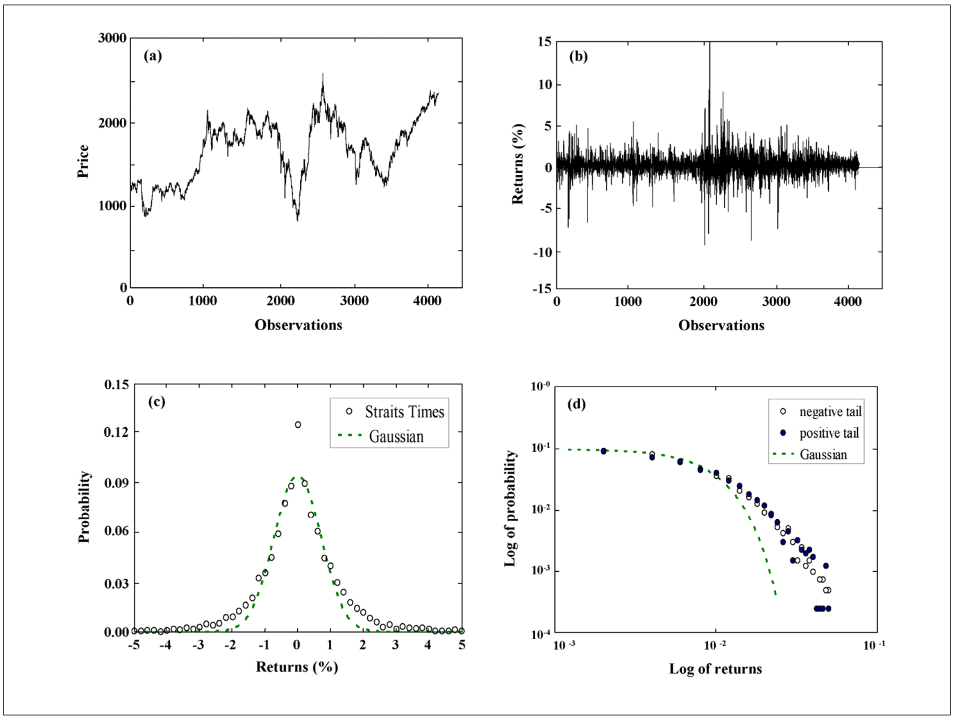
	Straits Times	KOSPI 200	KLSE	JAKCOMP	SET
Mean	0.016	0.013	0.011	0.025	-0.004
St. dev.	1.264	1.898	1.516	1.488	1.740
Minimum	-9.67	-12.74	-24.15	-12.73	-10.02
Maximum	14.87	8.42	20.81	13.12	11.35
Skewness	0.199	-0.044	0.484	0.261	0.234
Excess kurtosis	11.27	3.25	41.72	10.93	4.881
J-B	4814.6**	1925.8**	20168**	4584.7**	1572.6**
$Q(20)$	132.36**	88.99**	120.14**	244.97**	109.59**
$Q_S(20)$	1110.49**	2244.85**	2094.08**	1159.60**	1523.72**
BDS(10)	36.69**	53.19**	49.52**	47.99**	36.93**

Notes: J-B is the value of the Jarque-Bera statistic of the return series. The $Q(20)$ and $Q_S(20)$ are the Box-Pierce test statistics for the return and squared return series for up to 20th-order serial correlations, respectively. BDS(10) corresponds to the t-statistics of the BDS statistic with the embedding dimension m = 10. ** indicates rejection of the null hypothesis at a 5% significance level.

In addition, we examined the null hypothesis of a white noise (indepently and indentically distributed, with a mean of zero and a variance of one) process for sample returns using the Box–Pierce test of the return residuals ($Q(20)$) and the squared return residuals ($Q_s(20)$). Under the null hypothesis of iid, the test statistics are distributed asymptotically as a χ^2 (chi-squared) distribution with 20 degrees of freedom. Both the residuals and the squared residuals are highly correlated up to the 20th lag, so we reject the null hypothesis. Likewise, the $\text{BDS}(10)$ test statistics of Brock et al. (1996) also reject the null hypothesis that the residuals are white noise processes. Thus, these tests imply that the return residuals exhibit linear dependence, non-linear dependence, or chaos.

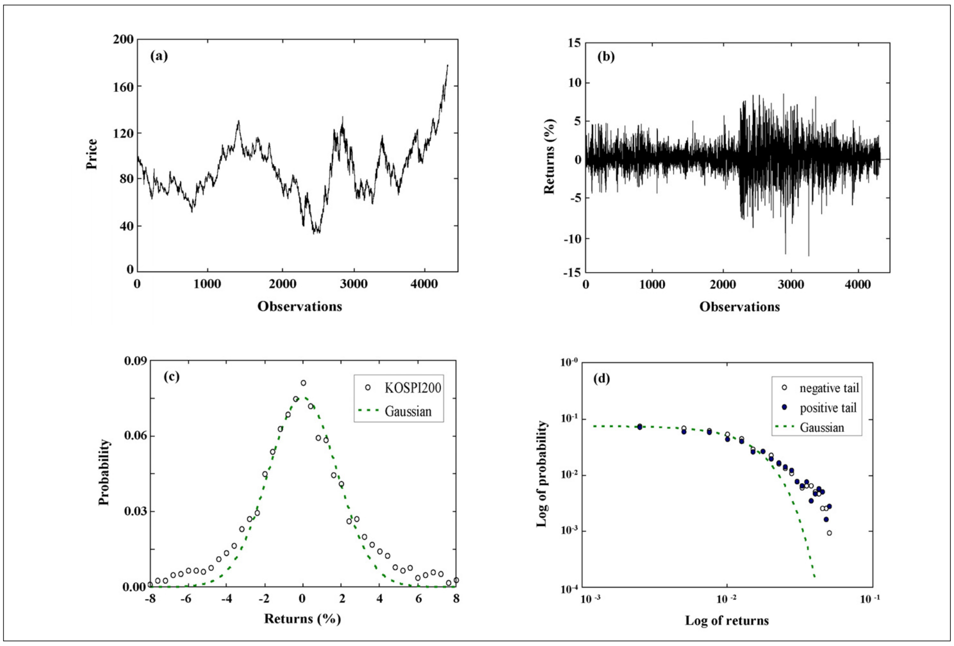
Volatility clustering was clearly observed in the daily returns for each of the five indexes (Figures 1–5, (a) and (b)). Graphs of the density versus the normal distribution, indicate that the returns distributions exhibit fat tails (Figures 1–5, (c)). In addition, from the tail distributions of returns are asymmetric, i.e., the negative tails are not symmetric with the positive tails (Figures 1-5, (d). The asymmetric and fat-tailed properties of the returns distribution motivate the use of non-normal distribution innovations.

[Figure 1] Dynamics and Characteristics of the Straits Times Index



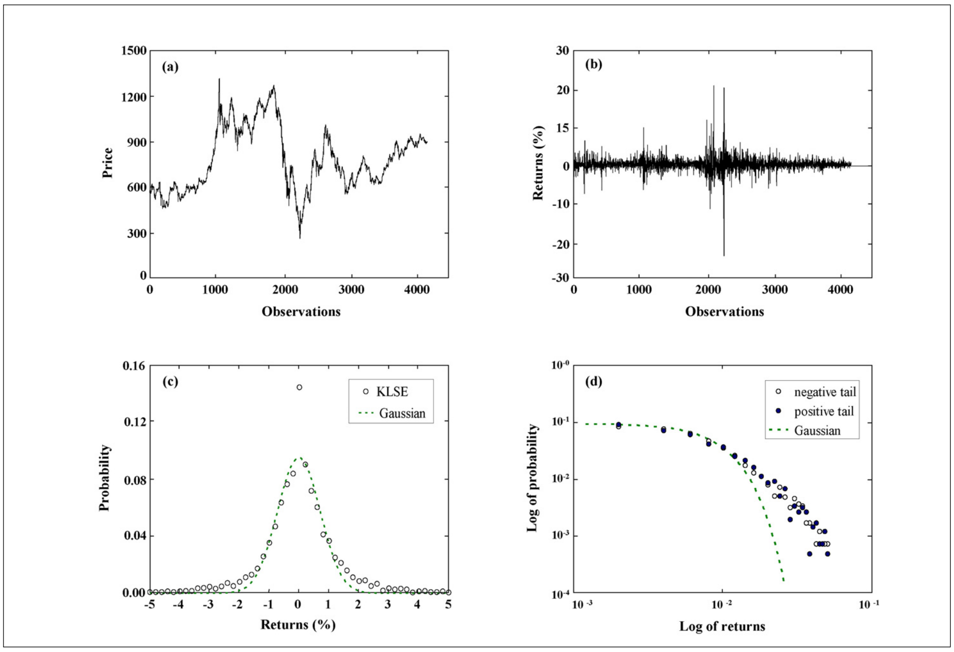
Notes: (a) Dynamics of the index; (b) Daily returns series; (c) Density of the daily returns vs. normal distribution; (d) Tail distributions of the density of the daily returns vs. normal distribution.

[Figure 2] Dynamics and Characteristics of the KOSPI 200 Index



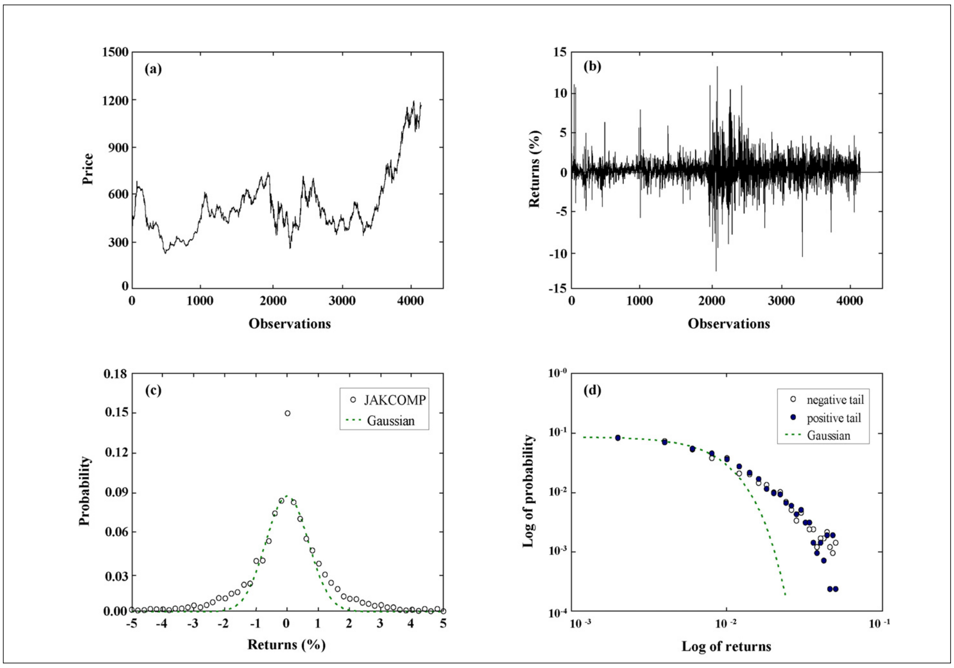
Note: See Figure 1.

[Figure 3] Dynamics and Characteristics of the KLSE Index



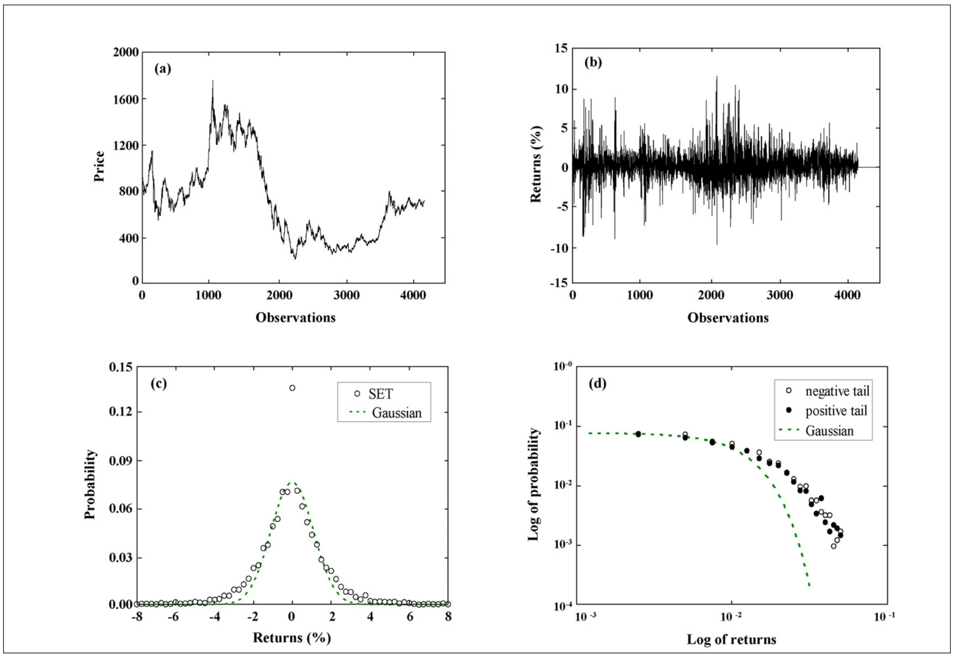
Note: See Figure 1.

[Figure 4] Dynamics and Characteristics of the JAKCOMP Index



Note: See Figure 1.

[Figure 5] Dynamics and Characteristics of the SET Index



Note: See Figure 1.

3.2. Estimation Results of APARCH Models

To perform the VaR analysis, we only estimated the APARCH model for the skewed Student's *t*-distribution (skewed Student's *t* $\mathrm{AR}(2)$ - $\mathrm{APARCH}(1,1)$ models; Table 2).[7] To check the relevance of the residuals, we used a set of diagnostic tests: Box–Pierce and BDS test statistics, the LM ARCH statistics of Engle (1982), and the stationary condition of the APARCH model (Table 2).[8]

The Box–Pierce statistics for the squared standardized residuals, $Q_s(20)$, are statistically insignificant, implying that the conditional variance equation is correctly specified in all of the cases. On the other hand, the Box-Pierce Statistics for the standard-ized residuals, $Q(20)$, in Table 2 are dramatically reduced in comparison with those in Table 1, they are still significant enough to reject the null hypothesis of independence, with the exception of the KOSPI 200. This implies that the AR(2) process cannot completely account for dependence in the returns. In general, the remaining temporal dependence may originate from non-linearity in any of the conditional moments (Beine, Laurent, and Lecourt, 2002).

To measure the remaining temporal dependence, we used the BDS test statistics of independence for the residuals from the AR(2)-$\mathrm{APARCH}(1,1)$ model. But according to these test statistics, the null hypothesis of iid residuals was not rejected for any models estimated under the assumption of a skewed Student's t-distribution. In addition, the AR-APARCH models generate insignificant ARCH(10) statistics, indicating that they can account for the dynamic conditional variance in the sample data.

The empirical results (Table 2) imply several findings. First, for all sample series, the values of the persistence coefficient (β_1) are highly significant and close to 1, which indicates a high degree of volatility

[7] The skewed Student's *t* APARCH model incorporates the normal and Student's *t* innovation models. In addition, the RiskMetrics model does not require any estimation for the conditional volatility specification (see Equation (3)).

[8] The LM ARCH statistics of Engle (1982) are used to test the presence of remaining ARCH effects in the residuals. The ARCH(10) statistics are the values of test statistics for the joint significance of lagged squared residuals in the regression of squared residuals on a constant and on 10th-order lagged squared residuals.

persistence. Furthermore, the values of the stationary condition in the bottom row of the table are lower than 1, implying that the APARCH models are stationary in all of the cases.

[Table 2] Estimation Results from the Skewed Student's *t* APARCH Model

Mean equation: $y_t = \mu + \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varepsilon_t$, $\varepsilon_t = z_t \sigma_t$, $z_t \sim SKST(0,1,k,\upsilon)$
Variance equation: $\sigma_t^{\delta} = \omega + \alpha_1(\varepsilon_{t-1} + \gamma_1 \varepsilon_{t-1})^{\sigma} + \beta_1 \sigma_{t-1}^{\delta}$,

	Straits Times	KOSPI 200	KLSE	JAKCOMP	SET
μ	0.010	-0.036	0.038	0.029	-0.008
	(0.020)	(0.030)	(0.024)	(0.022)	(0.028)
φ_1	0.179	0.035	0.184	0.230	0.108
	(0.019)**	(0.017)**	(0.019)**	(0.020)**	(0.018)**
φ_2	-0.025	-0.001	-0.001	0.057	0.059
	(0.017)	(0.017)	(0.017)	(0.017)**	(0.018)**
ω	0.035	0.025	0.024	0.493	0.093
	(0.008)**	(0.007)**	(0.006)**	(0.012)**	(0.022)**
α_1	0.128	0.101	0.136	0.250	0.133
	(0.016)**	(0.011)**	(0.017)**	(0.034)**	(0.016)**
β_1	0.866	0.904	0.877	0.803	0.838
	(0.018)**	(0.011)**	(0.016)**	(0.026)**	(0.019)**
γ_1	0.257	0.195	0.254	0.149	0.189
	(0.050)**	(0.041)**	(0.044)**	(0.038)**	(0.038)**
δ	1.467	1.559	1.511	1.329	2.038
	(0.179)**	(0.197)**	(0.163)**	(0.135)**	(0.245)**
υ	5.404	9.841	4.148	3.384	5.341
	(0.564)**	(1.356)**	(0.286)**	(0.208)**	(0.469)**
$\ln(k)$	0.030	0.056	0.067	0.053	0.056
	(0.021)	(0.021)**	(0.020)**	(0.019)**	(0.020)**
$\ln(L)$	-5994.34	-8194.42	-5938.08	-6232.49	-7511.45
AIC	2.876331	3.761771	2.849380	2.990414	3.603091
$Q(20)$	30.52**	28.20	38.87**	60.79**	37.89**
$Q_S(20)$	12.02	23.81	16.31	10.37	28.10
ARCH(10)	0.975	1.146	0.979	0.584	1.338
BDS(10)	0.558	1.953	1.307	1.125	-0.072
Stationary $\alpha_1 E(\lvert z\rvert - \gamma_1 z)^{\delta} + \beta_1$	0.975	0.989	0.981	0.977	0.972

Notes: Standard errors are indicated in parentheses below the corresponding parameter estimates. The $\ln(L)$ is the maximized Gaussian log likelihood. AIC is the Akaike information criterion. ARCH(10) represents the F-statistic of the ARCH test statistic with lag 10. The ARCH and BDS tests are based on the standardized residuals. See also Table 1.

Second, the power term (δ) for the case of SET is close to 2, indicating that a squared error term would correctly specify the conditional variance, whereas for the other series, δ ranges from 1.329 to 1.559, implying that modeling the conditional standard deviation would better fit the data than modeling the conditional variance. Additionally, the asymmetry coefficients (γ_1) of all stock indices is positive and statistically significant, indicating that the negative returns lead to higher volatility than do positive returns (called, the leverage effect).

Third, the Student's t parameter υ is greater than 2 and statistically significant for all of the cases, implying that the densities of all residuals exhibit fat tails. In particular, υ is the smallest for JAKCOMP (3.384), meaning that the probability distribution of JAKCOMP returns exhibit the fattest tails of all of the indexes. In addition, for the Straits Times, the estimated value of $\ln(k)$ is insignificant, indicating that its distribution is symmetric, whereas the conditional distributions of the other cases are skewed to the right side because the values of $\ln(k)$ are significantly positive. Thus, the Student's t-distribution is preferred for the case of Straits Times, whereas for the other cases-KOSPI 200, KLSE, JAKCOMP, and SET-it is preferable to use the skewed Student's t-distribution.

3.3. Empirical Results for VaR Analysis

We next computed not only in-sample VaR to examine the estimated model's goodness-of-fit, but also out-of-sample VaR to evaluate the forecasting quality of the estimated model. All models were tested at the 5% and 1% significance levels (α), and their performance was assessed by computing the failure rate from the stock index returns. If the VaR models are specified correctly, the estimated failure rate will be equal to the significance level.

3.3.1. In-sample VaR Analysis

The empirical results for the in-sample VaR analysis for the Straits Times, KOSPI 200, KLSE, JAKCOMP, and SET were determined for the 95% confidence level (Table 3) and 99% confidence level (Table 4). Interestingly, VaR models based on a normal distribution (RiskMetrics and normal APARCH models) sometimes demonstrate good performance for long and short positions at the 95% confidence level. For long positions, the RiskMetrics model achieves the lowest values of the Kupiec LR statistics in the case of the Straits Times, JAKCOMP, and SET. However, as the confidence level increases from 95% to 99%, the failure rates exceed the prescribed quantiles of all in-sample VaR analyses using a normal distribution. The null hypothesis of $f = \alpha$ is rejected by the Kupiec LR test for both long and short positions in all but one case using the normal distribution (Table 4). We conclude that models based on a normal distribution underestimate the VaR.

[Table 3] In-sample Performance for Different VaR Models $(\alpha = 0.05)$

	Short position			Long position		
VaR model	Failure rate	Kupiec LR	P-value	Failure rate	Kupiec LR	P-value
Straits Times						
RiskMetrics	0.0484	0.232	0.629	0.0507	0.053	0.817
APARCH-N	0.0422	5.699**	0.016	0.0409	7.638**	0.005
APARCH-St	0.0560	0.025	0.873	0.0450	2.242	0.134
APARCH-SkSt	0.0477	0.486	0.485	0.0467	0.973	0.323
KOSPI 200						
RiskMetrics	0.0567	3.872**	0.049	0.0529	0.788	0.374
APARCH-N	0.0551	2.244	0.134	0.0449	2.436	0.118
APARCH-St	0.0583	5.921**	0.014	0.0460	1.447	0.228
APARCH-SkSt	0.0546	1.858	0.172	0.0506	0.040	0.840
KLSE						
RiskMetrics	0.0532	0.868	0.351	0.0488	0.114	0.734
APARCH-N	0.0487	0.168	0.681	0.0400	9.408**	0.002
APARCH-St	0.0578	5.006**	0.025	0.0450	2.242	0.134
APARCH-SkSt	0.0518	0.262	0.608	0.0502	0.007	0.929
JAKCOMP						
RiskMetrics	0.0537	1.146	0.284	0.0498	0.002	0.957
APARCH-N	0.0463	1.281	0.257	0.0409	7.638**	0.005
APARCH-St	0.0568	3.863**	0.049	0.0500	0.000	0.985
APARCH-SkSt	0.0520	0.339	0.560	0.0550	2.207	0.137
SET						
RiskMetrics	0.0554	2.416	0.120	0.0486	0.168	0.681
APARCH-N	0.0523	0.425	0.514	0.0431	4.362**	0.036
APARCH-St	0.0580	5.314**	0.021	0.0488	0.114	0.734
APARCH-SkSt	0.0532	0.868	0.351	0.0524	0.521	0.470

Note: See Table 2.

[Table 4] In-sample Performance for Different VaR Models $(\alpha = 0.01)$

	Short position			Long position		
VaR model	Failure rate	Kupiec *LR*	P-value	Failure rate	Kupiec *LR*	P-value
Straits Times						
RiskMetrics	0.0144	7.098**	0.007	0.0203	34.81**	0.000
APARCH-N	0.0139	5.698**	0.016	0.0146	7.849**	0.005
APARCH-St	0.0103	0.037	0.846	0.0093	0.187	0.665
APARCH-SkSt	0.0089	0.567	0.451	0.0105	0.120	0.728
KOSPI 200						
RiskMetrics	0.0152	10.02**	0.001	0.0139	6.223**	0.012
APARCH-N	0.0145	7.647**	0.005	0.0105	0.128	0.719
APARCH-St	0.0097	0.061	0.804	0.0075	2.851	0.091
APARCH-SkSt	0.0074	3.445	0.063	0.0096	0.061	0.804
KLSE						
RiskMetrics	0.0204	34.81**	0.000	0.0203	34.81**	0.000
APARCH-N	0.0173	18.19**	0.000	0.0155	11.18**	0.000
APARCH-St	0.0120	1.548	0.213	0.0100	0.001	0.969
APARCH-SkSt	0.0084	1.166	0.280	0.0112	0.640	0.423
JAKCOMP						
RiskMetrics	0.0223	47.10**	0.000	0.0234	55.51**	0.000
APARCH-N	0.0163	14.00**	0.000	0.0162	14.00**	0.000
APARCH-St	0.0094	0.187	0.665	0.0067	5.174**	0.022
APARCH-SkSt	0.0077	2.501	0.113	0.0074	3.069	0.079
SET						
RiskMetrics	0.0223	47.10**	0.000	0.0177	20.46**	0.000
APARCH-N	0.0154	10.29**	0.001	0.0134	4.438**	0.035
APARCH-St	0.0111	0.423	0.515	0.0086	0.839	0.359
APARCH-SkSt	0.0099	0.013	0.906	0.0105	0.120	0.728

Note: See Table 2.

In contrast, the use of the Student's t innovation model leads to better in-sample performance than do the models with a normal distribution at the 99% confidence level (Table 4), but its performance is not satisfactory at the 95% confidence level (Table 3). For short positions, its performance is even worse than the models with a normal distribution. However, the skewed Student's t APARCH $(1,1)$ models generally improve on all other specifications for both long and short positions, especially at the 99% confidence level. According to the Kupiec LR test statistic, none of the models with skewed Student's t-distribution is rejected at the 95% or 99% confidence levels (Tables 3 and 4). Thus, the skewed Student's *t*-distribution VaR models predict critical loss more accurately than do the models with normal or Student's *t* innovation in-sample VaR analysis.

[Table 5] Out-of-sample Performance for Different VaR Models ($\alpha = 0.05$)

	Short position			Long position		
VaR model	Failure rate	Kupiec *LR*	P-value	Failure rate	Kupiec *LR*	P-value
Straits Times						
RiskMetrics	0.0500	0.000	1.000	0.0539	0.407	0.523
APARCH-N	0.0342	7.485**	0.006	0.0404	2.566	0.109
APARCH-St	0.0461	0.428	0.512	0.0420	1.762	0.184
APARCH-SkSt	0.0421	1.762	0.184	0.0428	1.419	0.233
KOSPI 200						
RiskMetrics	0.0493	0.016	0.896	0.0531	0.262	0.608
APARCH-N	0.0485	0.067	0.794	0.0484	0.067	0.794
APARCH-St	0.0548	0.584	0.444	0.0515	0.066	0.796
APARCH-SkSt	0.0470	0.272	0.601	0.0595	2.273	0.131
KLSE						
RiskMetrics	0.0556	0.791	0.373	0.0452	0.620	0.430
APARCH-N	0.0397	3.029	0.081	0.0238	22.38**	0.000
APARCH-St	0.0437	1.115	0.290	0.0261	18.06**	0.000
APARCH-SkSt	0.0366	5.306**	0.021	0.0293	13.17**	0.000
JAKCOMP						
RiskMetrics	0.0675	7.324**	0.006	0.0460	0.428	0.512
APARCH-N	0.0516	0.066	0.796	0.0404	2.566	0.109
APARCH-St	0.0739	13.19**	0.000	0.0571	1.296	0.254
APARCH-SkSt	0.0659	6.103**	0.013	0.0579	1.593	0.206
SET						
RiskMetrics	0.0516	0.066	0.796	0.0436	1.115	0.290
APARCH-N	0.0437	1.115	0.290	0.0300	11.07**	0.000
APARCH-St	0.0524	0.148	0.700	0.0317	10.09**	0.001
APARCH-SkSt	0.0445	0.849	0.356	0.0373	4.672**	0.030

Note: See Table 2.

3.3.2. Out-of-sample VaR Analysis

We further assessed the performance of the model with the normal, Student's *t*, and skewed Student's *t* innovations by computing out-of-sample VaR forecasts. Following the analysis procedure of Giot and Laurent (2003), we used the last five years (1,260 observations) of sample data to conduct out-of-sample forecasting for the period from January 2001 to December 2005. After we estimated the AR(2)-APARCH model using data from January 1990 to December 2000, which is the complete sample excluding the last five years, one-day-ahead we generated a VaR (for both long and short positions) by applying the rolling procedure for every 50 observations. Both results were recorded for later statistical assessment. We repeated this process of recursive model estimation and calculation of one-day-ahead forecasts, and re-examined 26 of every 50 observations for each estimated model using the above rolling approach. The empirical results for out-of-sample VaR analysis were determined for the 95% confidence level (Table 5) and 99% confidence level (Table 6). In general, the results for out-of-sample VaR analysis are consistent with those for the in-sample VaR analysis.

[Table 6] Out-of-sample Performance for Different VaR Models ($\alpha = 0.01$)

	Short position			Long position		
VaR model	Failure rate	Kupiec *LR*	P-value	Failure rate	Kupiec *LR*	P-value
Straits Times						
RiskMetrics	0.0104	0.012	0.910	0.0222	14.10**	0.000
APARCH-N	0.0096	0.029	0.864	0.0103	0.012	0.910
APARCH-St	0.0080	0.583	0.445	0.0055	2.996	0.083
APARCH-SkSt	0.0064	1.948	0.162	0.0071	1.153	0.282
KOSPI 200						
RiskMetrics	0.0143	2.063	0.150	0.0166	4.711**	0.029
APARCH-N	0.0127	0.853	0.355	0.0150	2.841	0.091
APARCH-St	0.0088	0.214	0.643	0.0103	0.012	0.910
APARCH-SkSt	0.0060	2.996	0.083	0.0150	2.841	0.091
KLSE						
RiskMetrics	0.0183	6.969**	0.008	0.0190	8.233**	0.004
APARCH-N	0.0112	0.151	0.696	0.0111	0.151	0.696
APARCH-St	0.0064	1.948	0.162	0.0055	2.996	0.083
APARCH-SkSt	0.0056	2.996	0.083	0.0079	0.583	0.445
JAKCOMP						
RiskMetrics	0.0223	47.10**	0.000	0.0234	55.51**	0.000
APARCH-N	0.0163	14.00**	0.000	0.0162	14.00**	0.000
APARCH-St	0.0094	0.187	0.665	0.0067	5.174**	0.022
APARCH-SkSt	0.0077	2.501	0.113	0.0074	3.069	0.079
SET						
RiskMetrics	0.0223	47.10**	0.000	0.0177	20.46**	0.000
APARCH-N	0.0154	10.29**	0.001	0.0134	4.438**	0.035
APARCH-St	0.0111	0.423	0.515	0.0086	0.839	0.359
APARCH-SkSt	0.0099	0.013	0.906	0.0105	0.120	0.728

Note: See Table 2.

At the 95% confidence level, the RiskMetrics model provides more accurate volatility forecasting than do the normal distribution and symmetric and asymmetric Student's *t*-distribution VaR models (Table 5). For example, the RiskMetrics model is successful in evaluating risk at the 95% confidence level except for the short positions of the JAKCOMP. Technically, as the RiskMetrics model strictly imposes a one-day-ahead VaR confidence level of 95%, and under a normal distribution, it provides better VaR performance at the 95% confidence level than under other distributions. However, for the higher 99% confidence level, the null hypothesis of the Kupiec test for the RiskMetrics model is rejected in the majority of cases (Table 6).

The accuracy of the skewed Student's *t* APARCH model's VaR predictions is statistically superior for both long and short positions at the 99% confidence level (Table 6).

In sum, the out-of-sample performance of RiskMetrics in estimating VaR is adequate for the confidence level of 95%, whereas the skewed Student's *t* distribution performs better at the 99% confidence level.

As an interesting note, our empirical results of out-of-sample VaR analysis are inconsistent with those of Giot and Laurent (2003), who found the APARCH model under the skewed Student's *t*-distribution to be the best at any confidence level for US stock indices.

IV. CONCLUSIONS

Recent studies regarding VaR have focused on establishing an accurate distribution for financial asset returns, which exhibit more skewed means and fatter tails than does the normal distribution. A correctly specified distribution might improve the estimated performance of the value-at-risk models and result in more effective risk management of or investment in financial assets.

We compared the performance of VaR models under the different distributions in Asian stock indices, i.e., Straits Times, KOSPI 200, KLSE, JAKCOMP, and SET. From in- and out-of-sample analyses, the skewed Student's *t* APARCH model generally provided the most accurate VaR estimates at both the 95% confidence level and the 99% confidence level. Interestingly, the RiskMetrics model based on a normal distribution sometimes provided good forecasting of the VaR at the lower confidence level (95%), but not at the higher confidence level (99%), The model accommodates volatility asymmetry and captures the skewness and excess kurtosis in stock returns, thus improving estimates of the VaR.

Based on our VaR analysis, risk-averse investors or portfolio managers may prefer to use the normal (skewed Student's *t*) distribution innovation at the 95% (99%) confidence levels. However, portfolio managers of long and short trading positions in Asian stock markets might build optimal margin levels using the VaR computation based on the skewed Student's *t* APARCH model. The main limitation to this approach is that a skewed Student's *t*-distribution cannot be extended to multivariate GARCH models. Future research should consider the S_U-normal distribution of Choi and Nam (2008), which develops a flexible distribution for multivariate GARCH models.

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