

A STRUCTURAL VECTOR ERROR CORRECTION MODEL WITH SHORT-RUN AND LONG-RUN RESTRICTIONS

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We consider structural vector error correction models (VECMs) in which permanent shocks are partially identified with a set of long-run restrictions, and fully identified with an additional set of short-run restrictions. An identification method with a combination of short-run and long-run restrictions has been studied in the vector autoregressive models literature, but not thoroughly applied to the VECM framework. There exists a separation in the literature; permanent shocks are identified with long-run restrictions while transitory shocks are identified with short-run restrictions. This paper's innovation is the identification of permanent shocks using both horizontal restrictions.

JEL Classification: E32, C32

Keywords: Cointegration, Identification, Estimation, Impulse Response, Forecast-Error Variance Decomposition, Money Demand

I. INTRODUCTION

This paper develops a method of identifying structural vector error correction models (VECM) using short-run and long-run restrictions. The

Received for publication: July 25, 2006. Revision accepted: Feb. 22, 2008.

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identification scheme with a combination of short-run and long-run restrictions was first developed by Gali (1992) for vector autoregressive (VAR) models. However, it has not been thoroughly studied in VECM literature. This motivates us to develop the method of identification with short-run and long-run restrictions for VECM.

Impulse response analysis has been used in various empirical studies within a framework of vector autoregressive models (VAR) since the publication of Sims (1980). Blanchard and Watson (1986), Bernanke (1986), and Blanchard (1989) discuss the identification method with contemporaneous short-run restrictions, while Blanchard and Quah (1989) use long-run restrictions for identification. Gali (1992) combines short-run and long-run restrictions using IS-LM models.

It is well known that VAR models in difference result in misspecification problems in the presence of cointegration. The IS-LM model of Gali (1992) is subject to this critique as long as money demand is stable or the real interest rate is stationary. Therefore, it is necessary to develop an identification method in VECM with short-run and long-run restrictions.

King *et al.* (1991; KPSW for short), first develop a method of identification of VECMs with long-run restrictions. This paper generalizes the study of KPSW. First, we use block recursive assumptions on the long-run effects of the structural permanent shocks. Second, we combine a set of long-run restrictions and a set of short-run restrictions to identify the structural permanent shocks.

Vlaar (2004), in his independent work, proposes a two-step estimation of structural VECMs in the framework of Giannini's (1992) C-model. He uses both short-run and long-run restriction' on the just- or over-identified structural VECMs and performs FIML estimation of structural parameters in the second step using the reduced-form model estimates in the first step. In contrast, this paper estimates the structural parameters of just-identified models vice simple computations without involving FIML estimation in the second step. This method is useful for constructing models that impose stylized facts in the short and long run: (i) money does not affect output contemporaneously; (ii) money does not affect output in the long run, etc.

The rest of the paper is organized as follows. Section II presents the identification problem and develops the method of estimation. Section III provides empirical results. Section IV concludes. Appendix A describes the data employed and Appendix B provides the algorithm for computation of structural parameters.

II. IDENTIFICATION WITH SHORT-RUN AND LONG-RUN RESTRICTIONS

2.1 The Model

To highlight the method of identification with long-run and short-run restrictions in VECM, we slightly modify the KPSW model. Consider the four-variable model consisting of output (y_t), nominal interest rate (R_t), inflation rate (Δp_t), and real money supply ($m_t - p_t$), in which p_t is the price level.¹ Let $x_t = (y_t, R_t, \Delta p_t, m_t - p_t)'$. Following KPSW, we assume that x_t is a vector difference stationary process, $I(1)$, and that the real interest rate is nonstationary.² We further assume that there exists only one cointegrating vector representing the stable money demand relation:

$$m_t - p_t = \beta_y y_t - \beta_R R_t + \eta_t. \quad (1)$$

In order to interpret how the structural shocks are propagated in the system, we also consider the conventional Fisher equation:

$$R_t = r_t + E_t \Delta p_{t+1}, \quad (2)$$

where r_t is the *ex ante* real interest rate and $E_t \Delta p_{t+1}$ is the expected inflation rate between t and $t+1$. The model is consistent with KPSW

¹ y_t , m_t , and p_t are in logarithms.

² KPSW also consider a model in which the real interest rate is stationary for their sensitivity analysis; yet, we stick to the assumption that the real interest rate is nonstationary to highlight the generalization of recursive assumptions on the long-run effects. Otherwise, we have two independent cointegrating vectors and the long-run effects of the structural permanent shocks become recursive.

except that we consider the four-variable system with one cointegrating vector instead of the six-variable system with three independent cointegrating vectors.³

2.2 Short-Run and Long-Run Restrictions

The present paper builds on KPSW, who extend Blanchard and Quah's (1989) method for VAR models to the VECM framework. Consider the structural Wold representation of the form,⁴

$$\Delta x_t = \Gamma(L)v_t \quad (3)$$

and its reduced form

$$\Delta x_t = C(L)\varepsilon_t, \quad (4)$$

where x_t denotes an n ($= 4$) dimensional vector of $I(1)$ time series, $\Gamma(L) = \Gamma_0 + \sum_{i=1}^{\infty} \Gamma_i L^i$, $C(L) = \Gamma(L)\Gamma_0^{-1}$ and $\varepsilon_t = \Gamma_0 v_t$. We assume that v_t is an $n \times 1$ vector of serially uncorrelated structural disturbances with a mean of zero and a covariance matrix Σ_v , and that ε_t is an $n \times 1$ vector of serially uncorrelated linear forecast errors with a mean of zero and a covariance matrix Σ .

Let r denote the number of cointegrating vectors and k denote common trends. In particular, $r = 1$ and $k = 3$ in our specific model. Having represented the model in (3), we can interpret the long-run restrictions imposed by the cointegrating relations according to Stock and Watson's (1988) common trends representation. They show that the cointegrated model can be represented in terms of a reduced number of common stochastic trends ($k = n - r$), and transitory components (r). These common trends are generated by permanent shocks so that v_t is decomposed into $(v_t^{k'}, v_t^{r'})'$, where v_t^k is a k dimensional vector of

³ The six-variable system includes consumption and investment in addition to the four variables, and the additional cointegrating vectors are given by the "great ratios" that represent the same growth rate of output, consumption, and investment.

⁴ For simplicity, we drop the deterministic terms.

permanent shocks and v_t^r is an r dimensional vector of transitory shocks. As described in KPSW, this decomposition ensures that

$$\Gamma(1) = [A \quad 0], \quad (5)$$

in which A is an $n \times k$ matrix and 0 is an $n \times r$ matrix with zeros representing long-run effects of permanent shocks and transitory shocks, respectively. The restrictions implied by cointegration separate the structural permanent shocks and the structural transitory shocks. As interpreted by KPSW, we consider three structural permanent shocks: the balanced-growth shock (v_t^y), the long-run neutral inflation shock ($v_t^{\Delta p}$), and the real interest rate shock ($v_t^{R-\Delta p}$).

Building on Blanchard and Quah (1989), KPSW use the long-run restrictions that the balanced-growth shock has long-run effects on the level of output, but the other two permanent shocks ($v_t^{\Delta p}$ and $v_t^{R-\Delta p}$) have no long-run effects on the level of output. In contrast with KPSW, we relax the assumption of no long-run effects of the real interest rate shock on the inflation rate.⁵ These long-run restrictions identify the first permanent shock from other permanent shocks, but can not identify the second or third permanent shocks. We impose either the short-run restriction that the contemporaneous price does not enter the money supply rule or the restriction that the inflation shock does not affect output contemporaneously, in order to identify these two permanent shocks. Gali (1992) employs this set of short-run restrictions in order to combine the short-run and long-run restrictions in the framework of vector autoregressive models.⁶ This paper can also be considered as the VECM version of Gali's (1992) study.

⁵ As KPSW noted, the standard model does not predict the responses of output to the shock in $v_t^{R-\Delta p}$. Therefore, it is reasonable to relax the assumptions on the effects of $v_t^{R-\Delta p}$, and to check the sensitivities.

⁶ Gali (1992) suggests two more alternative assumptions: that contemporaneous output does not enter the money supply rule and suggests contemporaneous homogeneity in money demand. Moreover, he considers a third set of restrictions: that the money supply and the money demand shocks have no contemporaneous effects on output. Note also that the interpretation of structural shocks in Gali (1992) is different from that in KPSW. In the present paper, we follow the interpretation of KPSW, and consider only one short-run restriction to make the model just-identified.

2.3 Identification

Following KPSW, decompose Γ_0 and Γ_0^{-1} as:

$$\Gamma_0 = \begin{bmatrix} H & J \end{bmatrix}, \quad \Gamma_0^{-1} = \begin{bmatrix} G \\ E \end{bmatrix} \quad (6)$$

where H, J, G and E are $n \times k$, $n \times r$, $k \times n$, and $r \times n$ matrices, respectively. Note that the permanent shocks are identified once H or G are identified, and that these two matrices have the one-to-one relation $G = \Sigma_v^k H' \Sigma^{-1}$, where Σ_v^k is the variance-covariance matrix of permanent shocks, v_t^k .⁷ Therefore, the above decomposition of Γ_0 does not generate additional free parameters.

The identifying scheme of the present paper basically follows that of KPSW, but leaves room to generalize their model as described below. Our identification uses the results of Engle and Granger (1987). Consider a structural model of the form

$$B(L)x_t = v_t, \quad (7)$$

where x_t is an $n \times 1$ vector of time series that are cointegrated of order (1,1). To be specific, each element of x_t is difference stationary, and there exist r linear combinations of x_t that are stationary. We assume that $0 < r < n$. The reduced form of (7) is given by

$$A(L)x_t = \varepsilon_t. \quad (8)$$

When the series are cointegrated, from $A(L) = A(1)L + A^*(L)(1-L)$, Engle and Granger (1987) show that there exists an error correction representation:

$$A^*(L)\Delta x_t = -A(1)x_{t-1} + \varepsilon_t, \quad (9)$$

⁷ One can easily derive this relation from the relation of $\Gamma_0^{-1}\Sigma = \Sigma_v\Gamma_0'$.

where $A^*(L) = I - \sum_{i=1}^{p-1} A_i^* L^i$, $A_i^* = -\sum_{j=i+1}^p A_j$, and $-A(1) = \alpha\beta'$ where α and β are $n \times r$ matrices with full column rank r . Engle and Granger (1987) show that $C(1)$ in (4) has the forms

$$\beta' C(1) = 0 \tag{10}$$

and

$$C(1)\alpha = 0. \tag{11}$$

Following KPSW, let $C(1) = \hat{A}D$ and $A = \hat{A}\Pi$, where \hat{A} is a known $n \times k$ matrix, Π is a $k \times k$ matrix, and $D = (\hat{A}'\hat{A})^{-1}\hat{A}'C(1)$. KPSW assume that Π is a lower triangular matrix with 1 on the diagonal, but we relax this assumption so that Π is a lower block triangular matrix:

$$\Pi = \begin{bmatrix} 1 & 0 & 0 \\ \pi_{21} & 1 & \pi_{23} \\ \pi_{31} & \pi_{32} & 1 - \pi_{23} \end{bmatrix}. \tag{12}$$

First, $C(1)\Gamma_0 = \Gamma(1)$ implies $C(1)H = \hat{A}\Pi$, so that we have a first set of restrictions of the form:

$$DH = \Pi, \tag{13}$$

which gives $\frac{k(k+1)}{2} - 1$ restrictions on H . Note that these long-run restrictions identify the first permanent shock (v_t^y) from the other permanent shocks separately, but the second and the third permanent shocks are not identified.

Second, (11) can be expressed as $\Gamma(1)\Gamma_0^{-1}\alpha = 0$, so that $G\alpha = 0$. Since $G = \Sigma_v^k H' \Sigma^{-1}$, we have a second set of restrictions of the form

$$\alpha' \Sigma^{-1} H = 0, \quad (14)$$

which gives kr restrictions on H .

Third, from $C(1)\varepsilon_t = \Gamma(1)v_t$, $\hat{A}D\varepsilon_t = \hat{A}\Pi v_t^k$. Therefore, we have a third set of restrictions of the form

$$\Pi \Sigma_v^k \Pi' = D \Sigma D', \quad (15)$$

which gives $\frac{k(k-1)}{2}$ restrictions on H provided that Σ_v^k is diagonal.

Finally, the short-run restriction that the contemporaneous price does not enter the money supply rule or the inflation shock does not affect output contemporaneously implies

$$G_{23} = G_{24} \text{ or } H_{12} = 0, \quad (16)$$

Respectively. This additional restriction identifies the second permanent shock ($v_t^{\Delta p}$) from the third permanent shock ($v_t^{R-\Delta p}$).

The above four sets of restrictions give nk restrictions, and the model is just-identified in the sense of solely identifying the matrix H .

2.4 Estimation

Having estimated the model (9), one can compute all the structural parameters sequentially. From $G = \Sigma_v^k H' \Sigma^{-1}$, the short-run restriction (16) implies that $H'_{\cdot,2} \Sigma_{\cdot,3}^{-1} - H'_{\cdot,2} \Sigma_{\cdot,4}^{-1} = 0$, where $M_{\cdot,j}$ denotes the j -th column of the matrix M . Combining this condition and the second columns of (13) and (14) yields

$$H_{\cdot,2} = \begin{bmatrix} D_{1:2,\cdot} \\ \alpha' \Sigma^{-1} \\ \Sigma_{3,\cdot}^{-1} - \Sigma_{4,\cdot}^{-1} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad (17)$$

where $M_{1:2}$ denote the first two rows of the matrix M and M_{i} denotes the i -th row of the matrix M . Using $H_{\cdot,2}$, we can compute π_{32} by

$$\pi_{32} = D_{3\cdot} H_{\cdot,2}. \tag{18}$$

Having computed π_{32} , one can compute Π from (15).⁸ The other columns of H ($H_{\cdot,1}$ and $H_{\cdot,3}$) are given by

$$H_{\cdot,j} = \begin{bmatrix} D \\ \alpha' \Sigma^{-1} \end{bmatrix}^{-1} \begin{bmatrix} \Pi_{\cdot,j} \\ 0 \end{bmatrix}, \tag{19}$$

for $j = 1,3$.

Therefore, the permanent shocks and the short-run dynamics are respectively identified by

$$v_t^k = G \varepsilon_t \tag{20}$$

and

$$\Gamma(L)^k = C(L)H. \tag{21}$$

Fisher *et al.* (1995) extend the KPSW method to identify transitory shocks. The first set of restrictions comes from conventional orthogonality conditions of the form

$$\begin{aligned} Cov(v_t^r, v_t) &= [E \Sigma G' \quad E \Sigma E'] \\ &= [0 \quad \Sigma_v^r], \end{aligned} \tag{22}$$

where Σ_v^r is $r \times r$ a diagonal matrix. This assumption gives $nr - \frac{r(r+1)}{2}$ restrictions on J (or E) provided that H (or G) is

⁸ We use the Newton method to get the solution. See Appendix B for the detailed algorithm.

identified. As these transitory shocks can not be identified from long-run restrictions, Fisher *et al.* (1995) suggest a recursive structure on the contemporaneous effects of the transitory structural shocks. For example, the sub-matrix of E that consists of the last r rows and the last r columns of E is lower triangular with 1s on the diagonal. With this set of additional $\frac{r(r+1)}{2}$ restrictions, the model is just-identified in the sense of identifying the matrix Γ . Similarly, the transitory shocks and the short-run dynamics are respectively identified by

$$v_t^r = E\varepsilon_t \quad (23)$$

and

$$\Gamma(L)^r = C(L)J. \quad (24)$$

In our specific model, we do not need additional assumptions other than normalization conditions for the identification of transitory shocks since $r = 1$. Thus, the transitory shock is identified and is computed from G . Having normalized $E_{14} = 1$, we can rewrite (22) as

$$\begin{bmatrix} E_{1,1:3} & 1 \end{bmatrix} \begin{bmatrix} \Sigma_{1:3,\cdot} \\ \Sigma_{4,\cdot} \end{bmatrix} G' = 0 \quad (25)$$

and $E_{1,1:3}$ is given by

$$E_{1,1:3} = -\Sigma_{4,\cdot} G' [\Sigma_{1:3,\cdot} G']^{-1}. \quad (26)$$

From $E\Sigma = \Sigma_v' J'$, J is given by

$$J = \Sigma E' (\Sigma_v')^{-1}, \quad (27)$$

where $\Sigma_v' = E\Sigma E'$ from (23). Therefore, the transitory shocks and the short-run dynamics are identified by (23) and (24), respectively.

III. EMPIRICAL EVIDENCE

3.1 The Data

The data are quarterly US observations over the period 1949:1-2000:4. These data are similar to those used by KPSW over 1949:1-1988:4, except we replace gross national product with gross domestic product.⁹ Output is measured by the logarithm of per capita private gross domestic product (y), and the price level is measured by the logarithm of its implicit deflator (p). The logarithm of M_2 is used for the money supply measure (m), the real balance is measured by $m - p$, and the three-month US Treasury Bill rate is used for the interest rate (R). See Appendix A for a detailed description of the data. Since the data cover the 1990s, we consider the possible breakdown of a stable money demand relation in 1990s due to financial innovations. Carlson *et al.* (1999) show that the M_2 relation breaks down in the 1990s according to the sharp increase of M_2 velocity between 1990 and 1994. They illustrate the upward shift in M_2 velocity due to financial innovations such as growth in stock and bond mutual funds that are close substitutes for time deposits. In order to capture these financial innovations, they include a dummy variable that is zero before 1990:1, one after 1994:4, and which increases linearly between them. As long as this dummy variable is included in the money demand regression, they find a stable relation. This paper adopts the same dummy variable as theirs, and runs the regression over 1954:1-1988:4 and 1954:1-1999:4.¹⁰

3.2 Unit Roots and Cointegration

Univariate properties of the four variables are summarized in Table 1. It shows that output, real balance, and nominal interest rate are unit-root processes with a 5% significance level over the two sample periods. The inflation rate is also characterized by the unit-root process with a 10%

⁹ This change does not affect the main results.

¹⁰ KPSW use data prior to 1954:1 as initial observations in lags to avoid the impact of price controls during the Korean War. Periods from 1989:1-1989:4 and 2000:1-2000:4 are used for observations in leads of regressors in the dynamic OLS.

significance level, except the results of Phillips-Perron tests. These imply that the price level and nominal money are $I(2)$ processes. The results of unit-root tests on the real interest rate are mixed. Augmented Dickey-Fuller tests show that the real interest rate is an $I(1)$ process over two sample periods, while Phillips-Perron tests, Park's (1990) $J(p,q)$ tests, and Park's (1990) $G(p,q)$ tests show that it is an $I(0)$ process. Johansen's (1995) maximum likelihood tests show that the real interest rate is an $I(1)$ process over 1954:1-1988:4, while an $I(0)$ process over 1954:1-1999:4 with a 10% significance level.

Table 2 shows the results of cointegration tests for the money demand equation. Even with inclusion of the dummy variables, Said-Dickey tests and Park's (1990) $I(p,q)$ tests cannot reject the null of no cointegration, while Johansen's (1995) tests cannot reject the null of cointegration with a 5% significance level. Park's (1990) $H(p,q)$ tests show the mixing results that the money-demand relation is not cointegrated over 1954:1-1988:4 in Panel A, but is (not) cointegrated over 1954:1-1999:4, when the dummy variable is (not) included in the regression in Panel C (Panel B, respectively). Most estimates of the coefficient on output are near 1 in Panel A and Panel C, while far from 1 in Panel B. The coefficient on the nominal interest rate even changes sign in Panel B. These results justify the inclusion of the dummy variable in the model.

The results of cointegration tests together with the univariate properties of the real interest rate imply that there might be, at most, two cointegrating relations among four variables. Table 3 shows the results of Johansen's (1995) rank tests. Maximum eigenvalue tests show that there is one cointegrating vector in Panel A, and two (no) cointegrating vectors in Panel C (Panel B, respectively). However, trace tests show less evidence of cointegration than the maximum eigenvalue tests.

From the preliminary unit-roots tests and cointegration tests, we include the dummy variable in the regression over 1954:1-1999:4 to consider the financial innovations in 1990s. The choice of cointegration rank is critical to structural VECMs. In order to compare these results with KPSW's, we choose one as the rank and the money demand equation as the cointegrating relation. As the dynamic OLS coefficients are not very different from other estimates, they are chosen as the estimates of the

cointegrating vector.¹¹

[Table 1] Unit Root Tests

| | Output | M ₂ | | Interest Rate | | Inflation Rate |
|-------------------------|-----------|----------------|-------------|---------------|--------------|----------------|
| | | Nominal | Real | Nominal | Real | |
| Sample: 1954:1 - 1988:4 | | | | | | |
| Null of I(1) | | | | | | |
| ADF | -3.12(1) | -3.13(8) | -2.15(1) | -1.64(7) | -1.58(17) | -1.34(15) |
| PP | -2.61(12) | -3.40(12)* | -1.81(12) | -1.87(12) | -5.35(12)*** | -4.26(12)*** |
| J(p,q) | 0.52(1,3) | 11.71(1,3) | 0.44(1,3) | 2.01(0,3) | 0.28(0,3)** | 1.34(0,3) |
| Null of I(0) | | | | | | |
| G(p,q) | 4.34(1,3) | 9.16(1,3)** | 2.99(1,3) | 6.76(0,3)* | 3.37(0,3) | 6.88(0,3)* |
| LRT($\chi^2_{(3)}$) | - | - | - | - | 7.65(0.054)* | - |
| Sample: 1954:1 - 1999:4 | | | | | | |
| Null of I(1) | | | | | | |
| ADF | -3.41(1)* | -2.42(13) | -2.14(1) | -1.95(17) | -1.89(17) | -1.09(17) |
| PP | -2.93(12) | -1.29(12) | -1.86(12) | -2.20(12) | -5.90(12)*** | -3.78(12)*** |
| J(p,q) | 0.54(1,3) | 6.56(1,3) | 1.26(1,3) | 1.31(0,3) | 0.21(0,3)** | 0.91(0,3) |
| Null of I(0) | | | | | | |
| G(p,q) | 5.79(1,3) | 10.08(1,3)*** | 6.64(1,3)** | 7.74(0,3)* | 3.24(0,3) | 7.32(0,3)* |
| LRT($\chi^2_{(3)}$) | - | - | - | - | 5.56(0.135) | - |

Note: The ADF test and Phillips-Perron statistics are computed by a regression equation with constant and trend terms for output and M₂, while with constant for interest rates and inflation rates. The number in each parenthesis of ADF denotes the lag length chosen following Campbell and Perron (1991) with maximum lag length as 20, while those of PP denote the lag length of serial correlation. The number in each parentheses of J(p,q) and G(p,q) denotes p and q chosen for each regression equation. LRT denotes Johansen's (1995) likelihood ratio test, and p-values are in parentheses. LRT is applied to test cointegration with a known vector. *, **, and *** denote that the null hypothesis is rejected with the significance levels 10%, 5%, and 1%, respectively.

¹¹ We choose four lags as the lag length in the following dynamic OLS:

$$m_t - p_t = \alpha + \beta_y y_t - \beta_R R_t + \sum_{i=-4}^4 \gamma_i \Delta y_{t-i} + \sum_{i=-4}^4 \delta_i \Delta R_{t-i} + \varepsilon_t$$

[Table 2] Cointegration Tests and Estimated Cointegrating Vectors

| Money Demand, $m - p = \alpha + \beta_y y - \beta_R R$ | | | |
|--|-----------|-----------|-----------------|
| | β_y | β_R | Test Statistics |
| Panel A: 1954:1 - 1988:4 | | | |
| Null of No Cointegration | | | |
| Said-Dickey | 1.027 | 0.013 | -2.62(18) |
| I(p,q) | 0.997 | 0.014 | 0.26(0,3) * |
| Null of Cointegration | | | |
| CCR, H(p,q) | 0.987 | 0.004 | 5.39(0,1) ** |
| DOLS | 1.035 | 0.012 | - |
| LRT($\chi^2_{(1)}$) | 1.108 | 0.016 | 2.07(0.150) |
| Panel B: 1954:1 - 1999:4 | | | |
| Null of No Cointegration | | | |
| Said-Dickey | 0.721 | -0.005 | -2.97(18) |
| I(p,q) | 1.014 | 0.013 | 2.47(0,3) |
| Null of Cointegration | | | |
| CCR, H(p,q) | 0.677 | -0.031 | 1.18(0,1) ** |
| DOLS | 0.686 | -0.013 | - |
| LRT($\chi^2_{(1)}$) | 0.522 | -0.051 | 0.73(0.392) |
| Panel C: 1954:1 - 1999:4, Dummy | | | |
| Null of No Cointegration | | | |
| Said-Dickey | 1.052 | 0.015 | -3.11(18) |
| I(p,q) | 1.076 | 0.014 | 0.42(0,3) |
| Null of Cointegration | | | |
| CCR, H(p,q) | 1.048 | 0.017 | 0.29(0,1) |
| DOLS | 1.051 | 0.015 | - |
| LRT($\chi^2_{(1)}$) | 0.942 | 0.001 | 2.61(0.106) |

Note: The Said-Dickey test statistics are computed by a cointegration equation with constant terms. The number in each parentheses of Said-Dickey denotes the lag length chosen following Campbell and Perron (1991) with maximum lag length as 20. The number in each parentheses of I(p,q) and H(p,q) denotes p and q chosen for each regression equation. CCR denotes Park's (1992) Canonical Cointegration Regression. LRT denotes Johansen's (1995) likelihood ratio test, and p-values are in parentheses. *, **, and *** denote that the null hypothesis is rejected with the significance levels 10%, 5%, and 1%, respectively.

[Table 3] Cointegration Rank Tests

| Eigen Value | max | Trace | Number of Cointegration (r) | k (=n-r) | Critical Value λ_{\max} | Trace |
|-------------------------------|------------|----------|--------------------------------|----------|------------------------------------|---------|
| Panel A: 1954:1-1988:4 | | | | | | |
| 0.1297 | 19.4433*** | 38.2288 | 0 | 4 | 17.8320 | 47.2080 |
| 0.0895 | 13.1276 | 18.7855 | 1 | 3 | 14.0360 | 29.3760 |
| 0.0253 | 3.5860 | 5.6579 | 2 | 2 | 11.4990 | 15.3400 |
| 0.0147 | 2.0720 | 2.0720 | 3 | 1 | 3.8410 | 3.8410 |
| Panel B: 1954:1-1999:4 | | | | | | |
| 0.0883 | 17.0151 | 33.6890 | 0 | 4 | 17.8320 | 47.2080 |
| 0.0579 | 10.9707 | 16.6739 | 1 | 3 | 14.0360 | 29.3760 |
| 0.0297 | 5.5414 | 5.7032 | 2 | 2 | 11.4990 | 15.3400 |
| 0.0009 | 0.1618 | 0.1618 | 3 | 1 | 3.8410 | 3.8410 |
| Panel C: 1954:1-1999:4, Dummy | | | | | | |
| 0.1100 | 21.4479** | 44.4835* | 0 | 4 | 17.8320 | 47.2080 |
| 0.0840 | 16.1377** | 23.0356 | 1 | 3 | 14.0360 | 29.3760 |
| 0.0291 | 5.4357 | 6.8980 | 2 | 2 | 11.4990 | 15.3400 |
| 0.0079 | 1.4623 | 1.4623 | 3 | 1 | 3.8410 | 3.8410 |

Note: The last two columns are critical values with a 5% significance level. *, **, and *** denote that the null hypothesis is rejected with the significance level 10%, 5%, and 1%, respectively.

Finally, we select nine lags as the lag lengths in the VECM. The lag length is chosen by the robustness of results. We check whether signs of the long-run multipliers of the VECM change as the lag length become smaller starting from 13. The signs are the same through lag lengths of 13 to 9, but different as the lag lengths become smaller than 9. Therefore, a lag length of 9 is robust, at least in terms of properties of long-run multipliers.¹²

3.3 Impulse Responses

Figures 1-4 show responses to the growth shock, the inflation shock, and the real-interest-rate shock. The responses are normalized by the

¹² The usual AIC and BIC criteria tend to select very small lags in this model (1 or 2 in this particular model), and results based on these small lags are not consistent with those implied by economic theories.

long-run effects that the growth shock increases output by 1 percent, the inflation shock raises the inflation rate by 1 percent, and the real-interest-rate shock raises the nominal interest rate by 1 percent. The lower and upper bounds are drawn by one standard-deviation, calculated by the Monte Carlo integration as described by Jang (2001). We consider four cases regarding the restrictions on the model. First, we examine the responses using the model employed by KPSW (KPSW-I), where all the permanent shocks are identified with long-run restrictions causally ordered by the growth shock, the inflation shock, and the real-interest-rate shock. Second, we switch the causal order of the inflation shock and the real-interest-rate shock (KPSW-II) to check the sensitivity of restrictions. Third, we consider the case where one permanent shock is identified by the long-run restrictions and the other two permanent shocks are identified by the short-run restriction that price does not enter the money supply (VLR-I). Finally, we consider an alternative short-run restriction that the inflation shock does not affect output contemporaneously (VLR-II). These short-run restrictions are parts of restrictions considered by Gali (1992).

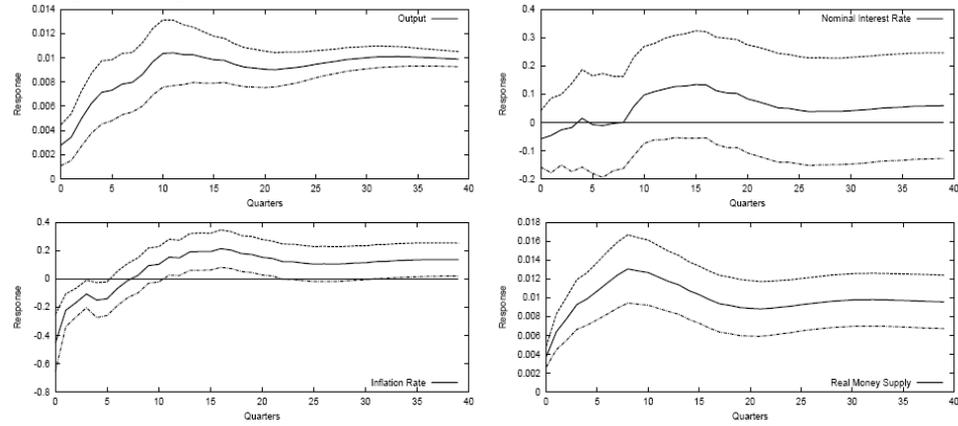
The point estimates of responses are very similar over two sample periods in three models-KPSW-I, KPSW-II, and VLR-II-while the responses in VLR-I show huge confidence intervals. This implies that the restrictions in VLR-I are not useful for identifying the last two permanent shocks. These results are consistent with those implied by the forecast-error variance decompositions in the next section.¹³

First, in response to the growth shock, output increases by 0.2% in the first quarter, and by 1% after four years. The nominal interest rate decreases by 0.15% in the first quarter and by 1% after six years, but not significantly different from 0 in most horizons. The growth shock lowers the inflation rate by 0.4% in the first quarter, and shows negligible effects after two years. Second, the inflation shock yields a rise in output up to 1% over the first few quarters, but the effects become negligible thereafter. The short-run responses of the nominal interest rate are also similar in the three cases. The nominal interest rate increases by 0.2% in the first quarter and reaches 1%-1.5% after one year. The long-run responses

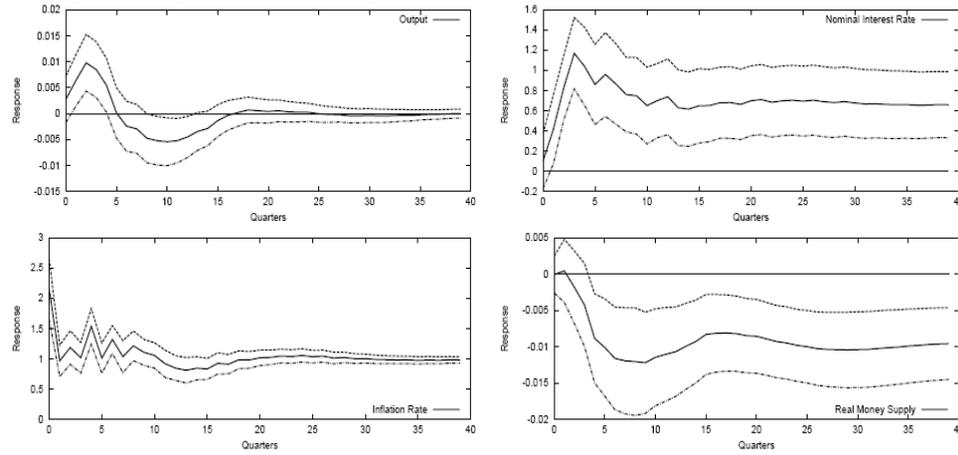
¹³ Figures for the sample period 1954:1-1988:4 are available upon request.

[Figure 1] Impulse Responses: KPSW-I, 1954-1999

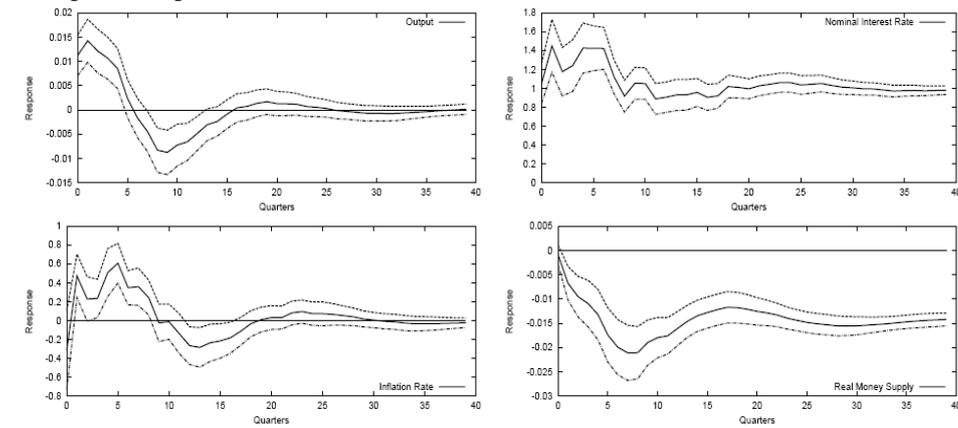
A. Impulse Responses to the Balanced-Growth Shock



B. Impulse Responses to the Long-Run Neutral Inflation Shock

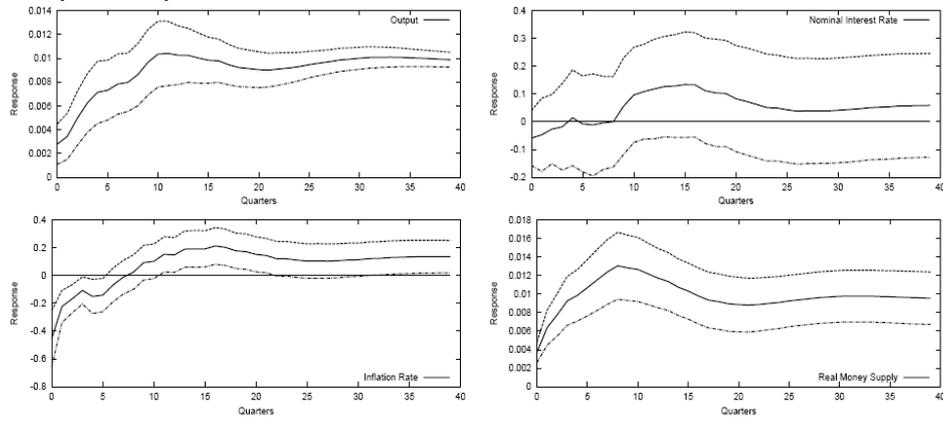


C. Impulse Responses to the Real-Interest-Rate Shock

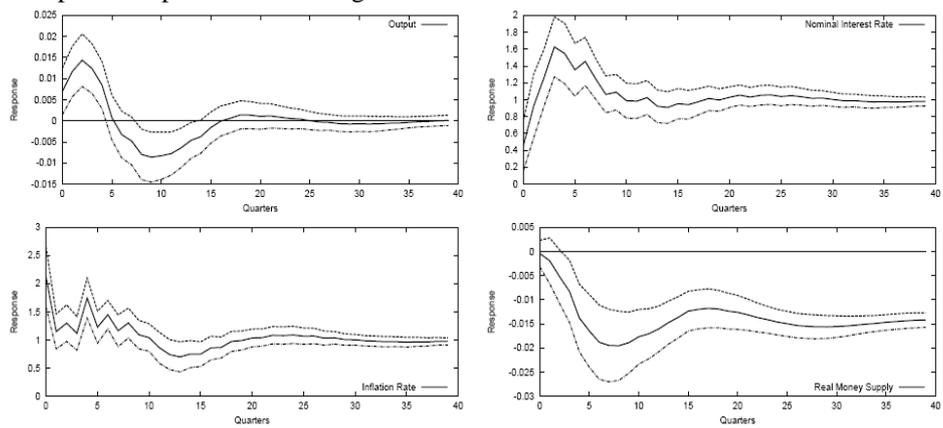


[Figure 2] Impulse Responses: KPSW-II, 1954-1999

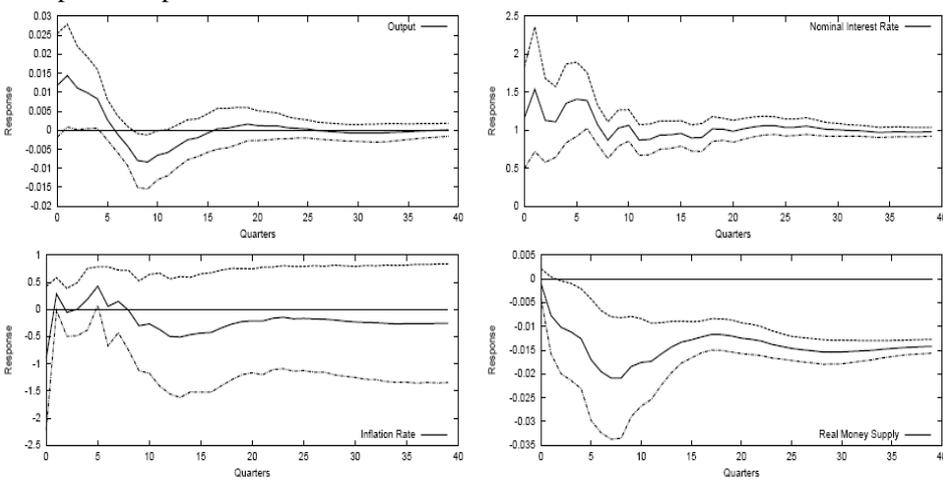
A. Impulse Responses to the Balanced-Growth Shock



B. Impulse Responses to the Long-Run Neutral Inflation Shock

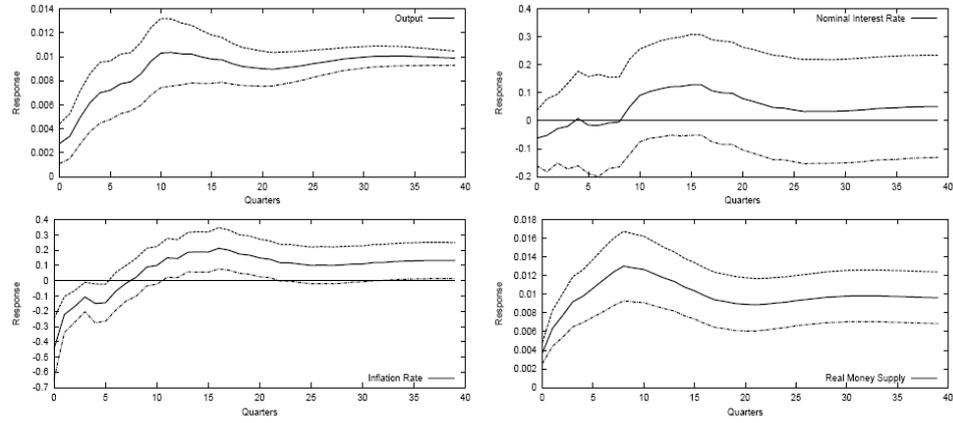


C. Impulse Responses to the Real-Interest-Rate Shock

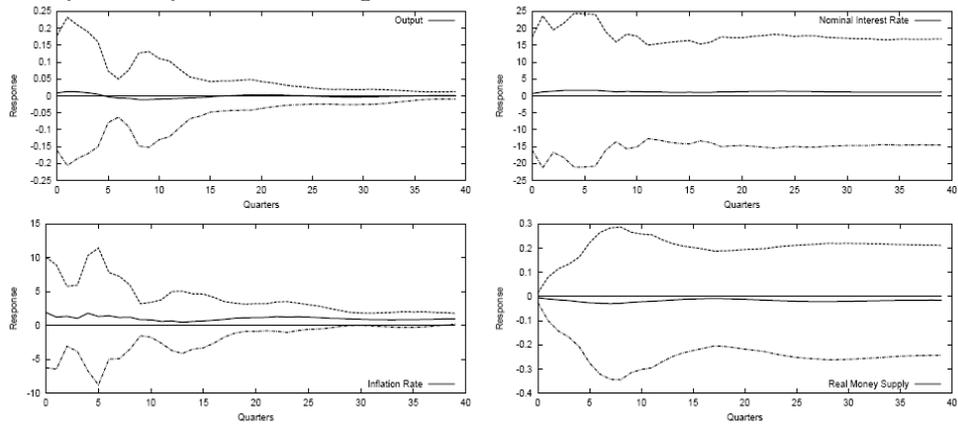


[Figure 3] Impulse Responses: VLR-I, 1954-1999

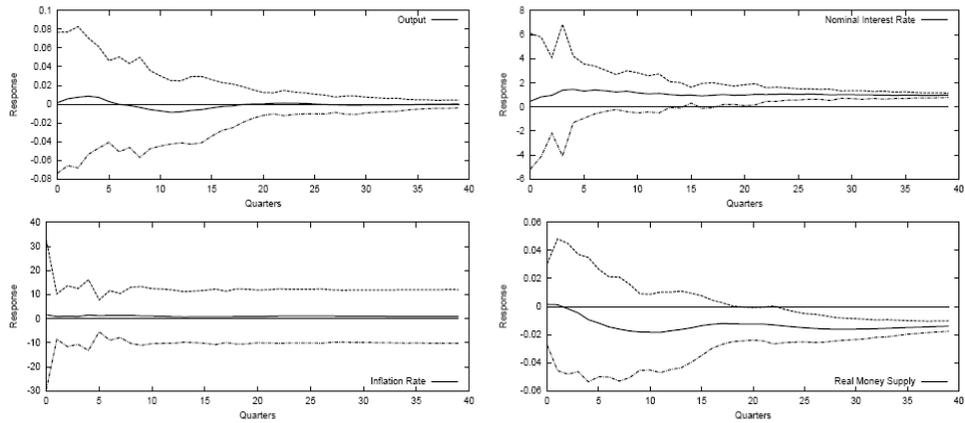
A. Impulse Responses to the Balanced-Growth Shock



B. Impulse Responses to the Long-Run Neutral Inflation Shock

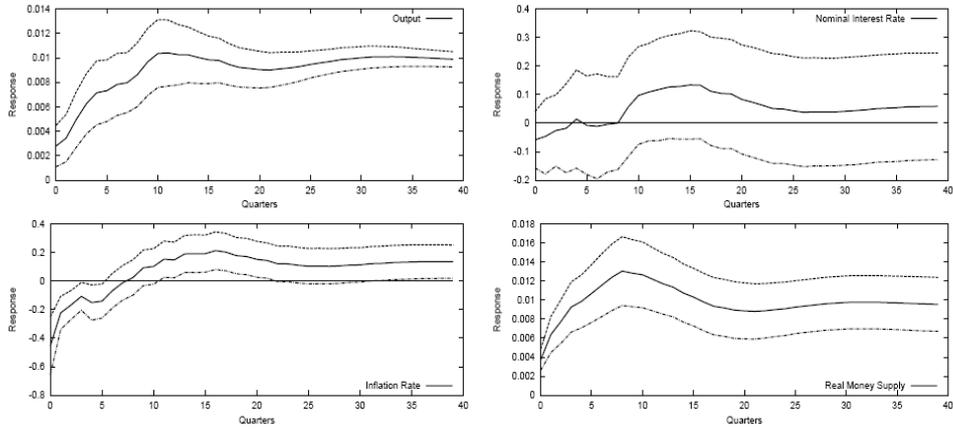


C. Impulse Responses to the Real-Interest-Rate Shock

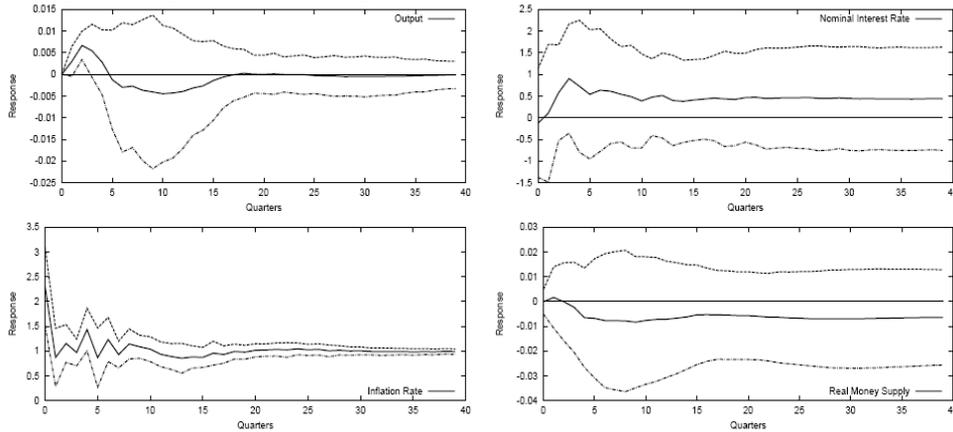


[Figure 4] Impulse Responses: VLR-II, 1954-1999

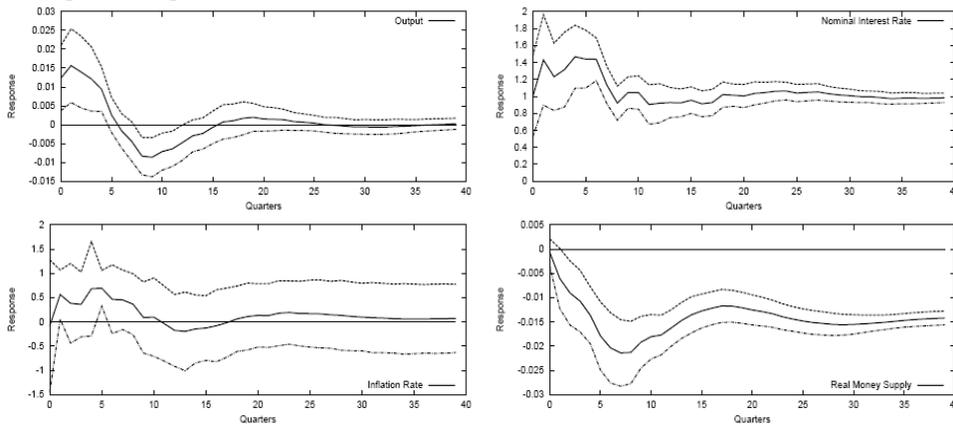
A. Impulse Responses to the Balanced-Growth Shock



B. Impulse Responses to the Long-Run Neutral Inflation Shock

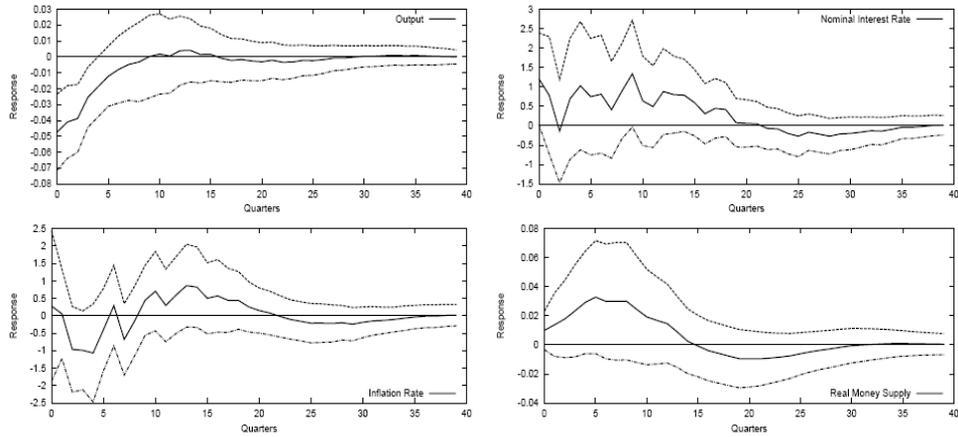


C. Impulse Responses to the Real-Interest-Rate Shock

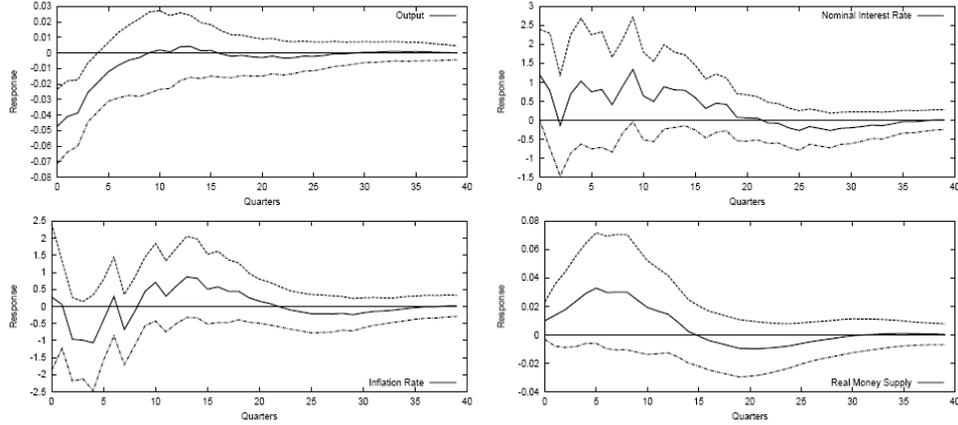


[Figure 5] Impulse Responses to the Money-Demand Shock

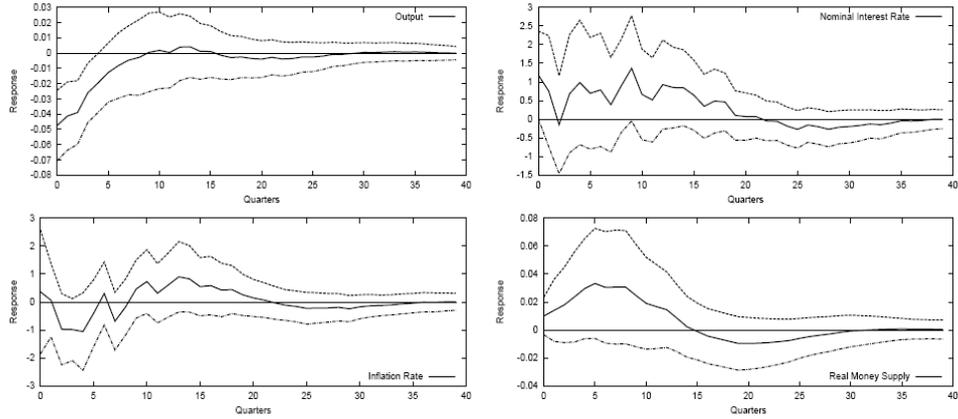
A. KPSW-I, 1954-1999



B. KPSW-II, 1954-1999

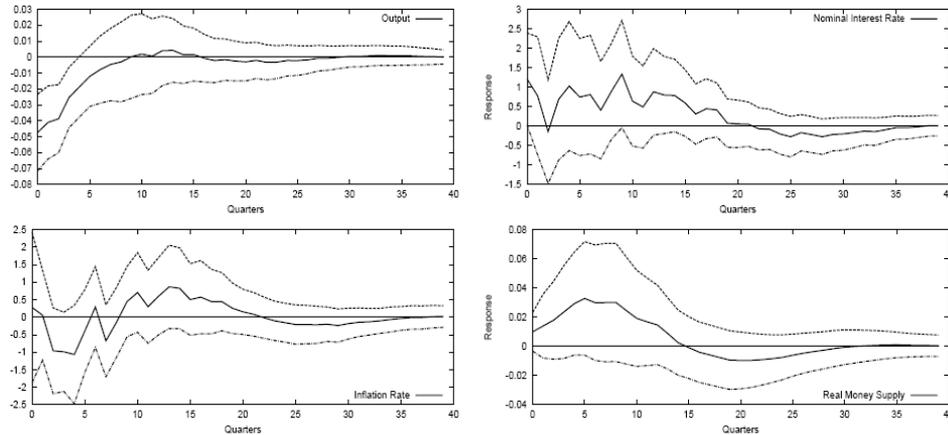


C. VLR-I, 1954-1999



[Figure 5] (Continued)

D. VLR-II, 1954-1999



depend on the restrictions. In KPSW-I and VLR-II, the nominal interest rate rises by less than 1% in the long run, implying a decrease in the real interest rate. In contrast, the real interest rate is not affected by the inflation shock in the long run by the restrictions in VLR-I. The inflation rate responds to its shock by rising 2% in the first quarter and by about 1% after a few quarters. The real balance increases over the first few quarters, but decreases after one year by 1%. Third, the real-interest-rate shock raises output by 1.5% in the first period, the effects become negative after five quarters, and become negligible after three years. This shock has the largest effect on output in the short run among the permanent shocks. The nominal interest rate rises by 1.1% in the first quarter. The response of the inflation rate depends on the model. The inflation rate decreases by 0.5% in the first quarter, but becomes positive up to 0.6% after one quarter in KPSW-I and VLR-II, while the negative effects are more persistent in KPSW-II. The response of real balance is very similar. It is negligible over the first few quarters and decreases 1.5%-2% after two years.

Figure 5 shows the responses to the transitory shock: the money demand shock. The shock is normalized so that it increases the real balance by 1% in the first quarter. The responses are similar over the four models. The money demand shock lowers output significantly up to 5%-8% over the first few quarters. This shock has much greater larger effects

than the real-interest-rate shock in the short run. It also raises the nominal interest rate by 2%, and the inflation rate by 1% in the first quarter, but those responses are not significantly different from 0. Increases of real balance over two years are not significantly different from 0.

3.4 Forecast-Error Variance Decompositions

Tables 4-7 show the forecast-error variance decompositions. As in the previous section, the results are similar over two sample periods in three models-KPSW-I, KPSW-II, and VLR-II-while the results in VLR-I imply that the short-run restriction is not useful in identifying the second and the third permanent shocks. For instance, the inflation shock has the largest contribution to the forecast-error variance of the nominal interest rate, while the real-interest-rate shock has the largest contribution to that of the inflation rate. This implies that the model VLR-I is misspecified and is not able to identify the structural shocks.¹⁴

First, the growth shock makes a small contribution to the variation of output, but the largest contribution to that of the real balance in the short run. Its contribution to output becomes dominant after eight quarters. Second, the inflation shock has the largest contribution to the forecast-error variance of the inflation rate, but the smallest contribution to that of output. Third, the real-interest-rate shock has the largest contribution to its variation. It also has the largest contribution to that of output in the short run among the permanent shocks. Fourth, the transitory shock (the money demand shock) has the largest contribution to the forecast-error variance of output over the first four quarters.¹⁵

¹⁴ Tables for the sample period of 1954:1-1988:4 are available upon request.

¹⁵ The fraction of the forecast-error variance attributed to the transitory shock is not listed in the table, but can be easily calculated from the table by noting that the sum of the fractions is 1.

[Table 4] Forecast-Error Variance Decompositions: KPSW-I, 1954-1999

| Fraction of the forecast-error variance attributed to balanced growth shock | | | | |
|---|----------------|----------------|----------------|----------------|
| Horizon | y | R | Δp | $m-p$ |
| 1 | 0.15 (0.12) | 0.02 (0.08) | 0.33 (0.14) | 0.97 (0.12) |
| 4 | 0.32 (0.14) | 0.01 (0.07) | 0.28 (0.12) | 0.84 (0.12) |
| 8 | 0.61 (0.13) | 0.00 (0.08) | 0.19 (0.10) | 0.72 (0.13) |
| 12 | 0.73 (0.10) | 0.01 (0.09) | 0.18 (0.07) | 0.72 (0.13) |
| 16 | 0.81 (0.08) | 0.02 (0.11) | 0.21 (0.08) | 0.73 (0.13) |
| 20 | 0.85 (0.07) | 0.03 (0.12) | 0.23 (0.09) | 0.73 (0.13) |
| 24 | 0.87 (0.06) | 0.03 (0.13) | 0.22 (0.10) | 0.73 (0.13) |
| B. Fraction of the forecast-error variance attributed to inflation shock | | | | |
| Horizon | y | R | Δp | $m-p$ |
| 1 | 0.01 (0.06) | 0.01 (0.06) | 0.64 (0.17) | 0.00 (0.05) |
| 4 | 0.07 (0.09) | 0.20 (0.12) | 0.64 (0.13) | 0.01 (0.04) |
| 8 | 0.04 (0.05) | 0.22 (0.13) | 0.68 (0.11) | 0.04 (0.05) |
| 12 | 0.04 (0.04) | 0.23 (0.13) | 0.71 (0.09) | 0.05 (0.06) |
| 16 | 0.03 (0.03) | 0.22 (0.13) | 0.68 (0.09) | 0.05 (0.06) |
| 20 | 0.02 (0.03) | 0.22 (0.13) | 0.67 (0.10) | 0.05 (0.06) |
| 24 | 0.02 (0.02) | 0.22 (0.14) | 0.69 (0.11) | 0.05 (0.06) |
| C. Fraction of the forecast-error variance attributed to real-interest-rate shock | | | | |
| Horizon | y | R | Δp | $m-p$ |
| 1 | 0.34 (0.18) | 0.89 (0.14) | 0.02 (0.06) | 0.00 (0.06) |
| 4 | 0.31 (0.16) | 0.78 (0.14) | 0.05 (0.04) | 0.13 (0.11) |
| 8 | 0.18 (0.10) | 0.76 (0.14) | 0.10 (0.04) | 0.21 (0.11) |
| 12 | 0.14 (0.06) | 0.75 (0.14) | 0.09 (0.04) | 0.21 (0.11) |
| 16 | 0.10 (0.05) | 0.73 (0.15) | 0.08 (0.04) | 0.20 (0.10) |
| 20 | 0.08 (0.04) | 0.73 (0.15) | 0.07 (0.04) | 0.20 (0.10) |
| 24 | 0.07 (0.04) | 0.73 (0.16) | 0.07 (0.04) | 0.21 (0.10) |

Note: Numbers in parentheses denote standard errors.

[Table 5] Forecast-Error Variance Decompositions: KPSW-II, 1954-1999

| A. Fraction of the forecast-error variance attributed to balanced growth shock | | | | |
|---|----------------|----------------|----------------|----------------|
| Horizon | y | R | Δp | $m - p$ |
| 1 | 0.15 (0.12) | 0.02 (0.08) | 0.33 (0.14) | 0.97 (0.12) |
| 4 | 0.32 (0.14) | 0.01 (0.07) | 0.28 (0.12) | 0.84 (0.12) |
| 8 | 0.61 (0.13) | 0.00 (0.08) | 0.19 (0.10) | 0.72 (0.13) |
| 12 | 0.73 (0.10) | 0.01 (0.09) | 0.18 (0.07) | 0.72 (0.13) |
| 16 | 0.81 (0.08) | 0.02 (0.11) | 0.21 (0.08) | 0.73 (0.13) |
| 20 | 0.85 (0.07) | 0.03 (0.12) | 0.23 (0.09) | 0.73 (0.13) |
| 24 | 0.87 (0.06) | 0.03 (0.13) | 0.22 (0.10) | 0.73 (0.13) |
| B. Fraction of the forecast-error variance attributed to inflation shock | | | | |
| Horizon | y | R | Δp | $m - p$ |
| 1 | 0.05 (0.10) | 0.09 (0.12) | 0.56 (0.18) | 0.00 (0.05) |
| 4 | 0.14 (0.12) | 0.37 (0.14) | 0.62 (0.14) | 0.02 (0.05) |
| 8 | 0.09 (0.07) | 0.41 (0.15) | 0.71 (0.11) | 0.09 (0.07) |
| 12 | 0.07 (0.05) | 0.42 (0.15) | 0.74 (0.09) | 0.10 (0.06) |
| 16 | 0.05 (0.04) | 0.41 (0.14) | 0.69 (0.10) | 0.10 (0.06) |
| 20 | 0.04 (0.03) | 0.41 (0.14) | 0.67 (0.11) | 0.10 (0.05) |
| 24 | 0.04 (0.03) | 0.41 (0.14) | 0.68 (0.11) | 0.10 (0.05) |
| C. Fraction of the forecast-error variance attributed to real-interest-rate shock | | | | |
| Horizon | y | R | Δp | $m - p$ |
| 1 | 0.30 (0.16) | 0.81 (0.18) | 0.10 (0.11) | 0.00 (0.06) |
| 4 | 0.24 (0.13) | 0.61 (0.15) | 0.07 (0.06) | 0.11 (0.10) |
| 8 | 0.14 (0.08) | 0.57 (0.15) | 0.07 (0.05) | 0.16 (0.10) |
| 12 | 0.10 (0.05) | 0.56 (0.15) | 0.06 (0.05) | 0.16 (0.09) |
| 16 | 0.08 (0.04) | 0.54 (0.15) | 0.08 (0.06) | 0.16 (0.09) |
| 20 | 0.06 (0.03) | 0.54 (0.15) | 0.08 (0.06) | 0.15 (0.09) |
| 24 | 0.05 (0.03) | 0.54 (0.15) | 0.07 (0.06) | 0.16 (0.09) |

Note: Numbers in parentheses denote standard errors.

[Table 6] Forecast-Error Variance Decompositions: VLR-I, 1954-1999

| A. Fraction of the forecast-error variance attributed to balanced growth shock | | | | |
|---|----------------|----------------|----------------|----------------|
| Horizon | y | R | Δp | $m-p$ |
| 1 | 0.15 (0.12) | 0.02 (0.08) | 0.33 (0.14) | 0.97 (0.12) |
| 4 | 0.32 (0.14) | 0.01 (0.07) | 0.28 (0.12) | 0.84 (0.12) |
| 8 | 0.61 (0.13) | 0.00 (0.08) | 0.19 (0.10) | 0.72 (0.13) |
| 12 | 0.73 (0.10) | 0.01 (0.09) | 0.18 (0.08) | 0.72 (0.13) |
| 16 | 0.81 (0.08) | 0.02 (0.11) | 0.21 (0.08) | 0.73 (0.13) |
| 20 | 0.85 (0.07) | 0.03 (0.12) | 0.23 (0.09) | 0.73 (0.13) |
| 24 | 0.87 (0.06) | 0.03 (0.13) | 0.22 (0.11) | 0.73 (0.13) |
| B. Fraction of the forecast-error variance attributed to inflation shock | | | | |
| Horizon | y | R | Δp | $m-p$ |
| 1 | 0.35 (0.20) | 0.90 (0.22) | 0.00 (0.18) | 0.00 (0.05) |
| 4 | 0.35 (0.18) | 0.87 (0.20) | 0.07 (0.17) | 0.13 (0.10) |
| 8 | 0.21 (0.11) | 0.87 (0.21) | 0.16 (0.19) | 0.23 (0.13) |
| 12 | 0.16 (0.07) | 0.85 (0.20) | 0.14 (0.17) | 0.24 (0.13) |
| 16 | 0.11 (0.06) | 0.84 (0.20) | 0.12 (0.15) | 0.23 (0.13) |
| 20 | 0.09 (0.05) | 0.83 (0.21) | 0.11 (0.15) | 0.23 (0.12) |
| 24 | 0.08 (0.05) | 0.84 (0.21) | 0.10 (0.15) | 0.23 (0.12) |
| C. Fraction of the forecast-error variance attributed to real-interest-rate shock | | | | |
| Horizon | y | R | Δp | $m-p$ |
| 1 | 0.00 (0.10) | 0.00 (0.17) | 0.67 (0.19) | 0.00 (0.05) |
| 4 | 0.04 (0.12) | 0.11 (0.17) | 0.62 (0.16) | 0.00 (0.05) |
| 8 | 0.02 (0.07) | 0.12 (0.19) | 0.62 (0.16) | 0.02 (0.06) |
| 12 | 0.02 (0.05) | 0.12 (0.19) | 0.66 (0.16) | 0.02 (0.06) |
| 16 | 0.01 (0.04) | 0.12 (0.19) | 0.65 (0.14) | 0.02 (0.06) |
| 20 | 0.01 (0.03) | 0.12 (0.19) | 0.64 (0.14) | 0.02 (0.06) |
| 24 | 0.01 (0.03) | 0.12 (0.19) | 0.66 (0.15) | 0.02 (0.06) |

Note: Numbers in parentheses denote standard errors.

[Table 7] Forecast-Error Variance Decompositions: VLR-II, 1954-1999

| A. Fraction of the forecast-error variance attributed to balanced growth shock | | | | |
|---|----------------|----------------|----------------|----------------|
| Horizon | y | R | Δp | $m-p$ |
| 1 | 0.15 (0.12) | 0.02 (0.08) | 0.33 (0.14) | 0.97 (0.12) |
| 4 | 0.32 (0.14) | 0.01 (0.07) | 0.28 (0.12) | 0.84 (0.12) |
| 8 | 0.61 (0.13) | 0.00 (0.08) | 0.19 (0.10) | 0.72 (0.13) |
| 12 | 0.73 (0.10) | 0.01 (0.09) | 0.18 (0.07) | 0.72 (0.13) |
| 16 | 0.81 (0.08) | 0.02 (0.11) | 0.21 (0.08) | 0.73 (0.13) |
| 20 | 0.85 (0.07) | 0.03 (0.12) | 0.23 (0.09) | 0.73 (0.13) |
| 24 | 0.87 (0.06) | 0.03 (0.13) | 0.22 (0.10) | 0.73 (0.13) |
| B. Fraction of the forecast-error variance attributed to inflation shock | | | | |
| Horizon | y | R | Δp | $m-p$ |
| 1 | 0.00 (0.00) | 0.00 (0.16) | 0.67 (0.18) | 0.00 (0.05) |
| 4 | 0.03 (0.03) | 0.10 (0.14) | 0.62 (0.17) | 0.00 (0.06) |
| 8 | 0.02 (0.03) | 0.10 (0.16) | 0.61 (0.16) | 0.02 (0.07) |
| 12 | 0.02 (0.03) | 0.11 (0.17) | 0.65 (0.15) | 0.02 (0.07) |
| 16 | 0.01 (0.03) | 0.10 (0.16) | 0.64 (0.13) | 0.02 (0.07) |
| 20 | 0.01 (0.02) | 0.10 (0.16) | 0.64 (0.13) | 0.02 (0.07) |
| 24 | 0.01 (0.02) | 0.10 (0.16) | 0.65 (0.13) | 0.02 (0.07) |
| C. Fraction of the forecast-error variance attributed to real-interest-rate shock | | | | |
| Horizon | y | R | Δp | $m-p$ |
| 1 | 0.35 (0.19) | 0.90 (0.17) | 0.00 (0.10) | 0.00 (0.06) |
| 4 | 0.36 (0.18) | 0.88 (0.16) | 0.08 (0.11) | 0.13 (0.11) |
| 8 | 0.21 (0.11) | 0.88 (0.18) | 0.17 (0.13) | 0.23 (0.12) |
| 12 | 0.16 (0.07) | 0.87 (0.19) | 0.15 (0.13) | 0.24 (0.12) |
| 16 | 0.12 (0.06) | 0.85 (0.20) | 0.13 (0.12) | 0.23 (0.11) |
| 20 | 0.09 (0.05) | 0.85 (0.21) | 0.11 (0.11) | 0.23 (0.11) |
| 24 | 0.08 (0.04) | 0.85 (0.21) | 0.11 (0.12) | 0.24 (0.11) |

Note: Numbers in parentheses denote standard errors.

IV. CONCLUDING REMARKS

The present paper generalizes the existing structural VECM literature by developing a method of identifying structural VECM with a combination of short-run and long-run restrictions. We consider a generalized block recursive VECM in which permanent shocks are partially identified with a set of long-run restrictions, and fully identified with an additional set of short-run restrictions. As an application, we slightly modify the KPSW model and adopt a set of short-run restrictions that are used by Gali (1992). The results show that the short-run restriction, that the price does not appear in the monetary policy rule, does not help identify the inflation shock (or the monetary policy shock, in a sense). In contrast, the other short-run restriction, that the inflation shock does not affect output contemporaneously, does well to identify the shock. With the latter restriction, the VECM with short-run and long-run restrictions are well specified, and give results that are consistent with those of KPSW.

Appendix A

Data Sources

All data for 1948:1-2000:4 except the earlier M_2 are obtained from FRED[®], The Federal Reserve Bank of St. Louis.

- *GDP*: Nominal gross domestic product. Seasonally adjusted. Billions of dollars.
- *GDP96*: Real gross domestic product in chained 1996 dollars. Seasonally adjusted. Billions of dollars.
- *GGE*: Nominal government consumption expenditures and gross investment. Seasonally Adjusted. Billions of dollars.
- *GGE96*: Real government consumption expenditures and gross investment in chained 1996 dollars. Seasonally Adjusted. Billions of dollars.
- *M₂*: M_2 money stock. Seasonally Adjusted. The earlier M_2 data for 1948-1958 are obtained from KPSW series that are reported in *Banking and Monetary Statistics, 1941-1970* (Board of Governors of the Federal Reserve System, 1976). The monthly data are averaged to obtain the quarterly data. Billions of dollars.
- *R*: The three-month Treasury Bill rate in the secondary market. Averages of business days. An annual percentage rate. The monthly data are averaged to obtain the quarterly data.
- *N*: Civilian noninstitutional population, 16 years and older. The monthly data are averaged to obtain the quarterly data. Thousands.
- *y*: Log real private output per capita formed by $y = \log(GDP96 - GGE96) - \log(N)$. Millions of dollars.

- m : Log money stock per capita calculated by $m = \text{Log}(M_2) - \text{Log}(N)$.
Millions of dollars.
- p : The GDP deflator calculated by $p = \log(\text{GDP} - \text{GGE}) - \log(\text{GDP96} - \text{GGE96})$. 1996 = 1.
- Δp : An annual percentage inflation rate. It is annualized by $\Delta p_t = 400 \times (p_t - p_{t-1})$.

Appendix B

An Algorithm to Solve the Models

Let $C = D\Sigma D'$. As $\pi_{12} = \pi_{13} = 0$ and Σ_v is diagonal, π_{21} , π_{23} and $\Sigma_{v,11}$ are given by

$$\Sigma_{v,11} = C_{11}, \quad \pi_{21} = \frac{C_{21}}{C_{11}}, \quad \pi_{31} = \frac{C_{31}}{C_{11}}.$$

Provided that π_{32} is known, we can rewrite (15) by

$$\Pi\Sigma_v\Pi' = \begin{bmatrix} 1 & 0 & 0 \\ \pi_{21} & 1 & x_1 \\ \pi_{31} & \pi_{32} & 1-x_1 \end{bmatrix} \begin{bmatrix} \Sigma_{v,11} & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{bmatrix} \begin{bmatrix} 1 & \pi_{21} & \pi_{31} \\ 0 & 1 & \pi_{32} \\ 0 & x_1 & 1-x_1 \end{bmatrix} = C.$$

Let $f(x)$ be the last three elements of $\text{vech}(\Pi\Sigma_v\Pi' - C)$ and $g(x)$ be the gradient of $f(x)$, $\frac{\partial f(x)}{\partial x'}$, where $x = (x_1, x_2, x_3)'$. Then

$$f(x) = \begin{bmatrix} \pi_{21}^2 \Sigma_{v,11} + x_2 + x_1^2 x_3 - C_{22} \\ \pi_{21} \pi_{31} \Sigma_{v,11} + \pi_{32} x_2 + x_1 (1-x_1) x_3 - C_{32} \\ \pi_{31}^2 \Sigma_{v,11} + \pi_{32}^2 + (1-x_1)^2 x_3 - C_{33} \end{bmatrix} = 0$$

and

$$g(x) = \begin{bmatrix} 2x_1 x_3 & 1 & x_1^2 \\ (1-2x_1)x_3 & \pi_{32} & x_1(1-x_1) \\ -2(1-x_1)x_3 & \pi_{32}^2 & (1-x_1)^2 \end{bmatrix}.$$

Having specified f and g , we can apply the Newton method to get the solutions of (15) by iterating

$$x^{(i)} = x^{(i-1)} + g(x)^{-1} f(x^{(i-1)})$$

with an arbitrary initial vector $x^{(0)}$ until changes in $x^{(i)}$ become 0.

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